# TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS 



Mathematics for ALL

# Editor-in-Chief 

Marta Civil
The University of Arizona

## Editors

Anthony Fernandes
University of North Carolina Charlotte
Ksenija Simic-Muller
Pacific Lutheran University
M. Alejandra Sorto

Texas State University
Eugenia Vomvoridi-Ivanovic
University of South Florida

## Associate Editor

Lawrence M. Lesser
The University of Texas at El Paso

## Layout Editor

Susie W. Håkansson<br>Mathematics Education Consultant

## Founding Editors

Cynthia Oropesa Anhalt, The University of Arizona
Miriam A. Leiva, University of North Carolina Charlotte, Emerita
Lawrence M. Lesser, The University of Texas at El Paso

Mathematics for ALL

## IN THIS ISSUE

From the Editors ..... 5
Equidad, Diversidad y Matemática Crítica ..... 6
Bradly Rivera-Muñiz, Robinson School, Puerto Rico
TODOS Position Statements ..... 16
Flags of Latin America: Culturally Relevant Learning Experiences With Technology to Enhance Geometry and Algebra Concepts ..... 17
Kelly Wamser Remijan, Illinois Mathematics and Science Academy - Statewide Educator Innovation
Poetry Corner ..... 27
No le Enseñes Sobre los ELs: Infusing Language Into Professional Development and Mathematics ..... 28
Vanessa Z. Mari, Nevada State College
Rachel S. G. Bower, Nevada State College
Still TEEM-ing with Enthusiasm: A History of TODOS’ esTEEMed Journal ..... 37
What Does 2SLGBTQIA+ Identity and Other Non-Normative Identities Have to Do With Mathematics Teaching and Learning? ..... 38
Brandie Waid, The Queer Mathematics Teacher
TODOS Mission and Goals and TODOS Live! ..... 52
TODOS 2022-2023 Leadership ..... 53
Apply for Awards to Attend the TODOS 2023 Conference ..... 54
TODOS 2023 Conference ..... 55

## From the Editors

This issue reaches you just a few months before our TODOS 2023 conference. We hope that many of you will attend what will be a great event to engage in critical conversations centered on the TODOS mission as well as a great opportunity to build community. Check out the information on the conference at: https://www.todos-math.org/conference.

This issue is also the first one for our most recent addition to the team of editors, Eugenia Vomvoridi-Ivanovic. We are very fortunate to have her join our journal because of her expertise in mathematics teacher education with a focus on social justice and culturally responsive teaching practices. As we write this editorial, we are also in the process of transitioning to an Open Journal Systems (OJS). This will enhance the management of many aspects of the journal, including communication with authors and reviewers. We are excited at having TEEM join the growing family of journals that use OJS. Check it out at: https://journals.charlotte.edu/teem.

As always, we encourage you to consider submitting an article to TEEM. If you have an idea and are not sure if or how it may fit, please contact us (teem@todos-math.org) and we will be happy to discuss it with you! We are especially interested in articles written by or with classroom teachers. Also email us if you have an idea for a special issue. Keep in mind the guidelines and scope for TEEM (http://bit.ly/3XHdDWH or http://bit.ly/3jbNIY0) and the TODOS mission (https://www.todos-math.org/mission-goals).

We hope you will enjoy the variety of articles in this issue. There are three peer-reviewed articles, including the first TEEM article published in Spanish (by Bradly Rivera-Muñiz). Our call for papers has long included both English and Spanish as acceptable languages for submissions and we look forward to receiving more future submissions in Spanish. This issue also has an invited article by Brandie Waid and a poem by our own Associate Editor Lawrence M. Lesser.
"Equidad, Diversidad y Matemática Crítica" (Equity, Diversity, and Critical Mathematics) is a theoretical article, in which Bradly Rivera-Muñiz proposes a model for integrating critical mathematics in the classroom. The author discusses how this model can serve as a framework for future research and development related to promoting equitable access to mathematics.

In "Flags of Latin America: Culturally Relevant Learning Experiences With Technology to Enhance Geometry and Algebra Concepts," Kelly Wamser Ramijan discusses ways in which flags from various Latin American countries may be used as a springboard for implementing culturally relevant mathematics learning experiences. The mathematics activities presented build connections between geometry and algebra concepts, incorporate the use of technology, and encourage dialog about different cultures.

In "No le enseñes sobre los ELs: Infusing language into professional development and mathematics," Vanessa Mari and Rachel Bower describe their approach to professional development of secondary teachers working with multilingual students in their classrooms. The authors outline how to build teachers' language awareness by engaging the teachers in three domains of language use - teacher domain, analyst domain, and user domain.

In the invited piece, "What does 2SLGBTQIA+ Identity and Other Non-Normative Identities Have to Do With Mathematics Teaching and Learning?," Brandie Waid argues for the need and urgency to consider queer and transgender identity in mathematics education. The author presents a framework of four trauma-informed priorities and illustrates how these priorities can be implemented in the teaching of mathematics. This article is based on a TODOS Live! Webinar held this past fall, archived at https://vimeo.com/761876288.

As editors, we are extremely grateful for the dedication and expertise of all our reviewers and authors. We are also very appreciative of the excellent editorial support provided by Associate Editor Lawrence M. Lesser and Layout Editor Susie W. Håkansson. TEEM gratefully acknowledges the support of all the leaders in our sponsoring organization, TODOS: Mathematics for ALL. We hope TEEM continues to serve the TODOS membership and that this issue serves as a resource for the community and a source of inspiration for future contributions to the journal.

Marta Civil, Anthony Fernandes, Ksenija Simic-Muller, M. Alejandra Sorto, and Eugenia Vomvoridi-Ivanovic

Equidad, Diversidad y Matemática Crítica<br>Bradly Rivera-Muñiz<br>Robinson School, Puerto Rico


#### Abstract

Resumen Los estudiantes de distintas etnias, razas, niveles socioeconómicos, grupos lingüísticos, de género, y otros subgrupos de la sociedad demuestran distintos niveles de rendimiento en matemáticas, lo que señala una diferencia que está relacionada con los factores sociopolíticos de la educación. En la sociedad global actual, la matemática figura como un agente de estratificación social que se ve influenciado por los problemas de falta de acceso equitativo a la educación matemática de alta calidad, el currículo oculto y las estructuras de poder que se promueven a través del acceso a ella. En este trabajo, se propone un modelo de integración de la matemática crítica al salón de clases con el propósito de promover un acceso equitativo a la matemática.


## Equity, Diversity, and Critical Mathematics


#### Abstract

Students of different ethnicities, races, socioeconomic levels, linguistic groups, gender, and other subgroups of society demonstrate different levels of academic achievement in mathematics, which points to a difference that is related to the socio-political factors of education. In the current global society, mathematics appears as an agent of social stratification that is influenced by the problems of lack of equitable access to mathematics education, the hidden curriculum, and the power structures that are promoted through it. In this work, a model for the integration of critical mathematics in the classroom is proposed with the purpose of promoting equitable access to mathematics.


## Preguntas de Discusión y Reflexión - Antes de la Lectura

1. ¿Cómo se reflejan los problemas de falta de acceso equitativo en la sociedad en su salón de clases, escuela o institución educativa?
2. ¿De qué manera podemos adaptar la forma en que enseñamos para proveer un espacio de acceso equitativo a la educación matemática de alta calidad?
3. ¿Cómo las matemáticas sirven como herramienta útil ante los problemas sociales?

Bradly Rivera-Muñiz (brivera@robinsonschool.net) is an Elementary School Division Head and STEM Program Lead at Robinson School in Puerto Rico. His experience as a mathematics teacher spans multiple grades in the K-12 grade band. As an education researcher his interests are critical mathematics and the use of technology in the teaching/learning of mathematics.

# Equidad, Diversidad y Matemática Crítica <br> Bradly Rivera-Muñiz 

## Introducción

En este artículo analizaré la situación de falta de acceso equitativo a la educación matemática de alta calidad, los factores que fomentan los problemas de inequidad, y presentaré un modelo de integración de la matemática crítica al salón de clases como herramienta para actuar en contra de estos problemas. Una manera común de definir la diversidad es en relación con el número de miembros de un grupo que provienen de distintos grupos étnicos, raciales, socioeconómicos, de género, de orientación sexual, entre otros subgrupos de la sociedad. Una definición más completa considera la participación de estos grupos en las prácticas de la comunidad, del hogar y de la sociedad, particularmente, la continuidad y discontinuidad de estas prácticas dentro y fuera de la escuela (Cobb \& Hodge, 2002). Entonces, la equidad en la educación está relacionada al estudio de cómo esta continuidad y discontinuidad entre las prácticas escolares y las no escolares se relacionan y sus efectos en el acceso a la educación de un estudiante o colectividad (Cobb \& Hodge, 2002).

Los estudiantes de distintas etnias, razas, niveles socioeconómicos, grupos lingüísticos, de género, y otros subgrupos de la población tienen distintos niveles de desempeño académico, lo que señala a una diferencia que
es causada por diversos factores que están relacionados a los aspectos sociopolíticos de la educación.

El acceso a la información y el conocimiento es un componente importante de la sociedad informacional y por lo tanto, la capacidad para generar nuevos conocimientos y recolectar información depende del posicionamiento geográfico y las fuentes de información a las que tiene acceso un estudiante (Skovsmose \& Valero, 2012). La sociedad informacional está definida como la unión de los conceptos de sociedad de la información y la sociedad del aprendizaje (Castells, 1999), donde se considera el conocimiento y acceso a la información como una fuente de valor y poder, y se trata el conocimiento como un proceso continuo de aprendizaje y adaptación. La sociedad informacional global trae consigo dos paradojas relacionadas con sus efectos en la educación (Skovsmose \& Valero, 2012). La paradoja de la inclusión estipula que la globalización y sus efectos sociales proponen un ambiente de acceso universal y de inclusión, pero a su vez, crean exclusión de algunos grupos sociales ya que no todos necesariamente se benefician de la globalización. La paradoja de la ciudadanía reconoce que la educación necesita ser condicionada a las situaciones sociales del momento (illustrado en la figura 1), lo que reduce el aprendizaje a la necesiad del individuo para adaptarse a unas exigencias

## Figura 1

Tendencia en puntuación promedio en NAEP en cuarto grado por razaletnia

sociales particulares. Debido a que el aprendizaje se ha convertido en una actividad de constante adaptación social, el problema de falta de acceso equitativo a la educación matemática cobra más importancia ya que la realidad social de un estudiante definirá el desarrollo de su habilidad para adaptarse a una sociedad global.

Por medio de este artículo, estaré proponiendo un nuevo enfoque que está fundamentado en la educación matemática crítica. Con este propósito se estará realizando una revisión de la literatura para proveer un marco teórico y práctico ante el problema aquí estudiado. El aspecto teórico provendrá de Freire (1970), Frankenstein (1983, 1990), Giroux (1979), Schoenfeld (2006), Sfard y Prusak (2005), Skovsmose (1990), y Skovmose y Valero (2012), mientras que el aspecto práctico provendrá del trabajo de Yeh y Otis (2019), y Gutstein (2006). Estaré analizando la falta de acceso equitativo a la educación matemática de alta calidad, la identidad matemática y el currículo oculto. A través del análisis teórico expondré cómo las estructuras de poder se ven reflejadas en estos problemas. Por medio del análisis práctico estaré proponiendo un modelo de integración de la matemática crítica al salón de clases, con el propósito de combatir un problema sumamente perjudicial y cuyos efectos continúan amplificándose al pasar el tiempo.

## Acceso a la Educación Matemática de Alta Calidad

Tradicionalmente se perciben las matemáticas como una disciplina que no está relacionada a lo político y social (Yeh \& Otis, 2019), pero esto no es la realidad. Las matemáticas escolares cada vez más son vistas como la armería utilizada para responder a las demandas políticas para el desarrollo económico y tecnológico (Brown et al., 2016). Se han observado estadísticas que demuestran una significativa brecha racial en el desempeño académico matemático de los estudiantes en Estados Unidos (Lee, 2002; Tate, 1997). Si analizamos datos de distintas pruebas estandarizadas que representen los niveles de desempeño en matemáticas, se observa una diferencia estando en desventaja los latinos, afroamericanos, nativos americanos, y poblaciones bajo el nivel de pobreza. Esto no debe ser tratado meramente como una coincidencia, sino como una situación que tiene raíz en las problemáticas sociopolíticas y de estructuras de poder que
fundamentan el sistema de educación en los Estados Unidos. Una estructura de poder es la manera en que se distribuye la autoridad y el poder entre los grupos; un factor importante que se convierte en problema cuando no existe una distribución equitativa de autoridad y poder.

Algunos adultos, aunque sean conscientes de la discriminación racial y por género, comúnmente ven las injusticias relacionadas como problemas personales y no que están entrelazados en las instituciones sociales (Frankenstein, 1990). El énfasis de las escuelas en medidas de control de calidad y la preparación profesional de los maestros enfocada en un currículo restringido continúa sirviendo los intereses económicos y políticos de las ideologías dominantes, y como consecuencia afectando negativamente a los estudiantes de algunos grupos sociales (Yeh, 2018). El posestructuralismo provee una percepción sociopolítica de este problema y establece que las personas no son producto de su propio razonamiento, conocimiento, y acción, sino que son producto de los discursos, siendo entonces su identidad matemática un resultado de los procesos políticos que se relacionan con los discursos sociales (Gutiérrez, 2013). Las matemáticas han sido utilizadas para reforzar la legitimidad de los intereses capitalistas de algunos sectores sociales, posicionando a los estudiantes y familias de color en lo más bajo de esta estratificación (Yeh \& Otis, 2019), siendo los negros, latinos, nativos, y razas mixtas, que históricamente han sido marginadas o excluidas de algunos procesos sociales. Es necesario establecer que la deprivación del conocimiento matemático promueve la falta de acceso a oportunidades de avance económico, por lo que la falta de acceso al conocimiento matemático en las comunidades que viven bajo pobreza ayuda que estas continúen en dicha situación social. La pobreza no solo implica deprivación material, sino también la imposibilidad del desarrollo total del potencial humano, por lo tanto, para erradicar la pobreza en el capitalismo se necesita no solamente una revolución en las fuerzas de producción, sino que un conjunto de instituciones históricamente nuevas (Isidro Luna, 2016). Una de estas instituciones históricamente nuevas, sin duda alguna, debe ser un nuevo sistema de educación.

Se ha observado la desaparición de brechas raciales cuando se utiliza un currículo que permite a los estudiantes relacionarse con matemáticas que sean de
valor basado en un estilo pedagógico fundado en la premisa que todos los estudiantes son capaces de aprender matemáticas y contribuir a la solución de problemas (Boaler, 2008). Esta suposición rompe con los márgenes del racismo, sexismo, clasismo, lingüismo, discriminación y prejuicio social que están presentes en los procesos de enseñanza, en ocasiones, a raíz de creencias estereotípicas de algunos maestros. Esto nos muestra una manera en la que podemos comenzar a erradicar las brechas raciales con una manera distinta de enseñar matemáticas, demostrando el fundamento de lo que debe ser la educación matemática que promueve la equidad.

En un estudio realizado en Colombia se observó que los maestros persistentemente utilizaban las identidades raciales, étnicas, y de clase social para naturalizar y justificar el pobre aprendizaje matemático, y a su vez utilizaban prácticas de enseñanza que se oponen al desarrollo de procesos de pensamiento matemáticos complejos en las escuelas con mayor presencia de estudiantes negros (Valoyes-Chávez, 2015).

## Identidad Matemática y Currículo Oculto

El concepto de identidad matemática se refiere a las disposiciones y creencias de como un estudiante se percibe en cuanto a su habilidad para hacer matemáticas (Martin, 2010). Las identidades pueden definirse como colecciones de historias sobre personas o como las narrativas sobre las personas, que se pueden materializar, endorsar y que son significativas, siendo las historias más significativas aquellas que implican afiliación, o exclusión, a varias comunidades (Sfard \& Prusak, 2005). Es por medio de ese sentido de pertenencia o exclusión a la comunidad que "hace matemáticas" que podemos potenciar el aprendizaje de la materia. Si un estudiante no se puede visualizar como capaz de aprender matemáticas, minimiza la probabilidad de que ocurra aprendizaje alguno.

Otra de las maneras mediante las cuales se sustentan o solidifican las estructuras de poder, por medio de la educación, es a través del currículo oculto. El currículo oculto se define como la transmisión y reproducción de la cultura, normas, valores y creencias que son transmitidas por los procesos de educación formal e informal (Giroux \& Penna, 1979). La manera en que se enseñan las
matemáticas como un proceso de seguir instrucciones trae consigo poco valor transferible al mundo real y los procesos de investigación o solución de problemas, y va más relacionado a regulaciones que las personas que participan de procesos rutinarios confrontan (Skovsmose, 1990). La manera en que se presentan los contenidos matemáticos por medio de situaciones o problemas estereotípicos, utópicos, o representativos de un grupo específico de la estratificación social, trae consigo un mensaje que influye en el desarrollo de la identidad matemática del estudiante y en su aprendizaje matemático. Yeh y Otis (2019) presentan un ejemplo de un salón de clases en donde los estudiantes critican problemas matemáticos del libro del curso, con el propósito de investigar problemas sociales, cuestionando qué dice el problema, qué significa lo que dice, y cómo esto es importante para las personas. Los estudiantes contextualizaron los problemas a su realidad y la realidad social de otros, tomando en consideración un problema de falta de acceso equitativo en la educación. Estos problemas matemáticos hacían alusión a la percepción social tradicional de la niña y el niño, y los objetos que relacionamos a estos. Para llevar a cabo la contextualización el grupo de estudiantes utilizó un organizador gráfico con las siguientes preguntas guías: ¿Qué dice el texto? ¿Qué significa el texto? ¿Por qué esto es importante para mí? La maestra selecciono tres problemas para conectarlos con la lectura actual de la clase sobre una niña transgénero y sus problemas siendo aceptada por su familia y amistades. La actividad llevo al grupo de estudiantes a establecer discusiones sobre género, pero a su vez trabajaron con distintas representaciones de fracciones. Esta actividad sirve para demostrar que es posible llevar a cabo actividades en la clase de matemáticas que provean la oportunidad para atender temas controversiales de manera crítica, sin desatender el contenido matemático que se está enseñando.

## Ideas Matemáticas Poderosas

Los estándares publicados por el Concilio Nacional de Maestros de Matemáticas (NCTM, por sus siglas en inglés) en el 2000, formulan de manera indirecta que adquirir conocimiento matemático es una condición para adaptase, sobrevivir, y por ende sostener el tipo de
desarrollo social que promueve la globalización y la sociedad de la información (Skovsmose \& Valero, 2012). Esto trae consigo la situación de determinar cuáles son aquellas ideas matemáticas que funcionan como el fundamento para ser adaptadas a distintas situaciones, y que permiten que el estudiante genere nuevas o más complejas ideas. Definir las ideas matemáticas poderosas es complejo y depende de la utilidad percibida de estas. Skovsmose y Valero (2012) analizan el concepto de ideas matemáticas poderosas por medio de cuatro lentes (presentados en la Figura 2); lógico, psicológico, cultural, y el sociológico. La intención de Skovsmose y Valero con este marco fue definir qué le da "poder" a una idea matemática desde cuatro perspectivas distintas. Por otra parte, en este trabajo se utilizan estas definiciones para determinar que acciones podemos llevar a cabo para conceptualizar ideas matemáticas poderosas.

Las ideas matemáticas poderosas determinadas a través de un lente lógico son aquellas que pueden ser relacionadas a otros conceptos, ideas o teorías previamente aprendidas, y que permiten desarrollar nuevos significados a aquello que fue previamente aprendido. Al utilizar un lente psicológico, se da suma importancia a las secuencias de contenido o ideas que evolucionan a través de los distintos niveles de complejidad que llegan hasta la abstracción y formalización de las ideas. Este lente considera el énfasis en lo afectivo y motivacional como aspectos importantes para generar ideas matemáticas poderosas. Las ideas matemáticas pueden convertirse en poderosas en la medida que proporcionen oportunidades para que un
estudiante visualice posibilidades deseables en su futuro. Este es el enfoque de lo que debe ser una idea matemática poderosa según el lente cultural. A través de este lente, la perspectiva del estudiante ante una idea matemática se convierte en una importante consideración. Por lo que debemos relacionar las ideas matemáticas poderosas con las oportunidades individuales de los estudiantes para participar en prácticas que las requieran en sus comunidades, convirtiéndose las matemáticas en una herramienta para interpretar e interactuar con el mundo.

Gran parte de nuestra sociedad es creada y apoyada en las matemáticas, por lo que la utilidad de las matemáticas es pertinente en el proceso de aprendizaje, siendo este el enfoque del lente sociológico. Si determinamos las ideas matemáticas poderosas a través de este lente, podemos establecer los principios más importantes para el proceso de aprendizaje-enseñanza en términos del aprendizaje matemático como actividad constructiva, social, cooperativa, añadiendo un enfoque en los contextos significativos para el estudiante. Considerando todas las interpretaciones anteriores podemos atenuar los efectos de las paradojas al determinar las ideas matemáticas poderosas, recordando que todos los estudiantes del mundo deben tener acceso a una educación matemática de alta calidad. Pero entonces nos encontramos con las siguientes problemáticas: ¿Quién define las ideas matemáticas que se deben aprender? ¿Cómo aseguramos que todos los estudiantes tienen acceso a ideas matemáticas poderosas y a un aprendizaje de alta calidad?

## Figura 2

## Ideas matemáticas poderosas según los cuatro lentes



## Educación Matemática Crítica

Varias perspectivas teóricas sobre los procesos socioculturales y cognitivos del aprendizaje se han considerado para abordar el problema de falta de acceso equitativo en la educación matemática, siendo uno de los retos en este tema el desarrollo de fundamentos teóricos y empíricos y la explicación del fenómeno de los distintos niveles de logro. Los investigadores que han estudiado el tema del antirracismo y de justicia social en la educación matemática se han distanciado de la perspectiva sociocultural, y se han estado enfocando en una perspectiva sociopolítica, basada en sus teorías, y exaltando el concepto de identidad matemática y poder (Gutiérrez, 2013). Una de las perspectivas teóricas que fomenta este giro es la educación matemática crítica, que se basa en las ideas de la pedagogía crítica. El trabajo de Paulo Freire ha servido como fundamento para el desarrollo de la pedagogía crítica. Sus aportes académicos y su popularización del concepto conscientização "aprendiendo a percibir contradicciones sociales, políticas, y económicas, y a tomar acciones en contra de los elementos opresivos de la realidad"- han provisto el fundamento de la pedagogía crítica (Stinson et al., 2012). La pedagogía crítica apoya las teorías y prácticas pedagógicas mediante las cuales los estudiantes y maestros desarrollan y comprenden la relación entre ideologías, poder, cultura, y se alejan de la idea de fundamentos universales de la verdad (Leistyna \& Woodrum, 1996). Además, valora las experiencias, la realidad cultural y social de los estudiantes, aportando un toque humanista a un proceso que en ocasiones carece de sentir humano y usualmente se valora al estudiante por un producto final esperado y predeterminado por ciertos grupos o expectativas sociales. Esta perspectiva pedagógica tiene como finalidad la concientización y desarrollo de la habilidad de un estudiante para conocer su posición en la sociedad, y de motivar a los estudiantes a actuar en contra de problemáticas sociales (Gutiérrez, 2013). La matemática crítica adopta las teorías y prácticas pedagógicas de la pedagogía crítica, mientras utiliza las matemáticas como instrumento analítico para examinar las injusticias sociales (Stinson et al., 2012). Por medio de
la matemática crítica contribuimos a solucionar los problemas de falta de acceso equitativo en la educación matemática al desarrollar ciudadanos que utilizan las matemáticas como entes críticos y activos de la sociedad. El maestro humanista revolucionario es el compañero del estudiante en el proceso de la enseñanza para la justicia social por medio del análisis crítico y la búsqueda de la humanización mutua (Freire, 1970). La matemática crítica requiere que el maestro vea las matemáticas como una herramienta para entender el mundo y deconstruir estructuras de poder que marginalizan a grupos en nuestra sociedad (Yeh \& Otis, 2019). Por medio del desarrollo del conocimiento matemático los estudiantes pueden convertirse en ciudadanos que analizan críticamente las situaciones actuales de su comunidad y la sociedad en general, siendo entonces participantes de la solución a estos.

Para Freire, el conocimiento es continuamente recreado a medida que reflexionamos sobre el mundo y actuamos en él (Frankenstein, 1983). Por esta razón, tenemos que ser conscientes que la educación matemática continuamente evoluciona y se adapta a la realidad social de la población a la que sirve. Una manera de replantear la educación matemática y el desarrollo de currículos de la materia es por medio del marco de referencia de las 3C (Gutstein, 2006), cuyos componentes son: conocimiento clásico, conocimiento comunitario, y conocimiento crítico.

Al contextualizar las matemáticas a la realidad social de los estudiantes, fomentamos el desarrollo de una identidad matemática por medio de la cual los estudiantes se perciben como capaces de hacer y aprender matemáticas, y a su vez, ayudamos a los estudiantes a hacerse más conscientes de los problemas que enfrenta la sociedad. Dedicando importancia a cada uno de los componentes (presentados en la Figura 3) durante el desarrollo de materiales curriculares, es posible replantear la manera en que se enseña matemáticas para que se convierta en un proceso que provea un mejor acceso equitativo a las matemáticas y las ideas matemáticas poderosas, y a su vez desarrolle el pensamiento crítico de los estudiantes.

## Figura 3

## Componentes del marco de referencia del contenido de Gutstein



## Implicaciones para la Sala de Clases

La matemática crítica provee un marco teórico para la enseñanza de las matemáticas que se fundamenta en la consideración de los factores sociopolíticos que influyen en todo lo relacionado al proceso de enseñanzaaprendizaje. Por medio del marco de referencia de las 3C (Gutstein, 2006), podemos visualizar la matemática crítica como el conjunto de tres tipos de conocimiento que debemos considerar al momento de enseñar la materia, mientras que las ideas matemáticas poderosas cuando son analizadas a través del lente lógico, psicológico, cultural y el sociológico (Skovsmose \& Valero, 2012) se pueden
identificar y relacionar a los tipos de conocimiento, de modo tal que reconozcamos qué debemos enseñar para promover el desarrollo de una consciencia críticamatemática en los estudiantes.

En la Figura 4, se presenta un modelo sugerido para conceptualizar las ideas fundamentales de la matemática crítica en la sala de clases, por medio de la acción intencionada de determinación de contenido y contextos que responden a los lentes lógico, psicológico, cultural y sociológico, los cuales a su vez se enmarcan en los tres tipos de conocimiento que propone Gutstein (2006) en torno a la matemática crítica. Tradicionalmente se

## Figura 4

## Modelo de Integración de la Matemática Crítica al Salón de Clases


enseña matemáticas con un enfoque en el conocimiento clásico a través de un lente lógico, y en ocasiones a través del lente psicológico. Esto permite que se desarrollen y se enseñen nuevas ideas matemáticas de modo secuencial y dependiente, en donde existe la posibilidad de establecer conexiones con el conocimiento previo, mientras que en otras ocasiones se integra el significado de los conceptos y el pensamiento matemático. Pero, debemos reconocer que estos dos componentes no consideran el espectro total de la matemática crítica. Además de lo anterior, la matemática crítica considera el conocimiento comunitario y crítico, lo que nos lleva a identificar la ideas matemáticas poderosas a través del lente cultural y el sociológico.

Si deseamos proveer una educación matemática que funcione como herramienta de cambio social en contra de los problemas de equidad, debemos de tomar en consideración cómo y qué enseñamos. La matemática crítica no propone disminuir los contenidos matemáticos, minimizarlos, o hacerlos más simples, sino tomar en consideración los factores sociopolíticos que enmarcan los procesos de aprendizaje y utilizaros como guía. La paradoja de la inclusión nos presenta como la globalización propone un acceso universal y de inclusión, pero esto no ocurre en beneficio de todos los grupos sociales. Es por esta razón que los contenidos matemáticos no se deben minimizar, de tal manera que todo estudiante tenga la oportunidad de aprender los mismos conceptos, evitando así reducir el aprendizaje a la necesidad de adaptación o de una exigencia social particular de un grupo, así como lo propone la paradoja de la ciudadanía. La clave de la educación matemática crítica se encuentra en la enseñanza de las ideas matemáticas poderosas por medio de la contextualización social.

El desarrollo crítico e integral matemático de un estudiante se fomenta por medio de las acciones que realizan en la sala de clases. Por lo tanto, propongo las siguientes ideas como fundamentales para transferir de teoría a práctica las ideas de la matemática crítica: Conexiones con conocimiento previo, nuevos conceptos matemáticos, significado de los conceptos matemáticos, pensamiento matemático, realidad social del estudiante, prácticas comunitarias, conocimiento integrado, y herramienta de acción social. Como se observa en la

Figura 4, las primeras cuatro ideas fundamentales provienen del conocimiento clásico visualizado a través del lente lógico y el psicológico. Estas ideas son la base para el desarrollo del conocimiento matemático por medio de la conexión continua de nuevos conceptos con el cocimiento previo a través de la adaptación de las ideas matemáticas poderosas. A su vez, mientras un estudiante continuamente adapta una idea matemática poderosa, desarrolla múltiples significados, usualmente más concretos, de los conceptos matemáticos. El pensamiento matemático, definido como el pensamiento basado en operaciones de naturaleza matemática y que puede ser aplicado en múltiples contextos, se promueve por medio de la práctica consciente de la reflexión sobre lo que hacemos y por qué lo hacemos (Burton, 1984), y funciona como una herramienta para ayudar a que cada estudiante comprenda el mundo a través de las matemáticas.

## Conclusión

Las oportunidades que tiene un estudiante son función de la continuidad y discontinuidad en las prácticas de distintas comunidades en las cuales el estudiante participa (Schoenfeld, 2006). Los problemas sociales no existen en una dimensión alterna a los procesos de educación, sino que están en constante intersección. Si buscamos humanizar los procesos pedagógicos, debemos de ver la escuela como un lugar en el cual se replican procesos sociales y en el cual existe la posibilidad de servir como un escenario de transformación (Yeh \& Otis, 2019). Por medio de este escenario de transformación debemos aspirar a la creación de una futura sociedad crítica, capaz de percibir y solucionar los problemas sociales que la afectan. La crítica y el diálogo son elementos de la comunicación que deben estar siempre presentes en los procesos de democratización del aprendizaje. La problematización es necesaria para establecer una ciudadanía crítica. Aunque la contextualización puede ser motivante, no es meramente eso; es una precondición para problematizar. Por medio de la contextualización se establecen discusiones sobre como las matemáticas operan como fuente de poder. Las matemáticas tradicionales asumen que las descripciones proporcionadas por los textos utilizados para impartir clases son exactas, verdaderas e incontrovertibles. Una de las funciones principales de la contextualización y
problematización es romper con este paradigma que no apoya la transferencia del conocimiento matemático a la realidad.

La era de la sociedad informacional global ha servido para aumentar los problemas de acceso equitativo a la educación matemática de alta calidad, que a su vez son un efecto secundario de las problemáticas sociales de racismo, sexismo, clasismo, lingüismo, discriminación y prejuicio social. Muchas personas no desarrollan la habilidad de ser conscientes sobre su situación, por lo que ven su condición como una causa de su propio fracaso y/o un ser supremo como un dios (Frankenstein, 1983). De esta forma, inconsciente o conscientemente, se han perpetuado las problemáticas sociales antes mencionadas influyendo así en la identidad matemática de los estudiantes y en el currículo oculto en la enseñanza de las matemáticas. Es por esta razón que es urgente actuar en contra de la falta de acceso equitativo en la educación matemática.

Las matemáticas pueden proveer estructura y formalización a las conexiones de un estudiante con el mundo (Brown et al., 2016). Es por medio de las matemáticas y las ideas matemáticas poderosas que entendemos muchas de las dinámicas que ocurren en la sociedad y comprendemos la realidad misma. Privar a ciertos grupos del acceso al conocimiento matemático, ya sea implícita o explícitamente, solamente fomenta la estratificación social que posiciona a unos grupos sociales en ventaja sobre otros. De esta manera creamos ambientes de vida poco favorables para una gran parte de la sociedad. La matemática crítica como herramienta para afrontar el problema de falta de acceso equitativo en la educación matemática por medio del desarrollo de ciudadanos críticos promete ser una ruta para seguir en la educación matemática. Aun así, el modelo sugerido como parte de este trabajo aún resta por ser evaluado a través de métodos cuantitativos y cualitativos que sugieran los próximos pasos a seguir en el desarrollo de la integración de la matemática crítica a en el salón de clases.

## Referencias

Allen, K. (2011). Mathematics as thinking. A response to "democracy and school math." Democracy \& Education, 19(2), 1-7.

Boaler, J. (2008). Promoting "relational equity" and high mathematics achievement through an innovative mixed-ability approach. British Educational Research Journal, 34(2), 167-194. https://doi.org/10.1080/01411920701532145
Brown, T., Solomon, Y., \& Williams, J. (2016). Theory in and for mathematics education: In pursuit of a critical agenda. Educational Studies in Mathematics, 92(3), 287-297. https://doi.org/10.1007/s10649-016-9706-7
Burton, L. (1984). Mathematical thinking: The struggle for meaning. Journal for Research in Mathematics Education, 15(1), 35-49. https://doi.org/10.2307/748986
Castells, M. (1999). A critical theory of informational society. In D. Macedo (Ed.) Critical education in the new information age (pp. 37-64). Rowman \& Littlefield Publishers.
Cobb, P., \& Hodge, L. L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. Mathematical Thinking and Learning, 4(2-3), 249-284. https://doi.org/10.1207/S15327833MTL04023_7
Frankenstein, M. (1983). Critical mathematics education: An application of Paulo Freire's epistemology. The Journal of Education, 165(4), 315-339.
Frankenstein, M. (1990). Incorporating race, gender, and class issues into a critical mathematical literacy curriculum. The Journal of Negro Education, 59(3), 336-347. https://doi.org/10.2307/2295568
Freire, P. (1970). Pedagogy of the oppressed. Herder and Herder.
Giroux, H. A., \& Penna, A. N. (1979). Social education in the classroom: The dynamics of the hidden curriculum. Theory \& Research in Social Education, 7(1), 21-42.
https://doi.org/10.1080/00933104.1979.10506048
Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. Journal for Research in Mathematics Education, 44(1), 37-68. https://doi.org/10.5951/jresematheduc.44.1.0037
Gutstein, E. (2006). Reading and writing the world with mathematics: Toward a pedagogy for social justice. Routledge.
Isidro Luna, V. M. (2016). The persistence of poverty in capitalist countries. Economía Informa, 400, 67-82. https://doi.org/10.1016/j.ecin.2016.09.005
Lee, J. (2002). Racial and ethnic achievement gap trends: Reversing the progress toward equity? Educational Researcher, 31(1), 3-12.
https://doi.org/10.3102/0013189X031001003

Leistyna, P., \& Woodrum, A. (1996). Context and culture: What is critical pedagogy. Breaking free: The Transformative Power of Critical Pedagogy, 1-7.
Martin, D. B. (2010). Not-so-strange bedfellows: Racial projects and the mathematics education enterprise. Proceedings of the Sixth International Mathematics Education and Society Conference, 1, 57-79.
Schoenfeld, A. H. (2006). Mathematics teaching and learning. In P. A. Alexander \& P. H. Winne (Eds.), Handbook of educational psychology. (pp. 479-510). Lawrence Erlbaum Associates Publishers.
Sfard, A., \& Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. Educational Researcher, 34(4), 14-22. https://doi.org/10.3102/0013189X034004014
Skovsmose, O. (1990). Mathematical education and democracy. Educational Studies in Mathematics, 21(2), 109-128. https://doi.org/10.1007/BF00304897
Skovsmose, O., \& Valero, P. (2012). Acceso democrático a ideas matemáticas poderosas. En P.Valero \& O.Skovsmose (Eds.), Educación matemática crítica. Una visión sociopolitica del aprendizaje y la enseñanza de las matemáticas ( $1^{\text {ra }}$ ed., pp. 25-61). Ediciones Uniandes.
Solomon, Y., Radovic, D., \& Black, L. (2016). "I can actually be very feminine here": contradiction and
hybridity in becoming a female mathematician. Educational Studies in Mathematics, 91(1), 55-71. https://doi.org/10.1007/s10649-015-9649-4
Stinson, D. W., Bidwell, C. R., \& Powell, G. C. (2012). Critical pedagogy and teaching mathematics for social justice. The International Journal of Critical Pedagogy, 4(1), 76-94.
Tate, W. F. (1997). Race-ethnicity, SES, gender, and language proficiency trends in mathematics achievement: An update. Journal for Research in Mathematics Education, 28(6), 652-679. https://doi.org/10.2307/749636
Valoyes-Chávez, L. (2015). Los negros no son buenos para las matemáticas: ideologías raciales y prácticas de enseñanza de las matemáticas en Colombia. Revista CS, 173. https://doi.org/10.18046/recs.i16.1909
Yeh, C. (2018). Democratic accountability in the neoliberal era: The politics of teaching and teacher education in mathematics classrooms. Policy Futures in Education, 16(6), 764-780. https://doi.org/10.1177/1478210318776470
Yeh, C., \& Otis, B. M. (2019). Mathematics for whom: reframing and humanizing mathematics. Occasional Paper Series, 41. https://bit.1y/3TLgXOm

## Preguntas de Discusión y Reflexión - Después de la Lectura

1. ¿Qué acciones actualmente usted lleva a cabo, de manera indirecta o inconsciente, que promueven el desarrollo o sustentan las estructuras de poder que minimizan el acceso equitativo a la educación matemática de alta calidad? ¿Qué puede hacer para evitar o no llevar a cabo estas acciones?
2. ¿Su currículo de matemáticas considera el conocimiento clásico, comunitario y crítico? De ser su respuest "si", ¿Cómo lo hace y que puede hacer para aumentar su impacto? De ser su respuesta "no", ¿Cómo puede integrar el conocimiento clásico, comunitario y crítico en su enseñanza de las matemáticas?

## TODOS Position Statements

TODOS has created (or co-created with the National Council of Supervisors of Mathematics, NCSM) several position statements which can be seen by scanning the QR codes below or by visiting https://www.todosmath.org/statements and https://www.todos-math.org/other-publications. The second website includes other TODOS statements.

Positioning Multilingual Learners for Success in Mathematics

A joint position statement from NCSM: Leadership in Mathematics in Education and TODOS: Mathematics for ALL (Fall, 2021)

THE MO(VE)MENT TO PRIORITIZE ANTIRACIST MATHEMATICS:
PLANNING FOR THIS AND EVERY SCHOOL YEAR Recipient of the 2021 NCTM Special Publication Award for Outstanding Journal Re-release Edition with Spanish Commentary Now Available!


## Mathematics Education Through the Lens of Social Justice: Acknowledgment, Actions, and Accountability

A joint position statement from the National Council of Supervisors of Mathematics and TODOS: Mathematics for ALL



# Flags of Latin America: Culturally Relevant Learning Experiences With Technology to Enhance Geometry and Algebra Concepts 

Kelly Wamser Remijan<br>Illinois Mathematics and Science Academy - Statewide Educator Innovation


#### Abstract

This article highlights flags from various Latin American countries as a springboard for implementing culturally relevant learning experiences within the mathematics curriculum. It provides middle school and high school math teachers, as well as college instructors, with examples of learning experiences to engage students in culturally relevant activities. These activities build a connection between geometry and algebra concepts while at the same time extending the knowledge of technology tools such as Google Sheets Pixel Art, Computer Numerical Control Machines, GeoGebra, and Desmos. Finally, discussing flags encourages dialog about different cultures and enhances global awareness and cultural competence.


## Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. How might flags of Latin America be used to connect geometry and algebra concepts?
2. What technology can be utilized to integrate the use of flags to create culturally relevant learning experiences within the mathematics curriculum?
3. How can teachers use flags to promote global awareness within the mathematics curriculum?
4. What are benefits of engaging students in culturally relevant learning experiences?

Kelly Wamser Remijan (wamser.remijan@yahoo.com) is Director-at-Large for the Illinois Council of Teachers of Mathematics (ICTM) and an active Board member of the Illinois Mathematics Teacher Educators (IMTE). Dr. Remijan is a former Professional Development and Curriculum Specialist with the Illinois Mathematics and Science Academy (IMSA) - Statewide Educator Innovation, former High School Mathematics Teacher in O'Fallon, IL, and former University Adjunct at McKendree University in Lebanon, IL. Dr. Remijan seeks to help students of all abilities and backgrounds learn and grow by sharing her knowledge, experience, and passion for mathematical connections with classroom teachers and teacher educators.

Acknowledgments: While most technology highlighted in this paper is free and available to those that have computer and internet access, a "school-friendly" Computer Numerical Control (CNC) Machine can cost up to $\$ 3000$. For this reason, it is important for math teachers to consider the technology resources available within one's district or community. As such, I would like to acknowledge that the flag of Chile described in this paper was replicated using a CNC machine located within the Illinois Math and Science Academy (IMSA) Makerspace. I thank the IMSA Makerspace Coordinator, Grant Bell, who machined the flag of Chile using the G-code that I provided. Additionally, I acknowledge the assistance of Southwestern Illinois College (SWIC) on various math and machining projects in which we have collaborated. I thank SWIC Precision Machining Technology Instructors: Mark Bosworth, Jerry Bonifield, and David Berry who have helped me learn about machining throughout the years and have brought CNC machines on site to various schools where students have seen their designs machined in front of their eyes.

## Flags of Latin America: Culturally Relevant Learning Experiences With Technology to Enhance Geometry and Algebra Concepts

## Kelly Wamser Remijan

Flags can allow students to explore and better understand the world around them. Through the exploration and discussion of the mathematics, symbolism, and history behind flag designs, flag-based learning experiences offer mathematics students the opportunity to make connections which aim to ignite student interest, increase global awareness, and build cultural competence. While global awareness involves having knowledge of geography, history, and culture (Werner \& Case cited in Burnouf, 2004), 60 percent of secondary students believe that the understanding of various cultures is the "most important skill", even more so than writing and math (Participate Learning, 2016). As global awareness is a critical element in building global competency in the $21^{\text {st }}$ Century (NEA, 2010), culturally relevant pedagogy is a critical need (Jones, 2016) and guiding principle for "Dismantling Racism in Mathematics Instruction" (EquitableMath.org, 2021). Since culturally relevant pedagogy "uses student culture in order to maintain it and to transcend the negative effects of the dominant culture" with the negative effects occurring as a result of "not seeing one's history, culture, or background represented in the textbook or curriculum or by seeing that history, culture or background distorted" (Ladson-Billings, 1994, p. 19), mathematics teachers should consider using flags within the mathematics curriculum as a means to engage students in culturally relevant learning experiences that
promote the discussion of history, geography, culture, and current issues.

Through the development and implementation of culturally relevant curricula and practices, cultural competence can improve when students are the given the opportunity to "appreciate and celebrate their cultures of origin while gaining knowledge of and fluency in at least one other culture" (Ladson-Billings, 2014, p. 75). While the use of flag designs of African countries (Remijan, 2021) illustrated how flags could enrich linear equations/inequalities using Desmos and provided students of African ancestry the opportunity to see their culture, history, or background represented within the math classroom, the integration of "Flags of Latin America" will showcase how flags can enhance additional algebra concepts as well as geometry concepts using not only Desmos, but also using GeoGebra, Google Sheets Pixel Art, NCviewer, and Computer Numerical Control (CNC) Machines. As such, these activities and corresponding technology tools can enhance the teaching and learning of algebra and geometry concepts as well as provide Latinx students the opportunity to engage in experiences that highlight their ancestry, history, and culture. With intentional planning for the integration of learning experiences involving flags within the mathematics classroom, teachers can use flags of Latin America as a springboard to provide opportunities for students to discuss global issues and/or their own

## Remijan

culture/heritage as well as the cultures of other people within their own community or around the world.

Having a desire to learn about other countries and their cultures, I have traveled to various countries and have shared my experiences with my students through personal pictures and the creation of related problems and activities. Incorporating flags of various countries within algebra and geometry activities across all levels, I have seen first-hand the discussions that can occur regarding culture, heritage, experience, history, geography, and/or current events. These discussions would often help me to learn more about the backgrounds and interests of students as well as recognize the lack of global awareness that exists among middle school and high school students. As such, with the U.S. Latinx population being one of the fastest-growing populations in the U.S. (Krogstad \& NoeBustamante, 2021) and identity of this group rooted in their (or their ancestors') country of origin (Cohn et al., 2021), this article provides middle school and high school math teachers, as well as college instructors, with examples of learning experiences involving a sample of flags from various Latin American countries. Flag-based experiences can promote various cultures and engage students in culturally relevant math activities and discussion topics that enhance mathematics learning, promote global awareness, and build cultural competence.

## Flag-Based Learning Experiences Enhancing Geometry and Algebra Concepts

This paper provides examples of flag-based learning experiences involving a sample of Latin American countries. It is important to note that these experiences can be implemented within the classroom as an introductory activity, warm-up review, lesson example problem, critical thinking activity, or extension project. Even though technology is highlighted in this paper, many of the activities can be done with or without technology. Considering the technology skills of one's students and the availability of technology, teachers can modify the activity to be done without technology or can choose to use the technology as a research, checking, or creating tool. As such, it is important to consider the

[^0]abilities, prior knowledge, and technology skills/access of the students before implementing the technology component of these activities. Furthermore, while this paper offers "talking points" about each country, natural conversations and discussions within the classroom are encouraged based on student knowledge and/or heritage as well as current events happening in the world.

## Mexico

For an introductory activity in geometry focused on dimensions, ratio, or area, the teacher can display the flag shown in Figure $1^{1}$ and ask students what they know about the flag. Some students may be able to identify that the flag is that of Mexico while some may simply describe the flag as having 3 colors represented by 3 rectangles with a bird of some kind and something in the bird's mouth. Students may wonder what country the flag represents, what the colors of the flag mean, what kind of bird is displayed in the flag, what is the bird sitting on, or what is the significance of the bird. Students could conduct internet research to find the answers to the various questions. While questions of "wonder" are open-ended and do not need to revolve around math (Ray, 2013), the math teacher could build on students' notes of what they see or what they wonder. If students do not wonder about any additional mathematical information about the flag, the teacher could transpose the flag on a grid background using PowerPoint, as shown in Figure 2, and pose questions focusing on the math concepts for a particular lesson such as "What are the dimensions of the flag?", "What is the aspect ratio [width/height] of the flag?", "What is the area of the green region?", or "What is the perimeter of the white region?"

## Figure 1

Flag of Mexico (Smith, 2001a)


## Figure 2

## Figure 1 With 4:7 Ratio on Grid



Beyond mathematics, discussion could occur regarding the connection of the flag's emblem to Aztec legend as well as the music, food, history, traditions, and multicultural heritage of the country which has been influenced not only by the Aztecs but also by the Mayans, European colonization, and the African slave trade (Schmal, 2020). Additionally, it can be pointed out that: (a) Mexico is recognized as having the largest Spanish population in the world, with the United States being second (Burgen, 2015), (b) There are 68 indigenous languages recognized in Mexico (Talley, 2021), and (c) Afro-Mexicans have only started to receive recognition by Mexico with the 2020 National Census being the first time in Mexico's 500-year history (Talley, 2021). As such, further discussion could occur regarding the discrimination and racism that Indigenous, AfroMexican, and dark-skinned Mestizos, people who are of mixed race with Spanish and Indigenous descent, face today (Vallejo, 2021), despite the efforts of Mexico "to create a national mixed-race identity that melded Hispanic, Indigenous and African ethnicities" (Varagur, 2016) after its independence from Spain in 1810.

## Cuba

For a warm-up review activity focused on area, the teacher could display the flag of Cuba and ask students "What do you see?". While some students may recognize the flag of Cuba, other students may observe that the flag consists of a red triangle with a white five-pointed star (concave decagon), two blue trapezoids, two white trapezoids, and one blue concave pentagon. After students have a chance to share what they notice, students can then share what they wonder regarding the flag such as the meaning behind the colors and overall design. Next, the teacher can provide students with a studentactivity worksheet as shown in Figure 3.

Working collaboratively or independently, students can answer the math-based questions and research additional extension questions that focus on topics involving flag symbolism, history, or geography. Discussion could occur regarding the change in meaning behind the design of the flag after the Revolution of Cuba in 1959 led by Fidel Castro (Cuba Flag Map, n.d.). Furthermore, the concept of Communism could be discussed which caused thousands of Cubans, along with their heritage, culture, and language, to leave their homeland for Miami, now known as "The Capital of Latin America" with the largest Latinx population outside of Latin America (Booth, 2001).

## Figure 3

Cuba Activity Worksheet


## Honduras

For a warm-up activity focused on linear equations/inequalities and domain/range, the teacher can display the flag of Honduras like the one found in a corresponding student activity worksheet (Figure 4). Students could again be asked what they notice and what they wonder. Student might describe what they see in terms of colors, numbers, and shapes such as two blue rectangles and five blue stars (concave decagons) found
within a white rectangle. Questions of wonder might pertain to the country the flag represents, the meaning behind the colors, or the significance behind the five stars within the design. Electronically displaying the flag of Honduras along with a map of the Western Hemisphere, a teacher can share that the blue stripes of the flag represent the two bodies of water in which Honduras lies, the Pacific Ocean and the Caribbean Sea. Furthermore, it can be pointed out that the name "Honduras", meaning "depths", was given by Christopher Columbus on his fourth and final trip to the Americas (History of Honduras, n.d.).

Figure 4
Honduras Activity Worksheet


With the Honduran flag having an aspect ratio of 1:2, the students can use the given flag shown in Figure 4 to identify the borders of the flag to be $\mathrm{x}=0$ where $0 \leq \mathrm{y} \leq$ $9, \mathrm{x}=9$ where $0 \leq \mathrm{y} \leq 9, \mathrm{y}=0$ where $0 \leq \mathrm{x} \leq 18, \mathrm{y}=9$ where $0 \leq x \leq 18$. Students could also mathematically identify the blue stripes as $0 \leq y \leq 3$ where $0 \leq x \leq 18$ and $6 \leq y \leq 9$ where $0 \leq x \leq 18$. A specific link in Desmos, such as http://bit.ly/3HhGklO, could be used by
students as a checking tool or challenge depending on student familiarity with Desmos as shown in Figure 4. It is important to note that if the position of the flag is moved on the coordinate plane or if the flag is made bigger or smaller, results could vary. As a result, this activity could be used to lead into other topics such as transformations with raising the flag up a flagpole; allowing for discussion on the traditions that occur with flag ceremonies associated with Honduran National Day. (As a side note, it might be worth sharing that Honduran National Day, September $15^{\text {th }}$, also happens to be the first day of Hispanic Heritage Month.) Furthermore, the controversy of "Columbus Day" versus "Indigenous Peoples' Day" can be discussed regarding Columbus "discovering" land that was already inhabited by the Mayan and other Indigenous Groups which historians now recognize Columbus enslaved and killed during his various voyages (National Geographic Kids, n.d.; Abrams, 2015; Willingham et al., 2021).

## Brazil

For an example problem involving a lesson pertaining to circles or parallelograms, students could be shown the flag of Brazil (Figure 5). Students could talk about the colors of the flag or what images they see in the flag...a circle (or planet) with stars inside of a rhombus. The teacher could ask students to write the equation for the circle representing the circle on the flag and students could further test their equation using Desmos where the flag image has been inserted onto a coordinate plane, found at https://bit.ly/3DSUXKU. Additionally, teachers could pose the question "How can we prove mathematically that the flag's design in fact involves a rhombus?" Students could then discuss the various methods of proving that a rhombus, in fact, exists. Following this, students could measure opposite angles, with a protractor by hand or electronically using GeoGebra, to show both pairs of opposite angles are congruent proving a parallelogram exists. Alternatively, students could measure the sides of the parallelogram to show that all sides of the parallelogram are congruent proving that the parallelogram is a rhombus.

Beyond mathematics, the flag can provide a starting point for the discussion of the geography, history, and language of Brazil. First, the significance of the design of

Figure 5
Brazil Activity Worksheet

the flag of Brazil could be discussed with the green color being representative of the Amazon Rainforest and the importance of the region to " 10 percent of all plant and animal species known on Earth" and "home to more than 24 million people...including hundreds of thousands of Indigenous Peoples belonging to 180 different groups" (Greenpeace, n.d.). Second, discussion can occur regarding: (a) Five million African slaves forcibly brought to Brazil during the Slave Trade compared to nearly 500,000 forcibly brought to the United States (Public Broadcasting Service [PBS], n.d.), (b) The impact of African culture on Brazil which has the second largest black population in the world outside of Nigeria (PBS, n.d.), and (c) The social inequality that exists with more than half of the Brazilian population considering themselves as Black or of mixed race (Darlington, 2013). Lastly, discussion can occur regarding the official language of Brazil being Portuguese, making it the only non-Spanish speaking country in Latin America.

## Bolivia

For a critical thinking activity involving geometry concepts such as ratio and area, students can research the
flag of Bolivia. Discovering that Bolivia has two official flags, students could replicate each flag either by hand using grid paper or with technology using Google Sheets Pixel Art found at https://tinyurl.com/skz55uup. After replicating the National Flag of Bolivia (Figure 6) as well as the Indigenous Wiphala Flag (Figure 7), students could further research the history and meaning behind each flag's design. The teacher could then ask students to compare the percentage of each flag being green, yellow, and red. Furthermore, discussion could occur regarding the social and political issues that occur with a country having two official flags.

Figure 6
Bolivia Flag (15:22 Aspect Ratio Without Coat-of-Arms)


Figure 7
Indigenous Wiphala Flag of Bolivia (1:1 Aspect Ratio)


## Colombia

For a critical thinking activity involving algebra concepts, teachers can play a song from the 2021 Disney movie Encanto found at https://bit.ly/3hvvVKc. Afterwards, students can be asked to research and replicate the flag of the country celebrated in the song using linear equations, linear inequalities, and domain/range. To begin this process, students can discover the flag has a 2:3 aspect ratio and consists of a top yellow stripe that is twice as wide as the blue and red stripes (Smith, 2001f), thus making the horizontal stripes in a 2:1:1 ratio. With this information, students could sketch the flag and its
dimensions on a hand-drawn coordinate plane and then electronically replicate the flag on a coordinate plane in GeoGebra as shown in Figure 8. (It is important to note that GeoGebra allows for a multitude of colors, including yellow, unlike Desmos which offers only 6 colors, none of which are yellow.) Students could research the meaning behind the colors, why the flag of Colombia is similar to the flags of Ecuador and Venezuela, and how similarity of flags can often help tell the history or culture of a country. Finally, discussion could occur on how Colombia is the first and only NATO alliance "partner" from Latin America (NATO, 2021), and how such affiliation may impact Colombia as well as other Latin American countries.

Figure 8
Colombia Flag in GeoGebra


## Chile

For an extension project involving coordinate geometry, students can be asked to replicate or re-imagine the flag of a country such as Chile. To do this, students can (a) Research the flag, (b) Insert the flag into Desmos (Figure 9), (c) Identify ordered pairs making up the image, (d) Write a computer program involving a Computer Numerical Control (CNC) programming language called G-Code to create the flag's design, and (e) Check their computer program using an online simulator program such as https://ncviewer.com/ (Figure 10). Furthermore, with access to a Computer Numerical Control (CNC) machine, students can machine their design, as shown in Figure 11, which can make math more meaningful for students and inspire students to consider a future career in machining, engineering, or design. (Note: Math teachers are encouraged to collaborate with Career Technology Education (CTE) teachers within their school/district who may have a CNC machine or teachers can connect

Figure 9
Chile Flag (2021) with 2:3 Aspect Ratio in Desmos


Figure 10
Chile Flag Using G-Code and NC Viewer


Figure 11
Chile Flag on 5.5 " $x 7.5^{\prime \prime}$ Piece of Wood

with a local college or business to gain accessibility to a CNC machine). Discussion can occur regarding the meaning behind the flag's design and the potential connection to the ancient Machupe Indigenous flag that was used during the Arauco War with the Spaniards (Flag of Chile, 2021). Additional dialogue can occur regarding Chile being the only country in Latin America that does not recognize Indigenous people in its constitution and the challenges that the Indigenous people face such as the lack of rights to the land that ancestrally belongs to them (International Work Group for Indigenous Affairs [IWGIA], n.d.).

## Other Countries Beyond Latin America

Flags from Latin America can be a springboard to the replication and discussion of flags from around the world. Students, for instance, could be asked to research the flag of a country which is not considered to be part of Latin America but is located within South America or the Caribbean. After identifying and researching such countries, students could replicate a flag, like that for Trinidad and Tobago (Figure 12), through the application of online graphing technology such as Desmos. Additionally, discussion could occur on why some countries are considered part of Latin America and others are not. Finally, students could share or research what it means to be Latinx and how one's culture may compare with someone who is from a non-Latin American country such as Trinidad and Tobago.

Flags from countries around the world that have historic ties to Latin American could also be utilized to connect geometry and algebra connections and promote global awareness through further discussion. A map of Latin America and a map of the world can provide visual aids on the location and proximity to the United States as well as other countries connected to Latin America through history. As the slave trade, for instance, involved the forced transportation of slaves from Africa to Latin America during the European Colonialization of Latin America, many Latinx may have African or European Ancestry; thus, the use of African flags for geometry and algebra concepts could be incorporated as additional activities or activity extensions. In addition, with European explorers, such as Christopher Columbus, reaching present day Latin America, believing that the East could be reached by traveling West, the flags of the
present-day countries of these explorers and the countries they were seeking could also be used as extensions of flags from Latin America.

## Conclusion

Culturally relevant learning experiences within the mathematics curriculum can make math more meaningful for students. As culturally relevant pedagogy "uses student culture in order to maintain it and to transcend the negative effects of the dominant culture" with the negative effects occurring as a result of "not seeing one's history, culture, or background represented in the textbook or curriculum or by seeing that history, culture or background distorted" (Ladson-Billings, 1994, p. 19), mathematics teachers must reflect upon their current curriculum and teaching methods and consider various ways to engage students in culturally relevant learning experiences. As such, teachers are encouraged to get to know their students and to incorporate flags from countries around the world, including flags that represent Indigenous groups, which recognize the diverse backgrounds of their students, community, and the world in which we live.
Through the exploration and discussion of the mathematics, symbolism, and history behind flag design, as well as the skills or creativity required in replicating (or reimagining) a flag design with technology, flag-based learning experiences offer mathematics students the opportunity to engage in mathematics activities with cultural connections and technology tools which ultimately enhance skill development and student learning. In conclusion, the intentional implementation of culturally relevant learning experiences involving flags

## Figure 12

## Trinidad and Tobago Flag in Desmos


within the mathematics classrooms has the potential to ignite student interest, enhance mathematics learning, and increase global awareness which makes students not only more mathematically competent, but also more culturally competent.

## References

Abrams, A. (2015). Columbus Day 2015: What is The Day of the Race? Latin America Celebrated Dia De La Raza instead of Italian explorer. International Business Time. http://bit.ly/3UKzzOs
Booth, C. (2001). Miami: The capital of Latin America. Time. http://bit.ly/3URGa9W
Burgen, S. (2015). US now has more Spanish Speakers than Spain - only Mexico has more. The Guardian. http://bit.ly/3toXpUv
Burnouf, L. (2004). Global awareness and perspectives in global education. Canadian Social Studies, 38(3), 1-12. https://files.eric.ed.gov/fulltext/EJ1073942.pdf
Cohn, D., Brown, A., \& Lopez, M.A. (May 14, 2021). Black and Hispanic Americans see their origins as central to who they are, less so for White adults. Pew Research. http://bit.ly/3EqQxw7
Cuba Flag Map and Meaning. (u.d.). Mappr. https://www.mappr.co/flag-maps/cuba/
Custodio, J. (2018). Afro-Mexicans fight for visibility and recognition. Pulitzer Center. http://bit.ly/3ULHRq5
Darlington, S. (2013). From Samba to Carnival: Brazil's thriving African culture. CNN. http://bit.ly/3TwokZh
EquitableMath.Org. (2021). A pathway to equitable math instruction: Dismantling racism in math instruction. http://bit.ly/3EsOXCl
Flag of Chile. (2021). Wikipedia. http://bit.ly/3XqpiJ8
Green Peace. (n.d.). Brazil and the Amazon Rainforest. http://bit.ly/3E55oL8
History of Honduras. (n.d.). CentralAmerica.com. https://www.centralamerica.com/honduras/history/
History of Mexico. (2020). History.com. http://bit.ly/3G6MOFi
History.com (2020). Why Columbus Day Courts Controversy. http://bit.ly/3WX1BbK
International Work Group for Indigenous Affairs [IWGIA]. (n.d.) Indigenous peoples of Chile. https://www.iwgia.org/en/chile.html
National Institute of Indigenous Peoples [INPI]. (2021). Constitutional reform on the rights of Indigenous and Afro-Mexican peoples. http://bit.ly/3Gb6SXc
Jones, S. (2016). Culturally relevant pedagogy in mathematics: A critical need. TEDx Talks. https://youtu.be/EjLOuUhN6xY

Krogstad, J. M., \& Noe-Bustamante, L. (September 9, 2021). Key facts about U.S. Latinos for National Hispanic Heritage Month. http://bit.ly/3E5f8Fz
Ladson-Billings, G. (1994). The dreamkeepers: Successful teachers of African American children. Jossey-Bass Publishers.
Ladson-Billings, G. (2014). Culturally Relevant Pedagogy 2.0: aka the Remix. Harvard Educational Review, 84(1), 74-84. http://bit.ly/3OqrVan
National Education Association [NEA]. (2010). Global competence is a $21^{s t}$-century imperative. http://bit.ly/3TVnN31
National Geographic Kids. (n.d.). Honduras. http://bit.ly/3UxCJpb
North Atlantic Treaty Organization [NATO]. (2021). Relations with Colombia. http://bit.ly/3fWohbs
Participate Learning. (2016). 7 shocking statistics illustrating the importance of global education. http://bit.ly/3A9cjSy
Public Broadcasting Service [PBS]. (n.d.). Q \& A with Professor Henry Louis Gates, Jr. Black in Latin America. http://bit.ly/3fWMiPq
Ray, M. (2013). Powerful problem solving: Activities for sense making with the mathematical practices. Heinemann.
Remijan, K. (2020). Travel the world through flags: Desmos, equations, and inequalities. 19-19-19 "COVIDeos 19" E-Learning Webinars. 11. http://bit.ly/3O1fBx5
Remijan, K. (2021). Flag designs of African countries: Enriching the graphing of linear equations and inequalities in algebra. The Lighthouse Almanac. 1219. http://bit.ly/3ElMr7t

Schmal, J. (2020). The Africans of Mexico: From slavery to the 2015 Intercensal Survey. Indigenous Mexico. http://bit.ly/3A7hMsU
Smith, W. (2001a). Flag of Mexico. Encyclopedia Britannica. http://bit.ly/3QXexLU
Smith. W. (2001b). Flag of Cuba. Encyclopedia Britannica. http://bit.ly/3GTLbcO
Smith, W. (2001c). Flag of Honduras. Encyclopedia Britannica. http://bit.ly/3j1oZW7
Smith, W. (2001d). Flag of Brazil. Encyclopedia Britannica. http://bit.ly/3Hpdf9B
Smith, W. (2001e). Flag of Bolivia. Encyclopedia Britannica. http://bit.ly/3J6Q4BR
Smith, W. (2001f). Flag of Colombia. Encyclopedia Britannica. http://bit.ly/3J7I16L
Talley, P. (2021). Results: 2020 Mexican Census Shows Afromexicans. http://bit.ly/3TtNc3O
Varagur, K. (2016). Mexico finally recognized its Black citizens, but that's just the beginning. Huffington Post. https://tinyurl.com/d9jbucc4

Willingham, A. J., Andrew, S., \& Andone, D. (2021). These states are ditching Columbus Day to observe

Indigenous Peoples' Day instead. Cable News Network. http://bit.ly/3A4OAmj

## Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Try This -
a. What do you notice and wonder about the given flag?
b. What is the aspect ratio of the flag?
c. If the dimensions of the flag are 10 inches x 15 inches, and the blue triangle is an equilateral triangle, what is the area of the triangle?
d. What percentage of the flag is not covered by a triangle?
e. What is the meaning behind the design of the flag?

2. Try this - While the flag of Chile contains a five-pointed star, the original inspiration of the flag was the Mapuche flag containing an eight-pointed star called the Star of Arauco.
(Sources: http://bit.ly/3TxwhgJ, http://bit.ly/3UpNWIp)
a. Write a computer program involving G-code that will cut an 8-pointed star (known as a 16 -gon or a concave hexa-decagon) out of a 2 -inch $x$ 3 -inch piece of material using a computer numerical control (CNC) machine as represented in the image to the right.
b. Check your program using https://ncviewer.com/
c. If you have access to a CNC machine and a 2 -inch $\times 3$-inch piece of material, machine the design using the program that you have written.

3. Try this -
a. Using Google Sheets Pixel Art, found at http://bit.ly/3XkxoDz, re-imagine the National Flag of Bolivia as a mix of the current National Flag and the Indigenous Wiphala Flag of Bolivia.
b. Explain the rationale behind your design.
c. What are the pros and cons of creating one flag that mixes these two flags together?
4. Try this - During the time of European expansion to the Americas, slave ships transported slaves and goods from Africa to areas throughout Latin America and often returned to Africa leaving from Brazil. As a result, there are neighborhoods throughout Ghana, Nigeria, Togo, and Benin that consider themselves ethnically Brazilian even though live in Africa.
(Source: http://bit.ly/3E2R3Px)
a. Research the flags of the African countries mentioned above
b. Replicate one of the African flags mentioned above using either Desmos.com or GeoGebra.org applying the concept of equations/inequalities and domain/range.
5. Discussion Question - Why should culturally relevant learning experiences involving flags of Latin America be utilized within the mathematics curriculum?

## POETRY CORNER

UTEP's Lawrence Lesser reflected on a recent major event in U.S. history by writing a poem that incorporates current views on significance testing (http://bit.ly/3Ezv03m) as well as well-known parts/properties of an ellipse.

## The Ellipse

Was he at the center of opposing foci on the major axis of power?

Many in his orbit knew he knew the line, the red line, thin blue line.

But like statisticians, judges decide
with context, lest we unduly discriminate between straddlers
of boundaries: monolithic
acrylic or chance
of error
we accept.

# No le Enseñes Sobre los ELs: Infusing Language Into Professional Development and Mathematics 

Vanessa Z. Mari<br>Nevada State College

Rachel S. G. Bower<br>Nevada State College


#### Abstract

This paper shares our experiences providing professional development to secondary mathematics teachers in a school district looking for ways to meet the needs of their increasingly diverse student population. Our professional development focused on building teacher language awareness by addressing the language domains that Lindahl (2019) outlines as the teacher domain, user domain, and analyst domain. Further explanation of what each domain entails will be discussed and recommendations on how to engage teachers within these domains are shared.


## Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Teachers should seek out professional development related to English learners (ELs) (Pettit, 2011). What experience do you have receiving professional development? Have you been able to choose topics that interest you?
2. Have you received professional development (or other experiences) that specifically addressed mathematics and English learners? What was it like? What did you learn from the experience? Did you implement or use any of the strategies? What did you find challenging about the experience?
3. How would you describe the relationship between mathematical content and language?

Vanessa Z. Mari, Ph.D. (vanessa.mari@nsc.edu) is an Assistant Professor of TESL at Nevada State College. She holds a doctorate degree in bicultural and bilingual studies from the University of Texas San Antonio. She has also worked as the English Language Fellow in Peru and was a Fulbright scholar in Uruguay.

Rachel S. G. Bower, Ph.D. (rachel.bower@nsc.edu) is an Assistant Professor of Mathematics Education at Nevada State College. She holds a doctorate degree in mathematics education from Texas State University. She prepares pre-service elementary school students each year in content and methods coursework. Her research interests include students and teachers in multilingual, secondary mathematics classrooms.

# No le Enseñes Sobre los ELs: Infusing Language Into Professional Development and Mathematics 

Vanessa Z. Mari and Rachel S. G. Bower

## Introduction

Professional development (PD) for mathematics teachers needs to strategically address what the demands of language are in mathematics. When PD focuses on building teacher language awareness (TLA) it helps mathematics teachers see themselves as language teachers. TLA is defined as, "the interface between what teachers know, or need to know about language and their pedagogical practice" (Andrews \& Svalberg, 2016, p. 2). It recognizes that all educators, regardless of their content areas, are users, analysts, and teachers of the language. In order to do these three roles successfully, teachers need "well-developed language proficiency plus conscious (declarative) TLA and the ability to draw on that declarative knowledge when enacting the curriculum in the language classroom" (Andrews, 2007 p. 232).

Lindahl (2013) discusses TLA in the teacher, user, and analyst domains, with each domain sharing similarities with the other. The teacher domain is composed of the knowledge teachers have of pedagogy, including L2 (second language) theory knowledge, and the empathy they might have towards the EL student experience. The user domain takes into consideration the teacher's language proficiency and the implicit and procedural knowledge they have about language. The analyst domain considers the teachers' knowledge about the language (forms and functions). It also accounts for their metalinguistic awareness and their knowledge of the linguistic sub-fields. The domains of TLA interrelate with each other to include features such as attitudes, beliefs, awareness of EL interlanguage, ability to solve language problems, teacher's life experiences, and their sensitivity towards the EL experience. Readers wanting more detail on the TLA domains and the relationships among them may consult the figure on page 32 of Lindahl and Baecher (2015).

Creating PD that specifically targets TLA development in mathematics teachers is important because it helps them understand how they can teach
language and mathematics in the content area. Through this article, we are going to discuss how we built TLA into the PD for a group of secondary in-service mathematics teachers. Initially the PD was to be offered to educators at a large urban campus but was ultimately delivered to educators at a smaller rural campus. The PD did not need to be significantly altered, which we feel demonstrates the adaptability of the content proposed here.

## No le Enseñes Sobre los ELs/ Don’t Teach Them About ELs

We were initially encouraged by a friend and EL coordinator to put together PD for the mathematics teachers at her large, urban high school. During a planning meeting, we were told by school representatives, "Actually we don't want you to talk about ELs because then the math teachers won't be receptive to the PD. Can you just talk about math stuff?" Our plan to provide PD at this campus fell through, but about the same time an EL coordinator in a rural school district invited us to present to her secondary mathematics teachers and we agreed. Thus, our PD took place in a school district with a population of just over 10,000 students. About $31 \%$ identified as Hispanic and almost $10 \%$ of the whole are classified as ELs.

## A Focus on Mathematics and Language

The National Council of Teachers of Mathematics' (NCTM, 2014) Principles to Actions: Ensuring Mathematical Success for All encourages classrooms where all students engage with mathematical content both orally and in writing. The Common Core State Standards for Mathematics (2010) include the standards for mathematical practice at all grades which expect that student will make sense of problems, reason, construct arguments and critique others. Aguirre and Bunch (2012) note that an emphasis on the five language modalities in English-- that is reading, listening, speaking, writing, and representing-- can be an advantage for ELs, "because it
facilitates students' mathematical learning and English language development" (p. 192).

Our position is that mathematics teachers who still claim that their content area isn't about language aren't adhering to the expectations for current classrooms as outlined by NCTM and the Common Core State Standards for Mathematics (CCSSM). Schütte (2018) said, "It is certainly desirable for all participating children to be introduced to formal and subject-specific mathematical language aspects, and for the teacher to act explicitly as a linguistic role model" (p. 34). The subject of mathematics is known to employ its own register. "Registers are specialized uses and meanings of a specific language for mathematical purposes (e.g., specialized meanings and purposes for vocabulary (words, phrases or expressions) as well as grammatical structures) that can be chosen by an individual to fit a situation or a context" (Schütte, 2018, p. 26). We believe a mathematics teacher is the best person to model this mathematical register. Mathematics teachers cannot expect the English department or EL support staff to be the sole responsible party to make it possible for their students to engage in the language of mathematics. Pettit's (2011) literature review found that in classrooms where EL students were successful in learning the content, the content teachers assumed responsibility for teaching all students, including ELs.

The language demand of mathematics can be significant, full of phrases not used elsewhere in school such as $x$-intercepts, hyperbola, modulo, and commutative (Aguirre \& Bunch, 2012). Even native English-speaking children are regularly encountering new words in the mathematics classroom. To increase the confusion, students are also challenged by nonmathematical words that have alternative definitions in mathematics (such as differentiate, slope, range, and kite) as well as false cognates. Even word problems which might not contain much academic language at all can challenge students, and has long been a focus for mathematics education researchers (Pimm, 2018). It takes multilingual students 5-7 years to master academic language so we should be prepared for their challenges in mathematics by making accommodations proactively for their language needs (Wright, 2015). In addition, research shows a focus on language in the mathematics class
benefits all the students in the class (Vogt, Echevarría, Short, \& Amy, 2013).

## Our Professional Development

Our PD consisted of three one-hour segments in one day that addressed one domain per segment. We began by sharing free and low-cost resources for teaching mathematics. Some of these included websites such as Khan Academy in Spanish (es.khanacademy.org), the Math Twitter Blog-o-Sphere MTBoS (mtbos.org) and Youcubed (youcubed.org). Afterwards, we shared upcoming local and national professional development opportunities that included conferences hosted by Nevadans Teaching English to Speakers of Other Languages (NVTESOL), National Association of Bilingual Education (NABE), National Council of Teachers of Mathematics (NCTM), TODOS: Mathematics for ALL, and Research Council on Mathematical Learning (RCML). As we were in a more rural part of the state, we also shared virtual PD opportunities like free book study materials offered by NCTM which are perfect for professional learning communities. We then shifted our focus to working with English learners by first sharing research findings from Pettit (2011), de Araujo, Roberts, Wiley and Zahner (2018), Sorto, Mejia Colindres and Wilson (2014), Yoon (2008), Penfield (1987), and Boaler (2016). Originally, when delivering the PD, we planned to delay talk of ELs since this was recommended by the EL coordinator. We also hoped that what we said about mathematics, language, and English learners would be fresh in their minds as they reflected on the PD during the closure. We decided to create one strategy for each of the three TLA domains: teacher domain, analyst domain, and user domain.

Teacher Domain: This first strategy discusses the teacher domain of the TLA. This domain addresses pedagogical knowledge that includes "general knowledge, such as how to manage a classroom or pace a lesson, as well as pedagogical content knowledge, which is your ability to present lessons in and about English in such a way that your students understand them and are engaged in your class" (Lindahl, 2015, para. 4). Additionally, part of the TLA teacher domain addresses the empathy that teachers have for the experiences EL students have. Because of all
of the layers that the teacher domain has, we developed an activity that would help teachers build empathy and observe pedagogical practices that help with language acquisition through the content area. We started our lesson by giving the teachers the following story problem only in Spanish. (We were inspired to do this after experiencing mathematics in Vietnamese as presented by Kien Sham (2014) at a regional NCTM conference.) "Pedro compró un auto a 16,430 pesos y despues de 3 años lo vendió a 12,315 pesos. ¿Cuánto diners se devaluó sa auto?" ["Pedro bought a car for $\$ 16,430$ and after three months he sold it for $\$ 12,315$. How much value did the car lose?"]. We wanted to demonstrate what language scaffolding looked like when done successfully (and not so well). For the first attempt, we gave the teachers the story problem and began to teach the lesson in Spanish with no scaffolding. Throughout the five minutes we gave them to work independently on this exercise, we were explicit that time was running out and that they needed to "hurry". After the time was up, we collected the papers. For the second part of this strategy, we gave the teachers a new story problem also untranslated. "En un aeropuerto aterriza un avión cada 10 minutes. ¿Cuantos aviones aterrizan en un día?" ["At an airport, a plane lands every 10 minutes. How many planes land in one day?"]. The second story problem was written on a piece of paper with the cognate words in Spanish underlined as shown in

Figure 2. It also had drawings that we made to represent what was going on in the problem.

We started the lesson by using total physical response (TPR) strategies that connected the vocabulary to their surroundings. For example, we modeled a plane landing with our hands and arms and then encouraged them to also model it with their bodies. With TPR, students are reacting physically to verbal cues or commands (Hounhanou, 2020). Additionally, we gave them access to an online dictionary they could use and many opted to use Google Translate. We learned from the first modeling that three out of the ten participating teachers understood Spanish. So, we paired the participants that understood Spanish with others that did not and asked them to provide support.
During this second attempt, the teachers completed the task with greater accuracy and confidence. We had a discussion of what were the language and pedagogical supports we provided the second time that allowed them to be successful. The teachers were able to point out many of those supports and discuss as a group how they could use similar ones to teach English in the content area. Even though this was a short exercise, it served as a way to build empathy for the EL students' experience in English only classrooms. Further, the experience prepared them to delve into the topic of this PD that had been problematized in the school culture.

## Figure 2

Example of word problem given to teachers to model best practices and scaffolding

2. En un aeropuerto aterriza un avión caa 10 minutos. ¿Cuantos aviones aterrizan en un die? = sands like?


Auión
Airplane

Analyst Domain: The second strategy helped teachers identify the language demands of their lessons. This activity addresses the analyst domain of TLA. Lindahl and Watkins (2015) defines the analyst domain as including, "knowledge about language, both its forms and functions. It encompasses (but is not limited to) knowledge about the structure of English, its phonemic systems, how context can change the meaning of certain words, or the way different expressions are used in context" (p. 782). Additionally, this domain includes metalinguistic awareness, which is defined as an ability to reflect on language use, compare features across two or more languages, or develop different language learning strategies (p. 179). In order to address the language demands of the analyst domain, we developed an activity where teachers analyze and reflect on a sample lesson plan, then transfer what we did to their own lesson plans. The teachers, working together with their colleagues, determine the language demands of their own lessons.

We first introduced them to Aguirre and Bunch's (2012) Language Demand in Mathematics Lessons Tool (LDML) and facilitated how to evaluate the language demands of a mathematics lesson. The LDML tool is a template that allows educators to reflect on the reading, writing, speaking, listening and representing demands on students during the launch, explore and summary of a math lesson:sper The Aguirre and Bunch chapter comes with downloadable materials including a sample mathematics lesson with a completed LDML. As a whole group, we discussed what language demands could look like in a mathematics classroom including how we can over rely on some modalities of language. Now that they were familiar with this tool, we asked them to use it in their own lesson plans.

The teachers brainstormed individually about a lesson plan that they had developed previously for their class. We asked them to use the same LDML tool to evaluate the language demands of their own lesson. As further language support, we also provided them with the "LO (language objectives) Menu" developed by Lindahl \& Watkins (2014). With this "LO Menu," the teachers can plan lessons with reading comprehension, grammar, and writing conventions taken into account. This is a reference they can continue to use and share after the PD is over because it details language demands and possible student needs. It also described how to write LO for each level of student language needs. With the LDML and LO tools,
the teachers worked in groups and supported each other in developing language objectives for their lessons. We believed that having the teachers consider lessons using the LDML tool would allow them to reflect more deeply on their students and the opportunities to interact with language that they experience in the classroom. From this experience, we learned that many of the teachers had never written or considered language objectives before, which led to an interesting discussion amongst the group as to why they are important.

User Domain: The final strategy addresses the user domain of the TLA. This domain "centers on your ability to use the language, or your language proficiency. It also includes all that goes along with being able to use a language proficiently, including knowing the sociocultural norms of the language, the different registers of the language, and how the context of some utterances can change their meaning" (Lindahl, 2015, para. 2). For this domain of the TLA, we chose to show the participants a video from our personal collection. We chose to do this because we felt this video would exemplify the domain, as well as provide another opportunity for discussion with peers. After all, we would be leaving, and they would return to relying on each other for feedback and advice. We also chose to show the video clip because it gave them a chance to see excellent language support for ELs in action and also to hear a voice other than our own.

The video we chose was from a seventh-grade mathematics classroom studying unit rate. Most of the students and the teacher are bilingual in Spanish and English. The teacher uses common grocery items to demonstrate what unit rate is and how it is calculated. In our video clip, the students ponder, in a whole class discussion, the unit rate per serving for two different sized containers of a chocolate drink mix. This video allows viewers to witness a teacher facilitating discussion with students at all levels of English language acquisition. This video prompts a discussion about how we can help students when we do not speak their language and how we can support them when we are explicitly told not to speak their native language as is often mandated in some school districts. This video highlights students struggling with words that have multiple meanings and false cognates. This video also features a teacher who speaks Spanish but struggles with academic vocabulary and has
to tease out meaning with her students. A brief excerpt appears below which demonstrates the teacher and students making meaning from language in a mathematics class. What appears in square brackets is our translation.

Teacher: So look at this 21 servings, what does servings mean? Servings, servings, servings. What does it mean? Carlos, que quiere decir servings?/ [Carlos, what does serving mean?] This one says 38 servings, ¿Qué quiere decir, servings?/ [What does serving mean?]
Carlos: [mumbles]
Teacher: ¿Qué palabra? [What word?] Carlos is from Honduras, right? Carlos is from Honduras and sometimes the words that he says in Spanish even me sometimes I don't recognize them because some words are different right. Ok, son diferentes las palabras. [Ok, the words are different].
Students: Gramos. [Grams]
Teacher: Gramos , ok gramos está acá, pero [Grams, ok but grams are over here] this is 38 servings. ¿Qué piensas tú? Mira esta,[What do you think? Look at this one], this one says 21 servings. 38 servings, 21 servings. What do you think it is, Carlos? Betti, ¿qué piensas tú? [Betti, what do you think?]
Betti: ¿Es el contenido? [Is it the content?]
Teacher: El contenido [The content]
Betti: Sí. [Yes]
Teacher: Más specific [more specific]
Betti: Um, es como todo lo quiere lleva en la caja. [Um, it's like everything that it fits in the box.]
Teacher: The word servings, servings. Ser-...
Students: Servi, servidas/ servings (note that servidas is a false cognate and it was a word made up by the teacher and students. They were referring to porción/servings)
Teacher: ¿Servidas? Treinta y ocho. ¿Servidas?/[Servings? Thirty-eight servings?]
Betti: No. [this student acknowledges that the word servidas is not a Spanish word]
Teacher: It says 38 servings and this one says 21 servings. Do you understand what it means? Lo entiendes? Quiere decir...[Do you understand? It means...]
Betti: ¿Yo se es que el contenido, no? [I know it is the content, right?]

Teacher: El contenido y cuantas personas puedes tú, you can serve [The content and how many people you can serve.]
Betti: Si las personas que puede service con las...[If the people can service with the...]
Teacher: There you go. Servings and servicio [service] are similar.
Betti: Sí. [Yes]
Teacher: Awesome. You see and that is what happens in a lot of the words, a lot of the words the meaning the translation is almost identical.

After watching the video, a few teachers commented that they were not bilingual and therefore could not see themselves offering the supports that the teacher offered the students in the video. We knew this would be a comment and showed a subsequent video of another teacher who spoke only English. In this video, the mathematics teachers focused explicitly on mathematical terminology and how to apply it to the mathematics problem on the board. We were also able to connect this to our earlier activity solving mathematics story problems in Spanish and what strategies were helpful. At this point of the PD, other teachers felt comfortable enough to join the conversation and discuss supports they considered important for the student. The belief that to be a good language teacher you have to be bilingual is both unrealistic and difficult to overcome. We like to remind teachers that with over 7,000 languages in the world, we would not expect them to know them all. Teachers usually laugh at this, but this realization is often forgotten. At this point we found it useful to share the user-friendly CUNYNYSIEB (2021) web series of video demonstrations for teachers to peruse and critically examine later on their own time.

We found the video we showed helpful to the teachers, but it is not available to the public so here are some video resources to consider in order to encourage critical observations among educators of ELs: jeffzwiers.org, colorincolorado.org, videomosaic.org, elllps.squarespace.com, and meld.sdsu.edu. Searching under the following queries on youtube.com will also yield good results; "teaching math to English learners" and "Classroom examples of English learners." In our experience, finding high quality videos that highlight classroom strategies continues to be a challenge for PD providers.

## Reflection

During the three hours of professional development, we shared numerous resources with the teachers such as NCTM's Catalyzing Change in High School Mathematics and Beyond Good teaching: Advancing Mathematics Education for ELLs. We also looked at NCTM's process standards (NCTM, 2000) and ways we encourage or discourage them through our instructional choices. Then we proceeded with the three strategies outlined in this chapter. As facilitators we felt that the teachers were respectful and interested. They appreciated the resources we shared with them which included many freely available resources from Instagram and Twitter. The teachers reported not actively utilizing social media as a way to learn about teaching resources and current research. For example, few of the teachers were familiar with growth and fixed mindset (Boaler, 2016), which has been trending in schools and mathematics for several years.

The teachers were given time to provide anonymous, written feedback to the facilitators about this PD experience. Several teachers specifically praised the opportunity to watch videos of teachers working with students, mathematics, and language. One participant asked for, "even more examples of teachers doing 'good' things." The teachers also appreciated being able to work with peers in their discipline area which was not a common practice at their campus. Another teacher shared, "I appreciate the gentle reminder that we are not teaching a subject; we are teaching students." This comment speaks directly to teacher beliefs and Yoon's (2008) findings.

Teaching mathematics equitably and maintaining high expectations for all students is at the core of the field of mathematics education. The eight Mathematics Teaching Practices allow for this and require rigorous application of language to mathematics as evidenced in, "Facilitate meaningful mathematical discourse," and, "Pose purposeful questions," just to name two (NCTM, 2014, 2018). The notion of equitable practices and high expectations is particularly emphasized for teachers of multilingual students (Pettit, 2011). Now it is time for teacher educators and administrators to have high expectations for the capacity of secondary STEM teachers
to connect with their ELs. We believe that providing PD opportunities around TLA is of utmost importance. Teachers must have opportunities to think about language critically and consider the implication that it has in the classroom.

We also recognize that developing TLA takes time and is an ongoing process. We understand that one PD is not enough to fully acknowledge all areas of TLA. It is important to continue providing PD using this framework as a base and understanding what other areas of key need our teachers have. TLA also touches upon areas such as beliefs and empathy that one PD cannot address. On reflection, we also recognize how important it is to continue to frame linguistic resources as positive and additive skills our students have. Schütte (2018) explains

It seems that one future task of mathematics teaching will entail using children's linguistic resources positively, for example allowing them to switch into their first language during group work, as well as providing them with opportunities to build linguistic competences in the principal teaching language. (p. 34)

With the current shift we are seeing in the field towards normalizing translanguaging in the classroom, we will be addressing this in future PDs. Translanguaging is defined by Garcia and Wei (2014) as "extend[ing] our traditional definitions of language and bilingualism. It refers to the ways in which bilinguals use their complex semiotic repertoire to act, to know, and to be" (p. 137). It accounts for the multiple language practices that students bring into the classroom and provides value to each of them. Translanguaging was the topic of a recent TEEM special issue of this journal.

An important takeaway was that this PD was applicable to our new group of secondary STEM teachers in a rural school district. This demonstrates the wide applicability of this PD to not only different school districts, but also to potentially different content area teachers and grade levels. Continuing the discussion on TLA and preparing teachers using this framework is the goal that we aim to further develop with our future PDs. We continue to learn from each PD given and look forward to modifying our content to better suit our teachers.

## References

Aguirre, J. M., \& Bunch, G. C. (2012). What's language got to do with it? Identifying language demands in mathematics instruction for English language learners. In S. Celedón-Pattichis \& N. G. Ramirez (Eds.), Beyond good teaching: Advancing mathematics education for ELLs (pp. 183-194). National Council Teachers of Mathematics.
Andrews, S. (2007). Teacher language awareness. Cambridge University Press.
Andrews, S., \& Svalberg, A. M-L. (2016). Teacher Language Awareness. In Cenoz, J. et al. (Eds.), Language Awareness and Multilingualism, Encyclopedia of Language and Education ( $3^{\text {rd }} \mathrm{ed}$.). Springer.
Boaler, J. (2016). Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching. Jossey-Bass.
CUNY NYSEIB (2021, April 2). Teaching bilinguals (even if you're not one): A CUNY-NYSIEB Webseries. https://bit.ly/3Y5vcjr
de Araujo, Z., Roberts, S. A., Willey, C., \& Zahner, W. (2018). English learners in K-12 mathematics education: A review of the literature. Review of Educational Research, 88(6), 879-919. https://doi.org/10.3102/0034654318798093
Garcia, O., \& L. Wei. (2014). Translanguaging: Language, bilingualism and education. Palgrave Macmillan UK.
Hounhanou, A. J. V. (2020). Promoting TPR (total physical response) method in teaching vocabulary for ELF beginners in Benin secondary schools. International Journal of Applied Linguistics and, English Literature, 9(6), 23-31.
https://doi.org/10.7575/aiac.ijalel.v.9n.6p. 23
Khan Academy. (n.d.). https://es.khanacademy.org/
Lindahl, K. (2019). Teacher language awareness development and its implications for new educators. The New Educator, 15(2), 85-100.
Lindahl, K. (2013). Exploring an invisible medium: Teacher language awareness among preservice K12 educators of English language learners. Unpublished dissertation. The University of Utah.
Lindahl, K. (2015). How aware of teacher language awareness are you? https://bit.ly/3W8IIC7
Lindahl, K., \& Baecher, L. (2015). Teacher language awareness in supervisory feedback cycles. ELT Journal, 1(70), 28-38. https://doi.org/10.1093/elt/ccv047

Lindahl, K., \& Watkins, N. M. (2015). Creating a culture of language awareness in content-based contexts. TESOL Journal, 6(4), 777-789. https://doi.org/10.1002/tesj. 223
Lindahl, K., \& Watkins, N. M. (2014). What's on the "LO" menu? Supporting academic language objective development. The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 87(5), 197-203. https://doi.org/10.1080/00098655.2014.918531
MTBoS.org. (n.d.). https://mtbos.org/
National Council of Teachers of Mathematics. (2018). Catalyzing change in high school mathematics: Initiating critical conversations. Author.
National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Author.
National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. Author.
National Governors Association Center for Best Practices \& Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Authors.
Penfield, J. (1987). The regular classroom teachers' perspective. TESOL Quarterly, 21(1), 21-39. https://doi.org/10.2307/3586353
Pettit, S. (2011). Teachers' beliefs about English language learners in the mainstream classroom: A review of the literature. International Multilingual Research Journal, 5(2), 123-147. https://doi.org/10.1080/19313152.2011.594357
Phem, K. (2014, Nov. 20). Teaching Math to English Learning Students. [Conference session]. National Council of Teachers of Mathematics 2014 Regional Conference, Houston, TX.
Pimm, D. (2018). Sixty years (or so) of language data in mathematics education. In J. Moschkovich, D. Wagner, A. Bose, J. Rodrigues Mendes, \& M. Schütte (Eds.), Language and communication in mathematics education. (pp. 11-23). ICME-13 Monographs. Springer, Cham. https://doi.org/10.1007/978-3-319-75055-2 2 2
Rowling, J. K. (1999). Harry Potter and the sorcerer's stone. Scholastic.
Schütte, M. (2018). Subject-specific academic language versus mathematical discourse. In J. Moschkovich, D. Wagner, A. Bose, J. Rodrigues Mendes, \& M. Schütte (Eds.), Language and communication in mathematics education (pp. 25-36). ICME-13 Monographs. Springer, Cham.
https://doi.org/10.1007/978-3-319-75055-2 3

Sorto, M. A., Mejía Colindres, C. A., \& Wilson, A. T. (2014). Uncovering and eliciting mathematical perceptions in linguistically diverse classrooms. Mathematics Teaching in the Middle School, 20(2), 72-77.
https://doi.org/10.5951/mathteacmiddscho.20.2.0072
Sorto, M. A., Wilson, A. T., \& White, A. (2018). Teacher knowledge and teaching practices in linguistically diverse classrooms. In J. Moschkovich, D. Wagner, A. Bose, J. Rodrigues Mendes, \& M. Schütte (Eds.), Language and communication in mathematics education (pp. 219-231). ICME-13 Monographs. Springer, Cham.
https://doi.org/10.1007/978-3-319-75055-2 16

Vogt, M. E., Echevarría, J., Short, D., \& Amy, W. M. (2013). Making content comprehensible for elementary English learners: The SIOP model (2 ${ }^{\text {nd }}$ ed.). Pearson.
Wright, W. (2015). Foundations for teaching English language learners: Research, theory, policy and practice ( $2^{\text {nd }}$ ed.). Caslon Publishing.
Yoon, B. (2008). Uninvited guests: The influence of teachers; roles and pedagogies on the positioning of English language learners in the regular classroom. American Educational Research Journal, 45(2), 495522. https://doi.org/10.3102/0002831208316200

Youcubed. (n.d.). https://www.youcubed.org/

## Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Which language skills do you find that your EL students need the most support within mathematics? What do you do to address this?
2. In Harry Potter and the Sorcerer's Stone, Dumbledore says, "There are all kinds of courage. It takes a great deal of bravery to stand up to our enemies, but just as much to stand up to our friends" (p. 306). How can we engage with colleagues that express negative attitudes towards teaching English learners?
3. The six principles of NCTM (2000) are Equity, Curriculum, Teaching, Learning, Assessment, and Technology. How can the needs of ELs be addressed in each principle?
4. Do you feel like you now have a good understanding of the three TLA domains? What can you do to develop them further?
5. The authors refer to being inspired by a powerful experiential ELL empathy demonstration at an NCTM regional conference. What can you learn from related demonstrations that have been published in TEEM (e.g., p. 24 of the Winter 2020 issue, p. 28 of the 2015 issue, or p. 10 of the 2013 issue)?


# Still TEEM-ing With Enthusiasm: A History of TODOS' esTEEMed Journal 

Lawrence M. Lesser

As we celebrate TODOS' $20^{\text {th }}$ anniversary this year, it's fitting to recall the evolution of its refereed journal, Teaching for Excellence and Equity in Mathematics (TEEM). Current Editor-in-Chief Marta Civil asked me to write this since I'm the only one who's had the pleasure and privilege of involvement with every issue, either as Editor or Associate Editor.

So far, TEEM spans 15 years, 14 volumes (one covered two years), 17 issues, 68 articles (not counting editorials, notices, poetry, etc.), and 730 pages! Beyond the numbers, TEEM has always been qualitatively distinctive by aligning with the TODOS mission, targeting diverse stakeholders (researchers, practitioners, and administrators of all levels), offering DARE questions for professional development, and attracting submissions from distinguished veterans in the field as well as from those early in their careers.

Also notable is how TEEM editors have made it a point to welcome queries, explicitly solicit submissions from teachers as well as researchers, welcome contributions in Spanish (see page 28) as well as English, and offer (via a TODOS Live session and workshops at the most recent several TODOS conferences) not just information about TEEM's process but also feedback on attendees' article ideas.

So how did it start? Cynthia Anhalt, Michael Matthews and I (we were then co-Editors of Noticias) submitted a proposal to the TODOS board in 2008 to launch a peer-reviewed journal for educators (since a separate monograph series had just launched for researchers) and then-President Nora Ramirez relayed that the Board was "in total favor." The proposed title was actually Mathematics Teaching for ALL: A TODOS Journal for Quality and Equity before my later inspiration of TEEM, which was more concise, catchy, and (we hoped) welcoming to teachers by eschewing the word "Journal."

And so, TEEM was born with Cynthia, Miriam Leiva, and I as its founding Editors. Cynthia and I handled editorial matters (with Cynthia also utilizing her graphic design talents to do layout/production) while Miriam made key contributions as a liaison to the Board and using her contacts as TODOS' founding president to secure initial funding from Pearson as well as create a database of referees. We were grateful that other TODOS publications helped build TEEM's foundation. The monograph series targeted only researchers, but established precedent for TODOS sponsoring high-quality peer-reviewed scholarship. Also, each issue of Noticias generally contained a featured article that went through a review process, and competitively-selected adapted versions of these actually comprised TEEM's debut issue (published 10/21/2009) while we launched a call for new papers to be double-blind reviewed for subsequent $T E E M$ issues.

The years have brought various changes in editors as well as in policy for the submission window/process, increased page limits, which issues are accessible to the public, starting an Editorial Board (in 2011), accepting papers in Spanish, and adding special issues (so far: social justice, multilingual learners, and antiracism). TEEM is in a period of growth, with multiple issues in 2020 and 2021 (despite the pandemic!) and will gain additional support on the Open Journal Systems platform: https://journals.charlotte.edu/teem.

TEEM has been one of my most meaningful involvements ever for professional and personal reasons (see my 2015 TEEM piece) and for our shared TODOS imperative to support excellence and equity together (for inspiration, reread the quotes inserted throughout issue \#1). It's synergistic that I joined TODOS the same year (2004) I joined UTEP, the only R1 university in the US to maintain a $100 \%$ undergraduate admission rate while building research excellence. This journal is a wonderful example of how diverse educators and scholars come together with their professionalism and passions and - without paid staff - produce a journal of such high quality and fast-growing impact. It has truly taken a village and I offer my huge gratitude to all editors, authors, and reviewers, who have served (or will serve) as terrific "teem" players! Also, advance thanks to readers who we hope will dare to share with us how they've used TEEM articles. Onward!

Mathematics for ALL

# What Does 2SLGBTQIA+ Identity and Other Non-Normative Identities Have to Do With Mathematics Teaching and Learning? 

Brandie E. Waid<br>The Queer Mathematics Teacher


#### Abstract

In the last decade or so, many educators, researchers, and national and local mathematics organizations have sought to create targeted programs, frameworks, and educational supports (for teachers and students alike) to address disparities in who we see as a "math person" and who has access to high quality mathematics. Too often, however, queer and transgender people are left out of these initiatives, frameworks, and conversations about equity and increased representation. Many times, the exclusion of queer and transgender identities from talks of equity, diversity, and inclusion comes from the ways in which queer and transgender identity is often rendered invisible, "divisive," or irrelevant in PK-12, and sometimes even PK-16, settings, particularly in mathematics. This article provides an argument as to why considering queer and transgender identity in mathematics teaching and learning is not only relevant, but essential. The article also provides considerations for making our schools and mathematics classes more queer and transgender inclusive.


Note: This article is an adaptation of a TODOS Live! session of the same title, given by Brandie E. Waid on September 21, 2022, available at https://vimeo.com/761876288.

## Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What is one question you have about the experiences of 2SLGBTQIA+ (Two Spirit, lesbian, gay, bisexual / bigender/ biromantic, transgender, queer, intersex, agender / asexual / aromantic, and other gender and sexual minorities) students in your mathematics classroom or in your school?
2. Think back to your PK-12 or PK-16 schooling. What aspects of your identity were deeply tied to the development of your mathematics identity and your learning of mathematics?
3. Write down one thought you have about how 2SLGBTQIA+ identity might be related to mathematics teaching and learning.

Brandie E. Waid, Ph.D. (they/she or elle/ella) is an independent math coach and activist-scholar. She is the founder of The Queer Mathematics Teacher (QMT) and co-founder of Radical Pedagogy Institute (RPI). She obtained her PhD in Mathematics Education from Columbia University and formerly served as an Assistant Professor of Teacher Education at Drew University. She has also taught middle and high school mathematics in both New York City and Central Florida. Brandie's research and work at QMT and RPI centers on using a queer lens to challenge traditional practices in mathematics teaching. To learn more about her work or to contact Brandie, visit the QMT website at www.TheQueerMathematicsTeacher.com or the RPI website at www.RadicalPedagogyInstitute.com

Content Warning: The author feels this article warrants a content warning, not because it deals with 2SLGBTQIA+ identity, but because it contains references to violence against queer and trans youth (as well as youth of other marginalized identities), trauma and self-harm, and state-sanctioned genocide and land theft against Indigenous peoples. If you or someone you love identifies as 2SLGBTQIA+ and/or Indigenous (or even BIPOC and/or disabled), or if you have past experiences of trauma or self-harm, please practice self-care as you engage with this article and know that the author believes in your beauty and brilliance.

# What Does 2SLGBTQIA+ Identity and Other Non-Normative Identities Have to Do With Mathematics Teaching and Learning? 

Brandie E. Waid

As I write from a coffeeshop near my home, I acknowledge that as a queer demi-woman ${ }^{1}$ of settlercolonial ancestry extending from Cuba, Lebanon, France, Spain, and England, I have benefited tremendously from living on Turtle Island, what many of us now know as the Americas. As I gaze out the window, I take in the beautiful ancestral lands of the Lenni Lenape, in an area known as Lenapehoking, shown in Figure 1.
As Figure 1 illustrates, the Lenape residing in Lenapehoking, pre-colonization, spoke three dialects, indicating three distinct, yet interrelated communities the Munsee (meaning "People of the Stony Country"), Unami (meaning "People Down River"), and Unalachtigo (meaning "People Who Live Near the Ocean") (Nanticoke Lenni-Lenape, n.d.). I reside on the ancestral lands of the Munsee Lenape, known as diplomats and warriors by many neighboring communities of Lenapehoking. some of the first Indigenous people to encounter the European colonizers. Due to violent and forced relocations Westward, as well as mass genocide, much of the present-day Lenape population resides in Oklahoma, Wisconsin, and Ontario, though there remain Lenape peoples in southern Lenapehoking. Those people are known as the Nanticoke Lenni-Lenape, with their name indicating a merging of the Nanticoke and Lenape peoples. Many of the Lenape people that remained in Lenapehoking survived by adapting to the dominant, settler colonial ways, becoming farmers and traders, while working to preserve their role as "keepers of the land."

[^1]Figure 1
Lenapehoking


Note. From Lenapehoking [Image], by Wikipedia https://en.wikipedia.org/wiki/Lenape

In reflecting on what I have come to learn about the history of the Nanticoke Lenni-Lenape peoples and their resilience and dedication to the land, community, and culture, I am drawn to the ways of a subgroup of the Lenape people-those known today as Two Spirit. To understand Two Spirit identities, we must first recognize the sacred role of land and spirituality in Indigenous cultures. The element of spirituality, rather than the body

[^2]Waid
or sex assigned at birth, was at the core of the pre-colonial Indigenous concept of gender. In line with this spiritual understanding of gender, Two Spirit persons embody both male and female spirits and are gifted with skills and abilities that transcend a gender binary. As such, many pre-colonial Indigenous cultures looked beyond a binary conception of gender, recognizing three or more genders. In those times, Two Spirit identified tribal members were considered honored members of the community and given important roles such as healers, keepers of language and culture, mediators, educators, and caregivers of orphaned children and elders (Bronski, 2019).

Unfortunately, the honored customs surrounding Two Spirit identity were considered "immoral" by European Christian colonizers, who began violently imposing their binary conception of gender and gender roles upon Indigenous communities. This led to the eradication of many Two Spirit tribal members and Two Spirit traditions, whether through direct genocide or in the forced "reeducation" of Indigenous peoples, through means such as American Indian Boarding schools. It was not until 1990 that Indigenous people of tribes across Turtle Island came together to create the unifying term "Two Spirit" ${ }^{2}$ as a broad replacement for the traditional terms that had been lost to colonization and to begin reclaiming the sacred position Two Spirited Indigenous people held within their communities (Bronski, 2019).

I begin my discussion here in this historical understanding of Two Spirit identity and Indigenous cultures for several reasons. First, I believe it is essential that we authentically engage with the historical context, especially that of Indigenous land theft and cultural genocide, of the lands on which we occupy. Second, I find that this historical account provides context for the continued violence that Two Spirit, lesbian, gay, bisexual / bi-gender / biromantic, transgender, queer, intersex, agender / asexual / aromantic, and other gender and sexual minority youth (which I will henceforth refer to by using the acronym 2SLGBTQIA +$)^{3}$ face in our PK-16 systems and greater society.

[^3]
## Experiences of 2SLGBTQIA+ Students

Over the last few decades, our institutions and governments have moved from more overt campaigns of state sanctioned violence against non-dominant / nonnormative identities (i.e., those who are not White, cisgender, heterosexual, Christian, able-bodied, and, to some extent, men) through measures such as cultural genocide (e.g., the use of American Indian Boarding Schools. As a means of mass genocide of Indigenous peoples and their cultures), land theft and slavery, to more covert mechanisms of violence that exist to erase or silence non-dominant / non-normative identities. Schools are not exempt from these covert mechanisms of oppression. In truth, our schools remain sites of violence for many queer and trans Black, Indigenous, and people of color (QTBIPOC), disabled individuals, and Black, Indigenous, and people of color (BIPOC) that do not identify as queer, trans, or disabled. To illustrate this, one only need to turn to the results of GLSEN's ${ }^{4}$ National School Climate Survey (Kosciw et al., 2020), administered every two years. As of 2019 (the most recent data available at the time of this writing), $86 \%$ of 2SLGBTQIA+ students surveyed in secondary schools indicated within the last year they had been harassed or assaulted at school "due to personal characteristics, including sexual orientation, gender expression, gender, and actual or perceived race and ethnicity, religion, and disability" (p. 28).

As we are mathematics educators, let's think about this in terms of total number of students. In 2019, the population of 2SLGBTQIA+ students in secondary schools are members of Generation Z. It has been estimated that approximately $21 \%$ of Generation Z identify as 2 SLGBTQIA+ (Jones, 2022) ${ }^{5}$. If there were approximately 15.3 million students enrolled in high schools across the United States in 2019 (Bouchrika, 2022) and $21 \%$ of those students were 2SLGBTQIA+ identified, then that would mean that we might expect approximately $3,213,000$ high school aged students to identify as 2 SLGBTQIA+. Calculating $86 \%$ of those

[^4]Waid
2SLGBTQIA+ students, we would estimate that some 2,763,180 2SLGBTQIA+ students were harassed or assaulted in U.S. secondary schools in 2019 based on "personal characteristics." That is a lot of students... and these are only the numbers for students who identify as 2SLGBTQIA+ students, not those who do not identify as 2SLGBTQIA+, but are BIPOC, and/or disabled, nor those who are perceived to be 2SLGBTQIA+ due to nonnormative gender expression ${ }^{6}$.

The educational impact of hostile school climates cannot be understated and is compounded for students whose identities exist at the intersections of multiple systems of oppression ${ }^{7}$. The cumulative impact of hostile school environments and discrimination leads to higher levels of school truancy, lower GPAs, and lower selfesteem (Kosciw et al., 2020). It has also been linked to the overrepresentation of 2SLGBTQIA+ students, especially QTBIPOC students and queer and trans students with disabilities, that experience adolescent homelessness (Robinson, 2021), engage in risky behavior (e.g., drug use, self-harm) (Garnett et al., 2014; Hatchel and Marx, 2018), and that enter the school-to-prison pipeline as adolescents (Snapp et al., 2015). Furthermore, we cannot assume that experiences of hostile school climates are limited to high-school aged 2SLGBTQIA+ students, since normative identities are taught and reinforced from the day students enter schooling. As Keenan (2017) notes, schools work to categorize children's bodies in various ways (by race, ethnicity, ability, gender, intelligence, behavioral traits, language proficiency, etc.) from the moment they set foot in preschool. Keenan writes that the words used for these categories "are used to sort out who is 'normal' and who is 'different'" (p. 540). In terms of the categories used to sort "normal" versus "different" gender identities (and later sexual orientation), Keenan writes,

Although Dick and Jane books and the practice of walking children down hallways in parallel boys' and girls' lines have faded from view in most US schools, children continue to be taught that being a girl means one set of behaviors and roles associated with growing up into a woman (e.g., playing with other

[^5]girls, being sexually attracted to boys, caretaking, expressing emotion, wearing dresses) and being a boy means another, quite different set of behaviors and roles associated with growing up into a man (e.g., playing with other boys, being sexually attracted to girls, being physically active and aggressive, never wearing dresses). All of this works together to teach children a script about which kinds of bodies are normal and which are not (p. 540).
In failing to address this hidden curriculum (or "script") as it develops at the earliest grades of schooling and beyond, we are creating educational environments in which 2SLGBTQIA+ identity is taboo, something unusual to be feared. As such, we should not be surprised that the result is a hostile and unsafe school experience for those that identify as 2SLGBTQIA+, are part of 2SLGBTQIA + families, or are perceived to be 2SLGBTQIA+.

While the results of GLSEN's 2021 National Climate Survey (http://bit.ly/3Y2XGKa) were not available at the time of this writing, it is anticipated that the finding for 2021 will be no better, given our current sociopolitical climate, which has become increasingly hostile toward 2SLGBTQIA+ identities, as well as other identities that are not White, cisgendered, heterosexual, and male. The result of these hostilities to non-normative identities has led to a rising number of:

- "Don't Say Gay" bans that limit or completely forbid the discussion of 2SLGBTQIA+ issues, people, and identity.
- "Critical Race Theory" bans that limit or completely forbid discussion of or inclusion of accurate representations of historical or presentday issues of race, racial justice, racial bias, and discussions of social and emotional learning.
- the repeal of Roe v . Wade and the accompanying concurring opinion from Justice Clarence Thomas that urged that various key 2SLGBTQIA+ civil rights cases (e.g., marriage equality, decriminalization of same sex relationships) be reconsidered and overturned.
- legislative attacks on families of transgender students who have supported their transgender
approximately $4 \%$ experienced racial harassment in the past school year. GLSEN also found that just over $60 \%$ of 2SLGBTQIA+ undocumented students felt unsafe at school because of their citizenship status in 2019, a sharp increase from the approximately $40 \%$ in 2017.
teens in their desire to socially or medically transition.
- transgender sports bans, which predominantly target transgender girls, prohibiting transgender students from participating in school sports that align with their gender identity.
- legislative attempts to increase Immigration Customs Enforcement (ICE) raids and remove the sanctuary status of institutions such as schools.

Some may assume that 2SLGBTQIA+ youth (or possibly all youth) are not paying attention to these issues, however, there is evidence to the contrary. The Trevor Project (2022) found that $94 \%$ of surveyed 2SLGBTQIA+ identified youth reported that the current political climate had negatively impacted their mental health and expressed particular concern about the rights of Black and trans folx ${ }^{8}$.

## Impact of Trauma on Learning and Cognition

Why does all this matter and what does it have to do with 2SLGBTQIA+ students in PK-16 mathematics? To understand these questions, one first needs to understand the impact of trauma on learning and cognition. I define trauma here in a similar fashion as that of the Substance Abuse and Mental Health Services Administration (2014), which states, "Individual trauma results from an event, series of events, or set of circumstances that is experienced by an individual as physically or emotionally harmful or life threatening and that has lasting adverse effects on the individual's functioning and mental, physical, social, emotional, or spiritual well-being" (p. 7). 2SLGBTQIA+ students who are disproportionally impacted by discrimination, harassment, and assault at schools, in addition to the rejection (and violence) that many 2SLGBTQIA+ students face from their families and communities, would fall under this definition of trauma (McCormick et al., 2018), as would students who experience forms of systemic trauma due to their Black, Brown, Indigenous, and/or disabled identities (Venet, 2021).

[^6]In response to this trauma, the brain develops trauma responses that serve to help an individual cope. Psychotherapist and trauma specialist Resmaa Menakem (2017) elaborates on this, writing,

Trauma responses are unique to each person. Each response is influenced by a person's particular physical, mental, emotional, and social makeupand, of course, by the precipitating experiences themselves. However, trauma is never a personal failing, and it is never something a person can choose. It is always something that happens to someone. A traumatic response usually sets in quickly-too quickly to involve the rational brain. Indeed, a traumatic response temporarily overrides the rational brain (pp. 7-8).
Contrary to popular belief, trauma responses are not only the result of a single traumatic event (though that is possible as well), but they can also develop from a series of smaller wounds, sustained over time ${ }^{9}$ (Menakem, 2017; Venet, 2021). This indicates that trauma responses might form in response to the "slow violence" (Gutiérrez, 2018) of multiple years of microaggressions on both the systemic and individual level.

In addition to allowing individuals to cope, trauma responses also serve as a protective mechanism in any situation that has the potential to retraumatize an individual, whether that potential is realized or merely perceived. To illustrate this, Hammond (2015) discusses what happens to our brains when we are in such an environment or situation. Hammond describes how our autonomic nervous system is programmed to scan our environment for threats and send that information to the Reticular Activating System and amygdala, which in turn send a "distress signal" to our body, which begins developing stress hormones. These hormones significantly hinder our ability to learn. Hammond writes,

Even if the environment isn't hostile but simply unwelcoming, the brain doesn't produce enough oxytocin and begins to experience anxiety. This anxiety triggers the sympathetic nervous system, making one think [they] are in danger because the brain doesn't experience a sense of community....When we look at the stress some
like us" (Kapitan, 2016). Thus, my use of this term intricately tied to my Latinx, Queer identity.
${ }^{9}$ This is known as Complex Post-Traumatic Stress Disorder (C-PTSD).
students experience in the classroom because they belong to marginalized communities because of race, class, language, [sexual orientation], or gender [identity], we have to understand their safety-threat detection system is already cued to be on alert for social and psychological threats based on past experience. It becomes imperative to build positive social relationships that signal to the brain a sense of physical, psychological, and social safety so that learning is possible (p. 45).
As we can see from Hammond's quoted text, if we are not providing environments where 2SLGBTQIA+, BIPOC, and disabled students can not only feel safe (physically, psychologically, and socially), but also that foster a sense of belonging, then the conditions necessary to foster learning have not been met.

During adolescence, we know that the brain is still in the early stages of development and considered to be most "plastic." As such, the impact of traumatic experiences in adolescence has very real consequences to our brain structures. It is nearly impossible to escape such brainaltering trauma for 2 SLGBTQIA+ students, who are bombarded with messages that who they are is unacceptable, unwanted, and even despised, both inside and outside of school. This also extends to BIPOC and disabled folx, whether they are 2SLGBTQIA+ or not. The impact of this trauma has significant consequences in terms of student learning. McInerney and McKlindon (2014) write, "Children who have experienced trauma may find it more challenging than their peers to pay attention and process new information, and evidence suggests that some of these children develop sensory processing difficulties ..." (p. 4). They also note that for adolescents that have experienced such trauma, there are noted impacts on the way that they interact with others, either withdrawing from social situations or becoming more negative and bullying other students. Past trauma may also lead students to become distrustful of teachers and authority figures in general and could impact the development of problem-solving skills, creativity, and curiosity (Nelson et al., 2022), which are skills and dispositions that align with competencies emphasized in high quality mathematics teaching (National Research Council, 2001).

## Impact of Trauma on Mathematics Learning and Cognition

Thinking in terms of mathematics specifically, Seda and Brown (2021) discuss how "math classrooms are especially [emphasis mine] prone to traumatizing events because of the emphasis on speed and accuracy, individual performances, student labeling, one-shot tests, and a long history of cultural negativity" (p. 69). Many of these practices point to a mathematics that is gendered (as masculine), heterosexual, and White (Esmonde, 2011; Gutiérrez, 2017; Leyva, 2017; Martin, 2012; Mendick, 2006; Waid et al., 2021; Yeh \& Otis, 2019; Yeh and Rubel, 2020). In addition to the above practices of pedagogical trauma, 2SLGBTQIA+ students also experience curricular violence / trauma in their PK-16 schooling by having their identities erased, portrayed as trivial or complicating things, or irrelevant to mathematics contexts (Bright, 2016; Parise, 2021; Rubel, 2016; Waid, 2020; Waid, 2021; Waid \& Turner, 2021).

The above examples of pedagogical and curricular violence / trauma that 2SLGBTQIA+ students experience in mathematics as a discipline might help to explain why 2SLGBTQIA+ individuals are underrepresented at every level of PK-16 mathematics and in the STEM workforce. In PK-12 schools, 2SLGBTQIA+ students are less likely to take Algebra II (Whipple, 2018), often seen as a gatekeeper to higher level mathematics; in post-secondary settings, 2SLGBTQIA+ students are less likely to major in STEM (Greathouse et al., 2018), and those who choose a STEM major are less likely to persist in that major (Hughes, 2018). Similarly, after graduating with a STEM degree, 2SLGBTQIA+ students are less likely to enter STEM fields (Freeman, 2020) and, if they do, are less likely to remain in those fields (Cech and Waidzunas, 2021). All of this points to a field of mathematics (and STEM more broadly) that is inherently hostile to 2SLGBTQIA+ identities.

## What Can Mathematics Educators Do?

So, what can mathematics educators do? My first recommendation is two-pronged. For current

Waid
2SLGBTQIA+, BIPOC, and disabled students ${ }^{10}$ in our classes and schools (i.e., those who have already experienced harm in our school and societal systems), we must adopt trauma-informed practices that center equity and social justice. Venet writes " $[\mathrm{t}]$ rauma-informed educational practices respond to the impact of trauma on the entire school community and prevent further trauma from occurring. Equity and social justice are key concerns of trauma-informed educators as we make changes in our individual practice, in classrooms, in school, and in district-wide and state-wide systems." (p. 10). Venet also states that centering these practices in equity means "ensuring that all students can access high-quality education, that they are fully included in their school communities, that they are able to engage in meaningful and challenging academic work, and that they can do all of this in an environment that values them as people" (p. 22).

Using this foundation of equity and trauma-informed practice, Venet identifies a framework of four traumainformed priorities that we might use in our decisionmaking as teachers: predictability, flexibility, empowerment, and connection. A description of each of these priorities, along with examples of what they might look like in a mathematics classroom, are depicted in the sections below ${ }^{11}$.

## Predictability

Predictability refers to structures or routines that are consistent, while also maintaining flexibility (the next priority). These structures help those impacted by trauma find grounding and comfort / safety. One example of predictability in mathematics that I have described elsewhere (Waid, in press) is the use of regular check-ins at the end of each class period or at the end of each week. The check in, which usually appears in the form of an exit ticket, asks students to reflect on the question "What are you thinking or feeling in relation to what we learned today?" When introducing this activity at the start of the school year, I inform students that their reflection can take whatever shape makes most sense to them, meaning they can reflect on the content of what they learned (e.g., "I'm

[^7]still struggling with..., "I feel really good about...."), my pedagogy (e.g., "I really like the ___ strategy you used because ...," "I didn't find ___ strategy helpful because ..."), classroom dynamics (e.g., "I'm finding it hard to work with _ because...," "I worked well with because..," "I liked working alone on this assignment because...," "I liked working in groups on this assignment because...), their affect (e.g., "I was having a bad day today," "I was struggling to pay attention because I didn't sleep well last night") or anything else they find relevant. I find that the predictability of this activity and the question structure provides students with a sense of comfort, while also allowing them to engage in authentic reflection and communicate their needs to me so that I might be more responsive in future lessons. Other examples of predictability in mathematics might include utilizing community agreements or group work norms, having standard procedures for retaking or resubmitting work, outlining exactly what student performance will be evaluated upon (through a rubric or some other means), the use of predictable math routines (e.g., Quick Images, Choral Counting, I Notice, I Wonder, Which One Doesn't Belong), and so on.

## Flexibility

Flexibility is about meeting students where they are and prioritizing their humanity over all else. While maintaining high academic standards is important, allowing for flexibility in how students meet those standards is also essential. Likewise, while providing structure (which might also fall under the above priority of predictability) is important, being flexible with policies in ways that are context-specific is a hallmark of traumainformed practice. Seda and Brown's (2021) discussion of the use of "Totally Ten Choice Boards" provides an example of flexibility in mathematics teaching practice. On a Totally Ten Choice Board, "[s]tudents choose the questions, worth varying number of points, they want to answer, as long as their points total to a certain number" (pp. 151-152). They further note that " $[t]$ eachers can specify that students choose at least one question for each level" (p. 152).
as 2SLGBTQIA+, BIPOC, and/or Disabled, in our classrooms, even if those identities have not been disclosed.
${ }^{11}$ Special shout out/thank you to Alex Venet for looking these examples over and providing feedback!

## Waid

In terms of policy in the mathematics classroom, one example of flexible policy might be the implementation of flexible or rolling deadlines that do not penalize students when they are not prepared to submit work by the suggested deadline date. Other examples of flexibility in mathematics might include differentiating instruction, utilizing low floor-high ceiling math tasks, eliminating zero tolerance policies, standards-based grading or ungrading, and implementing choice in other areas such as seating arrangements, options for collaborations, etc.

## Empowerment

The priority of empowerment, which is deeply tied to student agency, is nicely captured in a 2019 tweet by Venet, which says "I like to say: empowerment in a trauma-informed context doesn't mean 'feeling good about yourself.' It means literally giving power TO, sharing power WITH." One method of empowering students that I have utilized in my mathematics classrooms, is the use of routines such as "I Notice, I Wonder." In a summer camp I run for 2SLGBTQIA+ youth entering grades $9-12$, I have used this routine to develop an entire unit of study, based on students posed questions. For example, one summer I knew I wanted to have students learn about various types of functions, so I provided them with a Statista (2021) graph, which showed the results of a Gallup Poll that asked respondents if they identified as LGBT. The graph, shown in Figure 2, showed respondent answers, by generation, in the survey's 2014, 2017, and 2020 administrations. Knowing the students would pose a question related to the overall percentage of individuals that identified as LGBT in each of these years, I also had a second graph (Statista, 2022), shown in Figure 3, ready to complete a second round of the "I Notice, I Wonder" routine, one that would allow students to begin posing questions about predicting identification as LGBT in future years. By selecting these images, and steering students toward questions of future prediction, students felt empowered in their role in contributing to the overall design of the lessons that followed - lessons centering on various types of functions, function families (especially linear, polynomial,

[^8]exponential, and logarithmic, and rational functions), and transformations of those functions. ${ }^{12}$

Similarly, Seda and Brown (2021), discuss a modified use of the "I Notice, I Wonder" that might happen in a single lesson that is already planned (rather than a sequence of lessons as I describe in my previous example). This modified routine has students write five factual things about the graph or problem they are noticing and wondering about, as well as two questions about the graph or problem. Students' questions and observations are recorded on the board, without judgment. Students' observations and questions can then be used as a transition into the day's lesson, structured in a way that honors their observations and questions. At the end of the lesson, Seda and Brown recommend "revisit[ing] the original statements and ask[ing] students to respond by saying, 'I agree, because...' or 'I disagree because...'" (p. 127). Other examples of empowerment in mathematics might include co-constructing community agreements and group work norms with students, asking students for regular feedback on your teaching (what's

## Figure 2

Percentage of U.S. Adults, by Generation, who Identify as LGBT (Statista, 2021)


## Figure 3

Overall Percentage of U.S. Adults who Identify as LGBT (Statista, 2022)

working, what's not working?) and adapting accordingly, utilizing student interest to develop mathematics tasks and projects, implementing choice in assignments or other areas such as seating arrangements, options for collaborations, and so on.

## Connection

Connection refers to the building of community, collaboration, and relationships. One practice that fosters connection, which I have used in my own mathematics classroom and discussed elsewhere (Waid, in press), involves placing emphasis on community over content in mathematics teaching and learning. Currently, teachers feel constrained by the content they must "cover" in mathematics. This tension often leads teachers to spend the first week or so of their classes on community building activities and focusing on dispositions and then moving on to center mathematics content, without revisiting these essential community building practices. In my chapter for TODOS's forthcoming book centered on the themes outlined in TODOS's position statement The Mo(ve)ment to Prioritize Antiracist Mathematics (2020), I discuss this practice by drawing upon the following metaphor (manuscript p. 8),
"If these activities are the sole efforts of community building, then this can be likened to the process of digging a hole, placing a plant in the ground, and watering it once, with the hope that these actions will be enough to sustain the plant. For most plants,
such actions are not enough. The same is true for students when we are building community."

I then discuss how I use bi-weekly "community agreement check-ins" that allow students to reflect on which community agreements need greater attention and support (as well as if community agreements need modification). Based on the results of this check-in, I develop a participation quiz (similar to those described in Cohen \& Lotan, 2014) which lists behaviors that embody the identified community agreement. For example, if students have selected a community agreement that centers on active listening, I would provide a list of behaviors that are indicative of active listening such as, positioning one's body to indicate you are listening, asking follow-up questions for understanding, rephrasing someone's understanding, etc. These behaviors would then be used by the entire classroom community to provide feedback over the next few weeks, with constructive feedback being shared in all directions: from teacher to student, student to student, and student to teacher. Other examples of connection in mathematics might include using collaborative group work and discourse rich practices such as Smith and Stein's (2018) five practices structure, providing time and mechanisms to check-in with students and allow them to engage in self-reflection (possibly through the check-ins I described in my example under the priority of predictability), utilizing student interests and cultural funds of knowledge to develop mathematics tasks and projects, etc.

## Waid

In addition to the classroom practices mentioned above, teachers must also work toward shifting overall school culture and dismantling the mechanisms of systemic trauma that harm BIPOC, disabled, and 2SLGBTQIA+ students. For example, in terms of 2SLGBTQIA+ identity, GLSEN has consistently found the following four practices to support an affirming school climate (Kosciw et al., 2020):

- The presence of a school-sanctioned gender sexuality alliance
- Comprehensive anti-discrimination and antibullying policies that explicitly include 2SLGBTQIA+ identity as protected
- Hiring 2SLGBTQIA+ affirming and identified teachers and staff
- Implementing 2SLGBTQIA+ inclusive curriculum across all content areas. ${ }^{13}$

While there is evidence that simply affirming a student's 2SLGBTQIA+, disabled, and/or BIPOC identity can lead to the development of a stronger, more positive mathematics identity (Fischer, 2013; Aguirre et al, 2013), we must go beyond this. This is where my second, more mathematics-specific, recommendation comes in. While creating affirming, inclusive environments and implementing inclusive mathematics curriculum is essential, we also need to reconsider our preconceived notions about mathematics. Present notions are heavily influenced by European ideologies of White Supremacy, heteropatriarchy, and ableism, and have led us to a field of mathematics dominated by Cisgendered, heterosexual, able-bodied, White men (Leyva, 2017; Gutiérrez, 2013; Lambert et al., 2018).

How do we begin reimagining and expanding the limits of who and what is considered mathematical? For me, the answer lies in learning from Indigenous, disabled, and queer and trans communities (especially QTBIPOC and disabled queer and trans communities). Against all odds, these communities, particularly young people, and those living at the intersection of interlocking systems of oppression, have found ways to build community, find joy, and thrive. In the face of being told who they should be, less they face the wrath of society, many in these communities have continued to find ways to be authentic and forge new paths, while "failing" to align with societal
norms. In thinking about this type of "failing," Queer author Jack Halberstam writes, "under certain circumstances failing, losing, forgetting, unmaking, undoing, unbecoming, not knowing may in fact offer more creative, more cooperative, more surprising ways of being in the world. Failing is something queers do and have always done exceptionally well" (pp. 2-3). The same can be said of BIPOC and disabled folx, in terms of "failing" to bend to normative, dominant ideologies. Elaborating on this point, scholars Engel and Lyle (2021) write, "these alternatives regularly require that we get lost-as it were- and fail to walk prescribed lines in thought and in body. In short, failure means refusing to practice endorsed modes of thinking and doing and being to see what else comes into view when one detours nonnormativity" (p. 15).

What might mathematics look like if we follow the example of queer and trans, disabled, Indigenous, and QTBIPOC individuals, stepping off the well-travelled path that our past mathematical experiences taught us was "acceptable" and wandering into the weeds to see what new possibilities come into view? One such vision of mathematics comes from a series of lessons I developed for "Camp" of Mathematical Queeries, a camp I design and implement every summer for 2SLGBTQIA+ identified youth entering grades 9-12. Throughout "camp" experiences, I attempt to infuse cross-disciplinary lessons that allow students to see the connection between mathematics, history, English, and queer culture and identity. One such lesson began with the posing of the question, "How many ways are there to be queer?" This question, simple, yet profound, provides students an opportunity to gain a sociocultural and historical perspective of models that have been used by scientists (e.g., Kinsey Scale, Benjamin Scale, and Storms Axes) and gender theorists (e.g., Genderbread person and Gender Unicorn) to "mathematically" understand queer and trans identities. The question also provides multiple avenues of mathematical investigation such as:

- reviewing the number line and various number systems (e.g., real, irrational, rational, integers)
- introducing mathematical terms such as line, ray, segment, continuous, discontinuous, and hole

[^9]- exploring concepts of sets, including finite vs infinite sets, as well as cardinality
- developing an understanding of theoretical versus experimental sample spaces and probability
- investigating appropriate mathematical notations, including interval notation and inequalities.
For example, the Gender Unicorn Model, shown in Figure 4, depicts several rays that one might use to identify their gender identity, gender expression, physical attraction, and romantic attraction. Introducing this model a provides an opportunity to introduce or reinforce the vocabulary of a ray (as opposed to a line or segment), and an understanding of mathematical notations that might describe the scale (e.g., ranging from 0 , inclusive, to infinity). Next, students use their mathematical and sociocultural / historical knowledge to provide critiques
of the gender and sexuality models and propose new, more inclusive models that build on their mathematical understandings and their belief about how many ways there are to be queer.

As mathematics educators, we must collaboratively brainstorm ideas, such as the one offered above, which transcend the boundaries of what we consider "traditional" mathematics. We must also collaboratively brainstorm how such ideas might be realized in our classrooms, given the limitations of our current education system, as well as in our work of challenging and dismantling those systems. This sort of radical freedom dreaming (Love, 2019) is needed to create mathematics classrooms that are truly equitable for 2SLGBTQIA+, Black and Brown, and disabled students alike, as well as all PK-16 students.

## Figure 4

The Gender Unicorn.


Note. From The Gender Unicorn [Image], by Trans Student Educational Resources.
https://transstudent.org/gender/

## A Call to Action

For those interested in exploring other avenues of reimagining mathematics by honoring the wisdom of queer, trans, and Indigenous ways of being in the world, TODOS: Mathematics for ALL has two special interest groups, TOD $\forall \mathrm{XS}=$ (pronounced "toe dax equals") and Educators of Native American Students (EONAS), designed for us to come together and realize this potential. Come join us in the reimagining of mathematics education, as we work to better support all our students, especially those that identify outside the norm and dare to transgress societal boundaries. If you are already a TODOS member and want to join TOD $\forall \mathrm{XS}=$ or EONAS, please visit https://bit.ly/TODOS Add; otherwise, please visit https://www.todos-math.org/join-todos- and click "join now" and make sure to select TOD $\forall \mathrm{XS}=$ and/ or EONAS as you complete the application process.

## References

Aguirre, J. M., Mayfield-Ingram, K., \& Martin, D. B. (2013). The impact of identity in $K-8$ mathematics: Rethinking equity-based practices. National Council of Teachers of Mathematics.
Bouchrika, I. (2022, September 13). 101 American school statistics: 2021/2022 data trends and predictions. Research.com. https://bit.ly/3kxKTR1
Bright, A. (2016). The problem with story problems. Rethinking Schools, 30(4), 14-19.
Bronski, M. (2019). A queer history of the United States for young people (Vol. 1). Beacon Press.
Cohen, E. G., \& Lotan, R. A. (2014). Designing groupwork: Strategies for the heterogeneous classroom ( $3^{\text {rd }}$ ed.). Teachers College Press.
Cech, E. A., \& Waidzunas, T. J. (2021). Systemic inequalities for LGBTQ professionals in STEM. Science Advances, 7(3), eabe0933.
Engel, S. M., \& Lyle, T. S. (2021). Disrupting dignity: Rethinking power and progress in LGBTQ Lives. New York University Press.
Esmonde, I. (2011). Snips and snails and puppy dogs' tails: Genderism and mathematics education. For the Learning of Mathematics, 31(2), 27-31.
Fischer, D. J. (2013). Out 4 math: The intersection of queer identity and mathematical identity. (Doctoral dissertation, Drexel University).
Freeman, J. B. (2020). Measuring and resolving LGBTQ disparities in STEM. Policy Insights from the Behavioral and Brain Sciences, 7(2), 141-148.

Garnett, B. R., Masyn, K. E., Austin, S. B., Miller, M., Williams, D. R., \& Viswanath, K. (2014). The intersectionality of discrimination attributes and bullying among youth: An applied latent class analysis. Journal of Youth and Adolescence, 43(8), 1225-1239.
Greathouse, M., Brckalorenz, A., Hoban, M., Huesman, R., Rankin, S., \& Stolzenberg, E. B. (2018). Queerspectrum and trans-spectrum student experiences in American higher education: The analyses of national survey findings. Rutgers Tyler Clementi Center.
Gutiérrez, R. (2013). Why (urban) mathematics teachers need political knowledge. Journal of Urban Mathematics Education, 6(2), 7-19.
Gutiérrez, R. (2017). Why mathematics (education) was late to the backlash party: The need for a revolution. Journal of Urban Mathematics Education, 10(2), 824.

Gutiérrez, R. (2018). Introduction: The need to rehumanize mathematics. In I. Goffney, R. Gutiérrez, \& M. Boston (Eds.), Rehumanizing mathematics for Black, Indigenous, and Latinx students (pp. 1-10). National Council of Teachers of Mathematics.
Halberstam, J. (2011). The queer art of failure. Duke University Press.
Hammond, Z. L. (2014). Culturally responsive teaching and the brain: Promoting authentic engagement and rigor among culturally and linguistically diverse students. Corwin.
Harris, M., \& Fallot, R. (2009). Creating cultures of trauma-informed care (CCTIC): A self-assessment and planning protocol.
https://bit.ly/3iTXYUL
Hatchel, T., \& Marx, R. (2018). Understanding intersectionality and resiliency among transgender adolescents: Exploring pathways among peer victimization, school belonging, and drug use. International Journal of Environmental Research and Public Health, 15(6), 1289.
Hughes, B. E. (2018). Coming out in STEM: Factors affecting retention of sexual minority STEM students. Science Advances, 4(3), eaao6373.
Jones, J.M. (2022, February 17). LGBT identification in U.S. ticks up to 7.1\%. Gallup. https://bit.ly/3XPzFWJ
Kapitan, A. (2016, September 12). Ask a radical copy editor: Folx. Radicalcopyeditor.com. https://bit.ly/3J6ABli
Kean, E. (2021). Advancing a critical trans framework for education. Curriculum Inquiry, 51(2), 261-286.
Keenan, H. B. (2017). Unscripting curriculum: Toward a critical trans pedagogy. Harvard Educational Review, 87(4), 538-556.

Kosciw, J. G., Clark, C. M., Truong, N. L., \& Zongrone, A. D. (2020). The 2019 national school climate survey: The experiences of lesbian, gay, bisexual, transgender, and queer youth in our nation's schools. Gay, Lesbian and Straight Education Network (GLSEN).
Lambert, R., Tan, P., Hunt, J., \& Candela, A. G. (2018). Rehumanizing the mathematics education of students with disabilities; Critical perspectives on research and practice. Investigations in Mathematics Learning, 10(3), 129-132.
Leyva, L. A. (2017). Unpacking the male superiority myth and masculinization of mathematics at the intersections: A review of research on gender in mathematics education. Journal for Research in Mathematics Education, 48(4), 397-433.
Love, B. L. (2019). We want to do more than survive: Abolitionist teaching and the pursuit of educational freedom. Beacon Press.
Martin, D. B. (2012). Learning mathematics while Black. The Journal of Educational Foundations, 26(1\&2), 47-66.
McCormick, A., Scheyd, K., \& Terrazas, S. (2018). Trauma-informed care and LGBTQ youth: Considerations for advancing practice with youth with trauma experiences. Families in society, 99(2), 160-169.
McInerney, M., \& McKlindon, A. (2014). Unlocking the door to learning: Trauma-informed classrooms and transformational schools. Educational Law Center. https://bit.ly/3H2STkS
Menakem, R. (2017) My grandmother's hands: Racialized trauma and the pathways to mending our hearts and bodies. Central Recovery Press.
Mendick, H. (2006). Masculinities in mathematics. McGraw-Hill Education (UK).
Nanticoke Lenni-Lenape (n.d.) Our tribal history... https://nanticoke-lenape.info/history.htm
National Research Council, \& Mathematics Learning Study Committee. (2001). Adding it up: Helping children learn mathematics. National Academies Press.
Nelson, K., Peterson, K. McMillin, L., \& Clarke, K. (2022). Imperfect and flexible: Using traumainformed practice to guide instruction, Libraries and the Academy, 22(1), 177-197.
Parise, M. M. (2021). Gender, sex, and heteronormativity in high school statistics textbooks. Mathematics Education Research Journal, 33(4), 757-785.
Robinson, B. A. (2021). "They peed on my shoes": foregrounding intersectional minority stress in understanding LGBTQ youth homelessness. Journal of LGBT Youth, 2021 (May), 1-17.

Rubel, L. H. (2016). Speaking up and speaking out about gender in mathematics. The Mathematics Teacher, 109(6), 434-439.
SAMHSA Trauma and Justice Strategic Initiative (2014). SAMHSA's concept of trauma and guidance for a trauma-informed approach. Substance Abuse and Mental Health Services Administration. https://bit.ly/3XKoNdj
Seda, P., \& Brown, K. (2021). Choosing to see: A framework for equity in the math classroom. Dave Burgess Consulting, Inc.
Smith, M., \& Stein, M. K. (2018). 5 Practices for orchestrating productive mathematics discussion. National Council of Teachers of Mathematics.
Snapp, S. D., Hoenig, J. M., Fields, A., \& Russell, S. T. (2015). Messy, butch, and queer: LGBTQ youth and the school-to-prison pipeline. Journal of Adolescent Research, 30(1), 57-82.
Statista (2021). 5.6 percent of U.S. adults identify as LGBT. https://bit.ly/3GXMQOI
Statista (2022). Americans who identify as gay, lesbian, bisexual, or transgender 2012-2020. https://bit.ly/3DtWA2j
Stewart-Ambo, T., \& Yang, K. W. (2021). Beyond land acknowledgment in settler institutions. Social Text, 39(1), 21-46.
Thomas, H. (2021, October 8). Why our trauma-informed teaching must be more culturally responsive. EdSurge. https://bit.ly/3iX1sWy
TODOS Mathematics for All. (2020). The mo(ve)ment to prioritize antiracist mathematics: Planning for this and every school year. https://bit.ly/3j5Yvip
Venet, A. S. [@AlexSVenet]. (2019, February 13). I like to say: empowerment in a trauma-informed context doesn't mean 'feeling good about yourself.' It means literally giving power to, sharing power with. [Tweet]. Twitter. https://bit.ly/3wprmW1
Venet, A. S. (2021, April 10). Equity-centered, traumainformed teaching with Alex Venet (No. 90) [Audio podcast episode]. In Human restoration project. https://bit.ly/3J6RZ9x
Venet, A. S. (2021). Equity-centered trauma-informed education. W.W. Norton \& Company.
Waid, B. E. (n.d.). American identification as gay, lesbian, bisexual, transgender or something other than heterosexual. The Queer Mathematics Teacher. https://bit.ly/3Hbih9A
Waid, B. E. (n.d.). Mathematical queeries: A hub of free, QMT created math activities. The Queer Mathematics Teacher. https://bit.ly/MathQueeriesHub
Waid, B. E. (2020). Supporting LGBTQ+ students in K12 mathematics. Mathematics Teacher: Learning and Teaching PK-12, 113(11), 874-884.

Waid, B. E. (2020). Supporting LGBTQ+ students in K12 mathematics. Mathematics Teacher: Learning and Teaching PK-12, 113(11), 874-884.
Waid, B. E. (2021). "The condo problem": Musings of a queer mathematics educator. Mathematics Teacher: Learning and Teaching PK-12, 114(5), e12-e15.
Waid, B. E. (forthcoming). Liberatory, anti-racist mathematics education for Black, queer and trans youth. In M. Strutchen (Ed.), Advocacy and action steps toward antiracist mathematics teaching and learning. TODOS: Mathematics for ALL.
Waid, B. E., \& Turner, K. H. (2021). Inqu[ee]ry across the curriculum. English Journal, 110(3), 82-88.
Waid, B. E., Velamur, A., Moore, A. S., \& Whipple, K. S. (2021). Disrupting normativity in mathematics education: Meeting queer students at the intersection of their queer and mathematics identities. In

Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference (pp. 119-121). Tredition.
Whipple, K. S. (2018). LGBTQ secondary mathematics educators: Their identities and their classrooms (Doctoral dissertation, University of Minnesota).
Wikipedia (n.d.). Lenapehoking. https://en.wikipedia.org/wiki/Lenape
Yeh, C., \& Otis, B. M. (2019). Mathematics for whom: Reframing and humanizing mathematics. Occasional Paper Series, 2019(41), 85-98.
Yeh, C., \& Rubel, L. (2020). Queering mathematics: Disrupting binary oppositions in mathematics preservice teacher education. In N. Radakovic \& L. Jan (Eds.), Borders in mathematics pre-service teacher education (pp. 227-243). Springer, Cham.

## Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. In the DARE Pre-Reading questions, you were asked to record one idea for how 2SLGBTQIA+ identity might be related to mathematics teaching and learning. How has this thought evolved or changed as a result of reading this article?
2. What is one step you might take today to make your school or mathematics classes more inclusive for 2SLGBTQIA+ students?
3. Try this: the article shares several trauma-informed practices based on four trauma informed teaching priorities, proposed by Venet (2021). Select one strategy to begin implementing in your classroom today.
4. Try this: GLSEN's National School Climate Survey provides a picture of the experiences of 2SLGBTQIA+ in schools across the United States. If you're looking for more contextual data, you can check out GLSEN's "state snapshot" documents (http://bit.ly/3DmehAN) or conduct a school local climate study at your own school (https://localsurvey.glsen.org).

## Mathematics for ALL

The mission of TODOS: Mathematics for ALL is to advocate for equity and high quality mathematics education for all students-in particular, Latina/o students.

Five goals define the activities and products of TODOS: Mathematics for ALL

1. To advance educators' knowledge and ability that lead to implementing an equitable, rigorous, and coherent mathematics program that incorporates the role language and culture play in teaching and learning mathematics.
2. To develop and support educational leaders who continue to carry out the mission of TODOS.
3. To generate and disseminate knowledge about equitable and high quality mathematics education.
4. To inform the public and influence educational policies in ways that enable students to become mathematically proficient in order to enhance college and career readiness.
5. To inform families about educational policies and learning strategies that will enable their children to become mathematically proficient.

## TODOS Live!



TODOS Live! began with funds obtained through a NCTM Mathematics Education Trust grant. Through the years TODOS Live! has had many excellent sessions and presenters. These sessions are generally an hour in length and occur in the late afternoon when classroom teachers can participate. A list of previous recordings and upcoming sessions can be found by scanning the QR Code below.


## 2022-23 <br> ELECTED LEADERSHIP

| Florence Glanfield | University of Alberta |
| :---: | :---: |
| President | Edmonton, Alberta |
| Linda Fulmore | Mathematics and Educational Equity Consultant |
| Past-President | Cave Creek, AZ |
| Sylvia Celedón-Pattichis | University of Texas at Austin |
| Vice President | Austin, TX |
| Nicole Lindgren | Mathematics Education Consultant |
| Director | Asheville, NC |
| Luz A. Maldonado Rodríguez | Texas State University |
| Director | San Marcos, TX |
| Marian Dingle | Advanced Education Research and Development |
| Director | Fund |
|  | Peachtree Corners, GA |
| Theodore Chao | The Ohio State University |
| Director | Columbus, OH |
| Trevor Warburton | Utah Valley University |
| Director | Orem, UT |

## APPOINTED LEADERSHIP

Nora Ramirez
Appointed Executive Director

Mathematics Education Consultant<br>Tempe, AZ

## Apply for Awards to Attend the TODOS 2023 Conference

These awards honor teachers who are TODOS members working towards creating a high-quality math education for all students, especially Latina/o students. The awards will help defray costs for the TODOS Conference in June 2023.

Applications are due March 1, 2023


## -

## TODOS 2023 Conference

Actions in Mathematics Education (AiME) Catalyzing, Cultivating, and Expanding Critical Transformations.

Tune 21-23.2023
$\mathscr{H C}$ иquerque. NN

https://www.todos-math.org/conference


Mathematics for ALL \#TODOS2023


The publications of TODOS present a variety of viewpoints. The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official position of TODOS. TODOS is a $501(\mathrm{c})(3)$ non-profit organization.


[^0]:    ${ }^{1}$ See Smith (2001) entries for more information on each country flag

[^1]:    ${ }^{1}$ For me, the demi-woman (or demi-girl) identity means that I mostly identify as a woman, but there are also times when that

[^2]:    identity doesn't feel quite right. For me, in those moments, I feel more aligned with a non-binary gender identity.

[^3]:    ${ }^{2}$ Prior to 1990, many tribes had their own word for twospirited individuals.
    ${ }^{3}$ While many may be more familiar with other versions of this acronym that begin with the letters L, G, B, T, I use this version of the acronym to honor the fact that Two Spirit identities were in existence on Turtle Island long before other Western gender identities and sexual orientations. I credit TODOS president

[^4]:    Florence Glanfield for reframing my thinking in this area to adopt the version of this acronym that puts Two Spirit first.
    ${ }^{4}$ GLSEN https://www.glsen.org/ is the leading education organization for 2SLGBTQIA+ advocacy and research.
    ${ }^{5}$ This data is from 2021. When Gallup polled Generation Z in 2017, $11 \%$ had identified as 2SLGBTQIA + .

[^5]:    ${ }^{6}$ Gender expression refers to a person's outward expression of gender and may include things such as mannerism, hairstyle, clothing, make-up, voice, interests, etc.
    ${ }^{7}$ In 2019, GLSEN found that just over $80 \%$ of QTBIPOC individuals heard racist remarks at school in the past year and

[^6]:    ${ }^{8}$ There is a mistaken belief that folx is spelled with an $x$ to make it gender neutral. Folx is spelled with an $x$ here (and throughout this article) to honor the advocacy work of Latinx, Black, Indigenous, and other POC activists in the 2SLGBTQIA+ community, who use the term folx in solidarity to mean "folks

[^7]:    ${ }^{10}$ Here I do not mean to imply that we will always know which students identify in these ways, but to acknowledge that we should always teach as though there are students who identify

[^8]:    ${ }^{12}$ For a more detailed description of this task and the lessons / activities that followed, please see Waid, n.d.a:
    https://bit.ly/3Hbih9A

[^9]:    ${ }^{13}$ See bit.ly/MathQueeriesHub for examples of inclusive mathematics tasks (Waid, n.d.b).

