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Teaching for Excellence and Equity in Mathematics

Special Issue on Multilingual Learners: Multilingual Practices

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Special Issue on Multilingual Learners:
Multilingual Practices

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TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

IN THIS ISSUE

From the Editors of *TEEM* Special Issue
Multilingual Learners: Multilingual Practices Issues

This special issue of *TEEM* on teaching multilingual learners was created with a call for papers showing how mathematics teachers of multilingual learners can implement equitable, rigorous, and coherent mathematics instruction (TEEM, 2019). Though many teachers are aware of the need to implement such instruction, they are unsure of how to *act* on this awareness, as evidenced by their lack of confidence in their ability to teach diverse groups of students (Banilower et al., 2018). *TEEM* solicited manuscripts from classroom teachers and/or teacher educators that provided evidence-based examples of how to enact effective instructional strategies with multilingual students. The call specifically focused on highlighting assets-based approaches to teaching multilingual learners.

This is usually the point where the authors of the introduction frame the theoretical perspectives that motivated and undergirded the special issue. Yet, as we write this introduction, mass protests against racial injustice are ongoing, and we would be remiss not to acknowledge this moment here and now. The people of the United States—and the world—are witnessing and participating in uprisings against racism. These uprisings were sparked by state-sponsored violence against Black people, including George Floyd and Breonna Taylor, who were killed by police officers in summer 2020. The murders of Floyd and Taylor followed the murders of hundreds of other Black people at the hands of police or vigilantes: Ahmaud Arbery, Trayvon Martin, Michael Brown, Eric Garner, Natasha McKenna, Tamir Rice, and, unacceptably, too many others to name. It is sad that this uprising is necessary, and it is heartening to see so many people joining the effort to dismantle racism and racist systems. Of course, it is critical to note that these uprisings did not spring from thin air. Organizers have been laying the foundations for this moment for years, and the work of dismantling racism and racist structures will continue long after the summer of 2020 is over. It is equally important to remind ourselves that the aforementioned systems that have been built upon pernicious racism and white supremacy will not be overcome without persistent vigilance, education, interrogation of complicity, and, in the words of Dr. Martin Luther King, Jr., maladjustment (King, 1963).

In light of this moment, we reflect on the work of Black scholars such as Danny Martin, who have challenged fellow mathematics educators to consider how—and whether—reforms of mathematics teaching and learning can dismantle racism and racist structures (e.g., Martin, 2011). Additionally, scholars, such as Nicole Joseph, have presented case studies illustrating how teachers can create more humanizing mathematics classrooms for their Black and Brown students (e.g., Joseph, 2016). The works of these scholars, and that of many others, inspires us as we consider this special issue in light of this current moment. While this collection of papers is a small contribution, we hope it is part of the foundation that can build up a more equitable mathematics education enterprise.

In this issue we present papers that evidence how mathematics teachers of multilingual learners can engage in equitable, rigorous, and coherent mathematics instruction. Many learners are excluded from mathematics learning environments on the basis of their identities, including students' language(s) spoken, race, ethnicity, nationality, disabilities, gender, and sexuality. At a minimum, disrupting these patterns of exclusion, and building a more inclusive mathematics education requires that teachers *purposefully* include students from non-dominant communities in their classrooms. By creating linguistically inclusive mathematics classrooms, teachers can address one of the ways in which non-dominant students are excluded from mathematics learning environments.

Our call for papers resulted in many submissions—an encouraging sign! In response to the large number of submissions we received, we have created two issues of *TEEM* from this solicitation. In this issue we focused on papers that addressed classroom-based interventions and pre-service teacher education. In a companion issue published in Summer 2020, we have a set of papers that examined the use of language(s) while teaching mathematics using a translanguaging framing. One of the uniting features of the papers in this issue is that each paper included special discussion about multilingual mathematics classrooms that have been overlooked in prior research, as well as guidance for teachers.

The first three articles in the present issue centered on pedagogical practices with multilingual students within classrooms. To begin, Martínez Hinestroza examined the potential pedagogical value of silence in multilingual classrooms. While most classroom-based research examines talk, Martínez Hinestroza reminds us that silence from students and silence from the teacher may create learning opportunities for students who are learning mathematics and acquiring a new language simultaneously. This analysis from a Spanish-immersion setting included a unique theoretical idea, as well as a description of a pedagogical activity, which can be useful for integrating mathematics and language learning.

In their paper, Ji-Yeong I and colleagues present an analysis of a modeling activity that they implemented in a mathematics class for recent immigrants. While most "newcomer" math classes focus on low-level skills and procedural fluency, this paper illustrates several ways teachers can engage all learners, including newcomers, in mathematically challenging modeling activities. One noteworthy contribution in this paper is I and colleagues' focused on how the teacher set up the task and considered how to build on their students' assets when designing and implementing the modeling activity.

Silva examined the development of mathematical agency of Carlos, an emergent bilingual student who had a diagnosed learning disability. Through analyzing Carlos's participation in an out-of-class teaching experiment, Silva showed that Carlos developed in his participation and agency. Silva noted that this development took place when Carlos was exposed to bilingual problem-based discussions that were quite different from traditional teaching methods recommended for students with diagnosed learning disabilities. This analysis points to potential new directions for development.

The final two papers in this issue examined the learning of teachers and prospective teachers beyond the classroom. Wilson describes a teacher learning activity that was based on framework for teachers' knowledge for teaching multilingual learners. In the professional development activity, the teachers were tasked with designing assessment questions related to teaching math to multilingual students. The process of designing assessment items sparks discussions rooted in the framework related to obstacles and resources for teaching mathematics to multilingual learners.

Finally, the special issue closes with a paper by Krause, who describes an activity in which bilingual pre-service teachers practice communicating about multilingual students' strengths to their parents. This innovative project built on the activities developed in the TEACH Math project and includes consideration of how teachers learn to communicate with families. Because we know that connecting with families is a critical part of building a more just and humane mathematics education enterprise, we are excited to share this innovative work showing how bilingual teachers were apprenticed into the practice of communicating with families.

In sum, the papers in this special issue provide ideas for considering how teachers and teacher educators can transform mathematics education to create a more just and equitable future, especially for multilingual learners who have endured over a century of marginalization and discrimination (Donato et al., 2017). In addition to the articles in this issue, we hope readers will continue to learn about and advocate for much needed changes to the ongoing immigration crisis in the United States that has resulted in the unjust and inhumane treatment of people seeking to enter this country. We note that immigration issues greatly impact multilingual students and families, and, thus, supporting students requires us to learn about and address these issues. While this scholarly work feels removed from the direct action we are seeing on the streets and in the halls of government in response to state-sanctioned violence against Black people, we hope that this work will contribute to ongoing efforts to create a more humane mathematics education enterprise, and, ultimately, a more equitable and just world.

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“Hush it up!”: Silence as a Pedagogical Resource in a Language Immersion Mathematics Classroom

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Abstract

Frequently, the silence of students whose first language is not the language of instruction is interpreted as indicative of lack of knowledge or language proficiency. I propose an alternative interpretation, illustrating how both teacher and student silence can be a pedagogical resource that respects students’ sense-making, honors the multimodal nature of mathematical activity, and follows principles from research on second language learning. I draw on an example from a geometry task in a third-grade Spanish immersion classroom and invite readers to consider how teachers and students can use silence as a pedagogical resource in their bilingual contexts.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. How do you interpret student silence during mathematics class discussions and small group work?
2. What productive roles can silence play in mathematical activity, especially for multilingual students?
3. What productive roles can teacher silence play in mathematical activity?

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"Hush it up!": Silence as a Pedagogical Resource in a Language Immersion Mathematics Classroom

José Martínez Hinestroza

Including all students in the exploration of meaningful ideas is an enduring issue in mathematics education. Addressing this issue is particularly complex when working with bilingual students, as students who are proficient in the language of instruction tend to dominate discussions. Initially, that seemed to be the case in a Spanish-immersion classroom that I regularly visited as part of my collaboration with a third-grade teacher, Mrs. Abad (all names are pseudonyms). During one of my visists to this classroom as a participant observer, Mrs. Abad approached a small group and asked students to share ideas about the problem they were solving. One of the students, Haley, quickly raised her hand and immediately started talking. Mrs. Abad acknowledged Haley's strategy and asked for other ideas. After a brief silence, Haley proceeded to explain a second strategy. Mrs. Abad asked if anyone else had ideas to share. There was no response.

When Mrs. Abad and I discussed this and similar episodes, we concluded that we needed to develop teaching strategies to help all students engage with mathematics tasks. We looked for these teaching strategies in previous research on mathematics education in classrooms where students' first language is different from the language of instruction. The strategies we used included providing comprehensible input (Li, 2015), using visual representations (Escobar Urmeneta, 2013), and using multiple languages strategically (Martínez Hinestroza, 2018).

Although these strategies facilitated communication of mathematical ideas, this approach to supporting bilingual students overemphasized the teacher's and the students' language production. After all, the motivation driving these efforts was to *hear* ideas from all students during discussions. An alternative motivation that mitigated the tendency to overplay the role of spoken contributions is to *support all students' mathematical sense-making*. This is a subtle but important shift from considering student talk as the ultimate goal to considering it part of the multimodal process of exploring mathematical ideas. Other modalities involved in this process include gesturing, representing with symbols, drawing, and writing (O'Halloran, 2005).

In addition to these modalities, mathematical sense-making "also includes silences. Like in a piece of music, far from being accessorial, silence plays a crucial role: It is a constitutive part of the text" (Radford et al., 2007, p. 517). However, silence—defined as the absence of spoken utterances—is often vilified and related to an undesirable stage where students passively receive knowledge established by others (Becker, 1995). Student silence is equated with lack of confidence and it is seen as a symptom of marginalizing classroom cultures (Foote & Lambert, 2011; Lubienski, 2000). This was how Mrs. Abad and I initially interpreted the example above. Although still vigilant about silence that isolated students and hindered sense-making, over time we shifted our focus: We considered *student silence* that opened up opportunities to use multiple modalities and *teacher silence* that allowed for students to author mathematical ideas.

In this paper, I describe a geometry task that drew on these kinds of silence in a Spanish-immersion third-grade classroom. I discuss how drawing on silence was respectful of students' diverse sense-making, honored the multimodal nature of mathematical activity, and was consistent with research on second language learning. First, I describe the classroom and lesson context. Then, I illustrate how both student and teacher silence served as pedagogical resources.

The Context

The example I present comes from a third-grade classroom in the Midwestern US. This classroom was part of a language enrichment (Brisk, 2011) Spanish-immersion program. Spanish was the language of instruction for most of the day (all but one lesson a day was in Spanish), including mathematics lessons. There were 21 students in the classroom, most of whom (18 students) spoke English at home and were learning Spanish as a second language at school. The other three students spoke both Spanish and English at home. All students understood directions and followed conversations both in Spanish and English, and there were variations in students' Spanish speaking skills. Mrs. Abad was the Latina, Spanish-English bilingual teacher. I, a Latino, Spanish-English bilingual researcher and teacher educator, visited this classroom at least once a week and worked closely with the teacher for over three years.

The Mystery Shapes Task

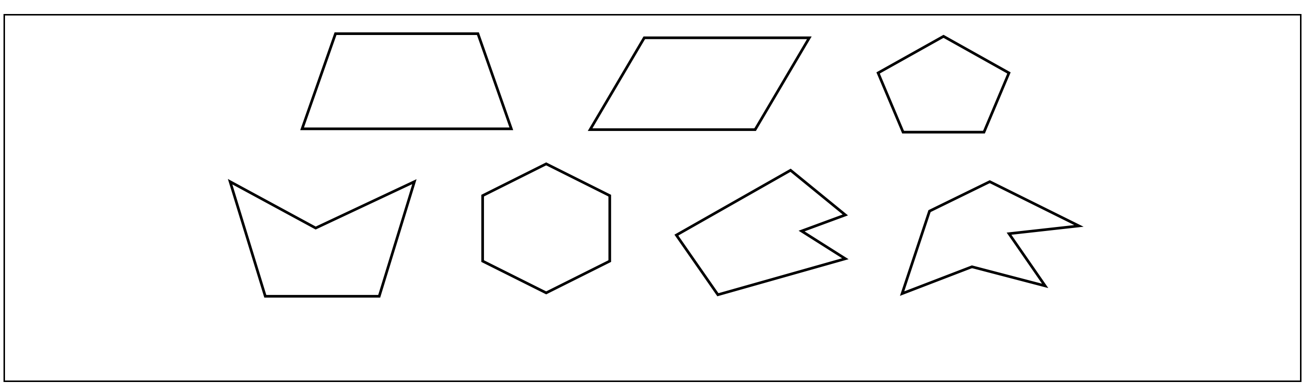
The mystery shapes task took place at the beginning of a geometry unit in the last quarter of the 2017-18 school year. The mathematical objectives were: (1) to identify defining attributes of 2D shapes, and (2) to build specific 2D shapes. The language objective was to use complete sentences to name and describe objects. Relating the mathematics and language objectives involved: (1) describing 2D shapes using informal language, and (2) using progressively more precise language to name 2D shapes and describe their defining attributes. There were two parts in this task. During the first part, students worked in groups of three. Each group selected a representative and the teacher and I shared directions with the representatives only. After representatives observed all the shapes in Figure 1, we assigned one of the shapes to each group. Only the representative, and not other group members, could see the shape throughout the task.

Representatives worked with their groups so that each group member could build the shape using pipe cleaners. Representatives could talk, gesture, and show the shape using objects (such as a pipe cleaner), but there were two restrictions: First, we asked representatives not to draw the shape. Second, we asked that no one touch anyone else's pipe cleaners. That is, once a student started working with a pipe cleaner, only that student could move, fold, or rearrange that pipe cleaner. These restrictions were intended to foster the use of multiple modalities. The teacher and I anticipated that representatives would use a combination of spoken language, gesturing, and manipulation of a pipe cleaner to make sense of the defining attributes of their shape and to communicate with their group. We anticipated that other group members would use a combination of listening, spoken language, observation, and manipulation of the pipe cleaner to build the shape. Rather than interpreting observation as meaningless imitation, multimodality fostered silence that made room for visualization and embodied sense-making.

The second part of the task was a whole class discussion where each group shared their shape with the rest of the class. The class discussed commonalities and differences among the shapes and ways of categorizing the shapes. In both parts of the task, the teacher and I intended to draw upon our own silence. Instead of telling students what to do, or guiding their exploration through questioning, we intended to unobtrusively and quietly observe. Our silence helped us to not take over children's ideas, while we made sense of students' problem-solving strategies.

Figure 1

Shapes included in the mystery shapes task.



This task was informed by the information gap tasks (commonly called 'info gaps' by educators) used in world languages classrooms that focus on communicative skills (Larsen-Freeman, 2011). In info gaps, students work together to complete a task but each student is missing some of the necessary information. Students communicate to fill in the gaps so that they all have all the information. In the mystery shapes task, each group representative had all the necessary information. They needed to communicate with the other two group members to share information that would allow all three students to build the shape. We adapted this kind of task to foster multimodality, including silence, instead of overemphasizing spoken communication.

From a language teaching standpoint, info gaps foster authentic communication that focuses on meaning. Communication is authentic because the students are interacting to find out information they need. The focus is on meaning because students use language to convey the necessary information, even if there are language inaccuracies. From a mathematics teaching standpoint, the mystery shapes task focused on relevant attributes of a shape. As the task unfolded, multiple modalities involved in mathematical activity emerged, including observation, gestures, and manipulation of the pipe cleaners. Student and teacher silence also emerged as important pedagogical resources supporting sense-making, as I describe next.

Silence Supporting Mathematical Activity

The following transcript illustrates how student and teacher silence emerged as an important pedagogical resource to support these bilingual students. The three students in this group (Willie, Calum, and Jimmy) all spoke English at home and Spanish was their second language. After listening to the directions, Willie, the group representative, explained the task to Calum and Jimmy. He then tried to describe the assigned shape, the convex hexagon in Figure 1. In the transcript, my English translation is italicized.

Table 1
Silence during mathematical activity

Student	Utterances	Non-spoken actions
Willie	OK. Hay uno, dos, tres, cuatro, cinco, seis… *(There are one, two, three, four, five, six…)* Oh boy… Es como un cosa de seis… (*It's like a thing of six…)*	Moves finger as if tracing a hexagon in the air. Traces with finger in the air, again.
Calum	¿Es como seis cosas? (*Is it like six things?)*	
Jimmy	¿Puedes dibujarlo para mi? (*Can you draw it for me?*)	
Willie	No puedes dibujarlo, pero puedes verlo porque es muy difícil describir. Es como esto. (*You can't draw it but you can see it because it's hard for me to describe it. It's like this.)*	Starts folding pipe cleaner. Calum and Jimmy lean over and they each start folding their pipe cleaners as they observe.
Calum	¿Es un triángulo? *(Is it a triangle?)*	
Willie	Es más o menos como un triángulo. Es como varios triángulo juntos. (*It's kind of like a triangle. It's like many triangles together.*)	All three students stop folding the pipe cleaners.
Calum	¿Como un cuadrado de dos triángulos? (*Like a square made out of two triangles?)*	
Jimmy	Hush it up! Let him show us.	Jimmy picks up Willie's pipe cleaner.

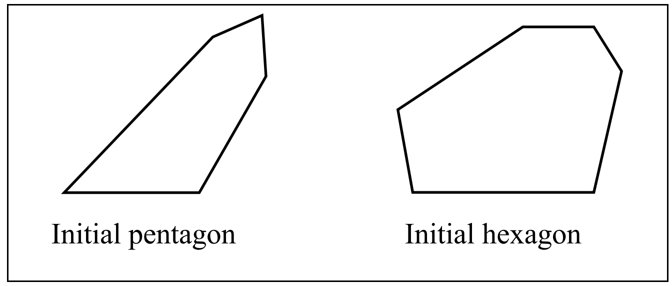
Student silence that fosters multimodality. The multimodal nature of mathematical activity is evident in this example as Willie combined talk and gestures when he simultaneously counted and traced the shape in the air. He, however, acknowledged that at this point in the lesson it was difficult for him to describe the shape. Silence followed. Far from being an idle moment, Willie diligently manipulated his pipe cleaner trying to make sense of the angles and sides that made up his hexagon. Calum and Jimmy coordinated observation and their own movements reshaping their pipe cleaners. Rather than indicating meaningless imitation, their actions show evidence of intent observation and embodied sense-making. In this case, silence opened up a space for Willie to use multiple modalities to show what he knew about the shape without vocal students interrupting. Willie's initial attempts to describe the hexagon could be interpreted as lack of language, and Jimmy's comment ("*Hush it up!"*) as a power move to silence his classmates. What happened next, however, suggests an alternative interpretation.

Willie continued folding his pipe cleaner. Unlike Calum who continued observing and folding his own pipe cleaner, Jimmy stopped folding and observed quietly. After a couple of attempts and some help (from me), Willie finished his shape. When Calum was done, he ended up with the pentagon represented in Figure 2. Willie tapped with his finger each side of Calum's shape, quietly counting the number of sides, and declared: "Esto no es la figura" (*this is not the shape*). Calum took up Willie's actions by tapping on each side of Willie's shape while saying out loud the numbers from one to six. Then, he did the same with his own shape and stated: "Oh! Son seis de estos líneas" (*Oh! It's six of these lines*), which Willie confirmed enthusiastically. Calum proceeded to rearrange his pipe cleaner and ended up with the hexagon represented in Figure 2. Observing Calum's hexagon Willie said: "Es como la figura, pero no es. Estos líneas son como igual" (*It is like the shape, but it's not. These lines are like the same.)*

In this interaction, when Willie and Calum counted the sides in each other's shapes, they reached an unspoken common understanding: A defining characteristic of their shape was the number of sides. Any other shape with a different number of sides was not their shape.

Figure 2

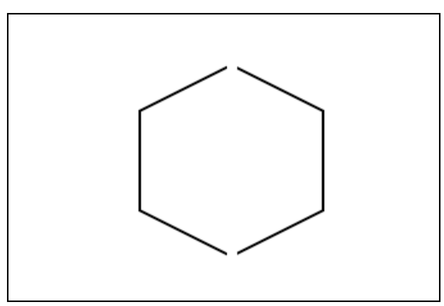
Calum's pipe cleaner shapes.



Student silence that enhances observation. While the previous interactions unfolded, Jimmy had continued to observe quietly. After listening to Willie's last comment challenging whether Calum's shape was their shape, Jimmy grabbed an extra pipe cleaner and announced: "Tengo un idea" (*I have an idea*). He folded one of the pipe cleaners in three parts, rearranging it until the three parts were approximately the same length. Then, he placed the second pipe cleaner over the first one and proceeded to fold it until both pipe cleaners looked the same. Finally, he placed them side by side, as shown in Figure 3. Willie and Calum observed this process. In the end, Willie said "Yes! Eso es la figura" (*Yes! That's the shape*) and Calum simply said "Wow!" as he started to follow a similar process.

Figure 3

Jimmy's hexagon from two pipe cleaners



Jimmy's silence during the part of the task when Willie and Calum counted the sides of their shapes was not an indication of lack of understanding. Instead, intent observation accompanied Jimmy's silent sense-making. Not putting his understanding in words did not interfere with Jimmy's ability to generate an efficient strategy. His strategy showed that he understood that the shape had six sides and that the sides were the same length. He extended this idea: By using two pipe cleaners, Jimmy showed an understanding of the shape's symmetry. Jimmy's was a silent and generative idea that filled the gap between the information that Willie had and Calum's search for a strategy to rearrange his shape.

Teacher silence that respects student sense-making. The role that the teacher and I played during these interactions involved silence. During small group work, children in this classroom were used to the teacher or I seating next to them and observing quietly witout necessarily adding our own thoughts. When I was observing this small group, it was tempting to intervene. For example, I could have asked Willie questions about the number of sides of his shape to focus the group's attention on defining attributes of their shape. Although questioning and other ways of supporting small group work can be productive, I remained silent. Instead of guiding their exploration of mathematical ideas and imposing language production, my own silence helped me avoid taking over these students' sense-making. In turn, students proved that, when given the chance, they can come up with their own multimodal strategies, and ways to develop and communicate mathematical ideas.

Teacher silence played an important role in reaffirming students' mathematical knowledge authority. Instead of the teacher and I having the final word when students found disagreement or unexpected solutions, we trusted them with the responsibility of reconfiguring their conflicting ideas. Our silence encouraged student-student interactions where children generated and assessed mathematical ideas. We did not, for example, tell Calum that the pentagon he created was the wrong shape and we did not interrupt Jimmy's silence by asking him to put his ideas in words. Teacher silence became part of the coordination of multiple modalities that also included student silence, observation, and movement.

Talking It Up

The second part of the mystery shapes task was a whole class discussion where each group shared and described their shape. Mrs. Abad and I asked questions for students to compare the different shapes. Our questions also inquired about mathematical terminology. It was during this discussion that some students began to put their understanding in words. For example, when Willie's group shared their shape, Willie said that "nuestra es seis de estos líneas" (*ours is six of these lines*). Mrs. Abad asked the class what "these lines" were called. With some scaffolding and modeling from classmates, Willie reformulated his explanation to "nuestra tiene seis lados y todos seis lados son iguales" (*ours has six sides and all six sides are the same*). Excitedly, Jimmy added: "es un 'seisángulo'!" (*it's a 'sixangle'*). Jimmy's coinage of a name for their shape was an entry point to a discussion about how to name shapes, and about the relationship between defining attributes of a shape and the shape's name.

There were several other instances of terminology emerging in this discussion. For example, the word "angle" emerged when Harper, looking at all the shapes that groups had presented, said "Todos son ángulos… Todos tienen ángulos" (*They are all angles… They all have angles*). The word "length" emerged when Juan, describing a trapezoid, said "Dos tienen el mismo longitud de lado y el otro dos la misma longitud" (*Two of the sides are the same length and the other two are the same length*). The idea of "parallel lines" followed when Rose stated that "casi todos [las figuras] tienen líneas paralelas" (*almost all [the shapes] have parallel lines*).

Rather than being a necessary precursor of engagement with powerful mathematical ideas, in the mystery shapes task formal and accurate mathematical language production followed the emergence of ideas in silent ways. Instead of postponing the exploration of mathematical concepts until an idealized stage when students are expected to have mastered a predetermined subset of language, students quietly engaged with and demonstrated nuanced understandings from the beginning of the task. The class broke the silence and collaborated in developing formal mathematical language when the need arose.

Closing Thoughts and Suggestions

Students who are simultaneously learning mathematics and the language of instruction creatively draw on multiple resources to support sense-making. Interactions around the mystery shapes task illustrated how silence can be one of these resources. Rather than condemning silence as an unequivocal indicator of lack of knowledge and confidence, Willie, Calum, and Jimmy purposefully and productively included silence as one of the modalities they used in this task. Their interactions highlight the importance of agentive use of silence that supports students' self-mediated learning (Blight & Drury, 2015). Teacher silence supported unobtrusive observation that is respectful of bilingual students' strategic use of silence in specific moments and with specific people (Drury, 2013). Teachers can learn from these uses of silence to intentionally decide when eliciting talk is productive and when silence is an alternative that honors students' diverse ways of making sense of ideas.

During the mystery shapes task, the teacher and I trusted students with decisions about their own language use and their own silence. Students decided when and in which language to speak, and the info gap task supported the emergence of other modalities. This flexibility is consistent with research on second language learning that acknowledges that students produce language when they feel an authentic need for it (Gass, 2018). In the mystery shapes task, the need to talk was more compelling during the whole class discussion than during small group work. Even then, teacher silence played an important role, as it was the students who did most of the talking and language modeling. Teacher silence, for example, opened up opportunities for students' lexical inventions (Dewaele, 1998), such as Jimmy's made-up word 'seisángulo' (*sixangle*) that skillfully connected mathematical ideas and terminology.

Ultimately, the mystery shapes task and the interactions around it support the call for teachers to attentively listen to students. In this case, the call is to listen not only to student talk but also to student silence and to avoid the tendency to mistrust interactions that are not regulated by the teacher. Teachers could focus efforts on disrupting specific kinds of student silence that marginalize bilingual learners, instead of attempting to eradicate all silence in favor of premature or unnecessary student talk.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. How would an info gap task that promotes productive silence look like when exploring other ideas, such as numbers and operations, measurement, or algebraic thinking?

2. How can teachers intentionally and explicitly incorporate silence as a pedagogical resource in the mathematics tasks they design?

3. Which other pedagogical resources that could support multilingual learners' mathematical sense-making do we tend to overlook?

4. How can teachers differentiate productive silence (i.e., silence as a pedagogical resource) from silence that needs to be interrupted (i.e., silence that marginalizes)?

Fencing in the Goats: Adaptation of Modeling Framework for Emergent Bilinguals

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Abstract

Through collaboration between a teacher and a researcher, we developed 5-Act Task Framework by utilizing modeling as vehicle, aiming to support Emergent Bilinguals (EBs) to engage in high-quality mathematics. A classroom episode is described to illustrate how 5-Act Task-based lessons were implemented effectively for EBs by connecting the students' language and real-life context. With the belief that EBs are capable of mathematical modeling with appropriate support, we implemented the 5-Act Task Framework that provides EBs with access to rigorous mathematics while developing language throughout the modeling process.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. How do you feel when your student speaks a language in class that you don't understand? What do you do (or would you do) when you hear a language that you don't understand?

2. How do you think Emergent Bilingual students engage with mathematical modeling? How can mathematical modeling be fully utilized by Emergent Bilingual students?

3. How do you think mathematics teachers usually support Emergent Bilingual students to solve mathematical modeling tasks? How do you want to support your Emergent Bilingual students in engaging in mathematical modeling tasks?

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Fencing in the Goats: Adaptation of Modeling Framework for Emergent Bilinguals

Ji-Yeong I, Kait Ogden, Richardo Martinez, and Betsy Araujo Grando

In order to have an excellent mathematics program, students need access to a high-quality curriculum along with high expectations (National Council of Teachers of Mathematics, 2014). However, it is no secret that teachers experience difficulty when guiding Emergent Bilinguals (EBs, a.k.a. English learners) to understand word problems. Interviews conducted by I et al. (2019) revealed that some teachers only want to use computational tasks for EBs because of the belief that simple computations do not pose a language barrier. However, with only easy computation tasks, EBs cannot engage in problem-solving or reasoning that cultivates a deeper understanding of high-level mathematics. For this reason, researchers (e.g., Moschkovich, 2010; Celedón-Pattichis & Ramirez, 2012) assert that EBs should have an equal opportunity to learn challenging mathematics. Making connections between mathematics and EBs' lives has been recommended to help EBs make sense of mathematics (e.g., Chval & Chavez, 2011).

Responding to these demands and recommendations, we designed modeling-based lessons for teaching high-quality mathematics to EBs in order to investigate how modeling can be used as a powerful tool to teach mathematics to EBs. The discussions around modeling imply that modeling must include challenging tasks while incorporating situations connected to students' lives, which complements recommendations for teaching EBs. However, teachers may hesitate to use modeling when teaching EBs due to the heavy language demand associated with modeling tasks. However, well-designed modeling lessons can engage EBs by utilizing social interactions and real-life contexts that are particularly crucial for EBs (Anhalt, 2014). With the belief that EBs can do modeling with appropriate support, we aim to develop a framework that empowers EBs to have access to rigorous mathematics.

5-Act Task: Modeling for Ebs

The Common Core State Standards for Mathematics (CCSSM) includes modeling as a standard of mathematical practice as well as a high school domain. Modeling is defined as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (NGA & CCSSI, 2010, p. 72). These inclusions of modeling represent two different functions of modeling: modeling as vehicle and modeling as content (Galbraith, 2012). Modeling as vehicle is using modeling as a tool to teach mathematical concepts and procedures while modeling as content treats modeling as a curriculum itself and enables students to use mathematics to solve an ill-defined, authentic, and real-life problem. These two functions of modeling may be complementary in a lesson when a teacher has a goal of teaching a specific concept and using a modeling activity to teach that concept. Our focus in this study is modeling as vehicle because our goal is to implement modeling as an instructional approach that can be used on a daily basis by teachers. And throughout this paper, the term *modeling* refers to modeling as a vehicle to learn and teach a mathematical concept, which is more related to the modeling standard of mathematical practice, “apply the mathematics they know to solve problems arising in everyday life, society, and the workplace” (NGA & CCSSI, 2010, p. 7)

To develop the modeling framework, we adapted the following 3-Act Task framework by Dan Meyer (2011):

> Act 1: A story is given and students identify the conflict of a real-life story.
> Act 2: Students choose tools, resources, and make assumptions they need to solve the conflict.
> Act 3: Students build models and find a solution to the problem and continue with an extension.

Instead of heavy text, 3-Act Tasks often utilize multimedia and visuals to describe a problem within a real-life story. We added more actions to this framework to help EBs make sense of the given story (which may include cultural biases or language barriers) and to support EBs in expressing their mathematical thinking. Hence, we developed 5-Act Task by adding Act 0 to support EBs’ ability to make sense of a problem before starting Act 1 and by adding Act 4 at the end to ensure EBs share their process (see Appendix).

Act 0 is crucial in that it reflects a set-up stage needed to engage in teaching EBs (I & de Araujo, 2019), where teachers support EBs’ understanding of both language and mathematics embedded in Act 1. For example, teachers assess EBs’ prior knowledge related to the story and provide appropriate scaffolding. Act 0 benefits all students because it helps them better comprehend the problem context (Jackson et al., 2013), but it is essential for EBs because unknown language or cultural biases may prevent them from finding mathematical entry points. Act 1 begins with a story, either shown in a video or demonstrated in class and ends when the questions/problems are posed. The real-life stories have conflicts that provoke students’ curiosity, leading to questions being posed with or without the teacher’s guidance. The creation of a question builds on quantities in the conflict of the problem, which leads to the development of the models used in the next phases. In Acts 0 and 1, sharing and valuing language and culture is crucial for EBs. Revisiting Bane (1992), Nieto (2018) reminds us that multilinguals like to hear their multiple languages in school: “when culture and language are acknowledged by the school, students are able to reclaim the voice they need to continue their education successfully” (p. 120). Hence, we should not just focus on delivering meaning(s) of a word, but also ask EBs to share their stories, using their language as a means to connect to their culture. When a teacher acknowledges students’ multiple languages, students can begin to see their language, culture, and themselves as part of the classroom (Macedo & Bartolomé, 1999).

In Act 2, students are asked to find information and make assumptions needed to answer the question created in Act 1. Act 2 provides students with another chance to have ownership over how they approach the task because they can select and explore whatever tools/information they think they need to solve the problem, rather than being given all necessary information and quantities. Act 3 represents key parts of the solving process in that students formulate, compute, interpret, and validate their models. Based on the result of Act 3, students may return to Act 0 (e.g., to be reminded the meaning of a word), to Act 1 (e.g., to justify the result in the context), or to Act 2 (e.g., to gather more information). The role of Act 0 is reemphasized in Act 3 in that Act 0 gives EBs the background to figure if both the model and solution make sense in the context, and if not, they modify either their models or the previous steps, which reflects the iterative process of modeling. Finally, Act 4 starts when EBs communicate their solutions and reasoning behind their models. Because Act 4 involves productive language modes (writing and speaking), this step can be challenging for EBs. However, adding this step is critical because it provides an opportunity for students to develop their bilingual proficiency. To reduce EBs' anxiety speaking English, teachers should design Act 4 carefully by applying various strategies such as having group/pair presentations, allowing EBs to explain with visuals and gestures, giving enough wait time for students to prepare what to say in English, and writing sentence frames on the board.

Setting and Task Selection

In this project, collaboration between a teacher and a researcher plays a crucial role in that the collective expertise balances each other's needs: The teacher's knowledge of students complements the researcher's knowledge of modeling. The middle school teacher (Ogden) acted as a researcher-teacher, and the researcher (I) acted as a teacher-learner-investigator, as they co-developed, co-taught, and co-analyzed the lessons (Jung & Brady, 2016). The study was conducted in a classroom consisting of 11 EBs in grades 7 and 8, who came from Sudan, Myanmar, Thailand, Congo, Guatemala, and Honduras. Eight students arrived in the U.S. as refugees, and seven of 11 students spoke more than two languages. All the students had been in the U.S. for less than two years and were assessed by their school district to be at the beginning level of English proficiency. Also, most of them had not experienced formal education for at least two years, and three students were illiterate in their own language.

The teacher chose a mathematical concept that she was required to teach in the semester and the researcher suggested tasks, and then they discussed, selected a task to be used, and co-developed a detailed lesson plan. While co-developing the lesson introduced in this paper, a pattern task was chosen because it encompassed algebraic reasoning and generalization that were connected to the target concept, creating equations. The task has been posed with various contexts such as combinations of tables and chairs, cows and pens, and goats (or other animals) and fence sections. Selecting a context was imperative because a real-world integration is a core of modeling, and familiarity with a context provides EBs with access to mathematics. Goats were chosen because the teacher noticed goats were the animal closely related to the EBs' prior experiences. Pigs were not considered to respect some students' religious beliefs, and we excluded chickens because chickens do not usually reside alone when fenced. Although authenticity was considered, we cannot deny the base case of the scenario of fencing goats in a row may sound artificial, but this setting was necessary to achieve the objective of generalizing an expression, following our view of modeling as vehicle.

5-Act Task Lesson

In this section, we introduce one of the 5 Act Task lessons developed and implemented in this study. The mathematics and language objectives were "write expressions/equations from a real-life situation using increasing patterns and functional patterns" and "use precise terms to explain their algebraic thinking using the expressions they developed," respectively. We started the lesson by grouping students and placing a shared large piece of chart paper and markers in each workspace. Then, we began Act 0 by showing the class a picture of a goat and asked questions to initiate a conversation. The following dialogue illustrates this moment:

Teacher: Does anybody have goats in their country? [one student raised a hand and all said "Yes!"] and you had to take care of them?
Students: [all answered] Yes! [one student said "My country has it, but not own."]
Researcher: What do you call goats in your language?
Teacher: Raise your hand [she gestured by raising her hand] if you know how to say *goat* in your language.
Students: Me, Me, Me [With hands raised, students' body language shows excitement]
Student 1: *mbuzi*
Researcher: *m-buzi* [repeating what Student 1 said]
Student 2: *cabra*
Teacher: *cab… cabra*
Researcher: In Korean, it is called *yumso* [multiple students repeat *yumso*]

All students were excited to share their languages and experiences related to goats. The word *goat* was said in Arabic (maeiz ماعز), Burmese (sate ဆိတ်), Korean (yumso 염소), Spanish (cabra), and Swahili (mbuzi). To introduce the story in Act 1, Researcher (this paper's first author) used a document camera to show a goat paper cutout and four popsicle sticks around the goat (Figure 1, left) in saying, “If you have one goat, you need four sections of fence.” Afterward, Teacher (the second author) wrote *G* and *S* on the whiteboard and also wrote 1 and 4 associated with each letter. Then, we asked what the letters *G* and *S* were for, and students replied that *G* represented the number of goats, and *S* the number of fence sections.

We continued to demonstrate more cases with goat cutouts and popsicle sticks, but this time, we asked a question, “If you have two (or three) goats, how many sections of fence do you need?” without placing all popsicle sticks (Figure 1, middle and right). Then, we gave a more challenging question, “What if you have six goats in a row?” “What if you have 20 goats?,” without showing the case. After students answered with many numbers, we asked the following questions for Act 2: “What do you need to find an answer?” and “What information/kind of stuff do you need?” We provided various tools, such as counters, popsicle sticks, multiplication charts, and goat cutouts, and said, “You can use any tools you want.” Each group chose their tools and wrote out their ideas on the shared chart paper. However, we did not provide more than four goat paper cutouts and six sticks to prevent students from counting. Then, Act 3 began. We walked around to monitor and help students. We observed one group arranged goats and then sticks around them, and another group placed only sticks without goats.

Figure 1

Three cases with one to three goats and the sections of fence needed

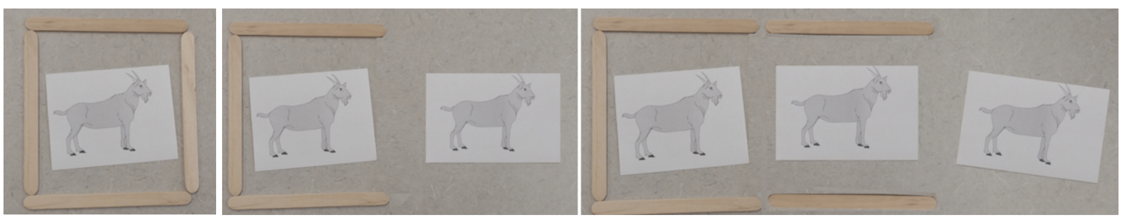
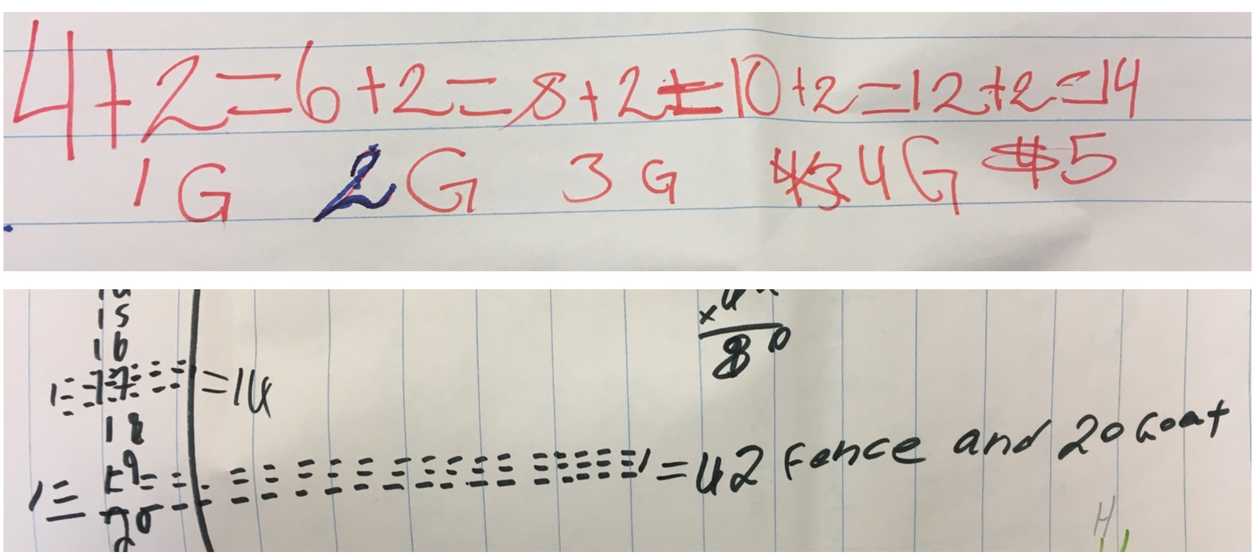


Figure 2

EBs’ initial modeling before teachers’ guidance



In other groups, one student started writing equations on the chart paper without using manipulatives (see Figure 2 top). Although he misused equal signs (4+2 is not equal to 6+2), the student found adding two was the repeating pattern for the number of fence sections. Then, one of his group members stopped him and showed his drawing model (Figure 2 bottom), using line segments to represent both goats and fence sections. This student found a correct answer by counting the line segments. To promote algebraic reasoning beyond rote counting, we gave the final task, "What if you have *G* goats?" The students first struggled with this generalization process, so we provided some guidance, such as "why don't you use a table?" and "why don't you list equations showing the relationship from one goat?" With guidance and a reminder of Acts 0 and 1, the students could build models for *G* and finally figure out the expression of the relationship between *G* and the number of fence sections needed (Figure 3).

The model in the left side of Figure 3 used a list of equations with *G* for the number of goats and *S* for the number of fence sections. Note that in this notation, the equals sign (=) does not represent the quantity equivalence, but instead shows the same circumstances or conditional situation (e.g., for 2 goats, we need 6 sections). The right side of Figure 3 shows another model, a two-column table. The first row has two variables and the numbers below were used to find a pattern between *G* and *S*. From the table, the EBs saw the repeated pattern of variant and invariant parts and figured out how the variant part can be replaced with *G* – 1.

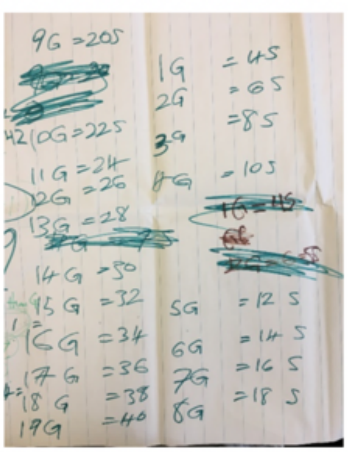
At the end of the lesson (Act 4), students reflected on their learning by writing in their journal where they were encouraged to use the language of their choice, as well as drawings and mathematical expressions. Since some EBs were not yet comfortable writing in English or even in their own language, we provided students with keywords and sentence frames on the whiteboard. One student wrote in his journal, "I like this math, challenging math today. I learned how to do expression." This particular student also verbally expressed his excitement while finding expressions with variables because he had hardly been given challenging tasks in prior math classes. Others wrote sentences with words (goats, fence, expression, number) and the expression they found, using either English or their native language.

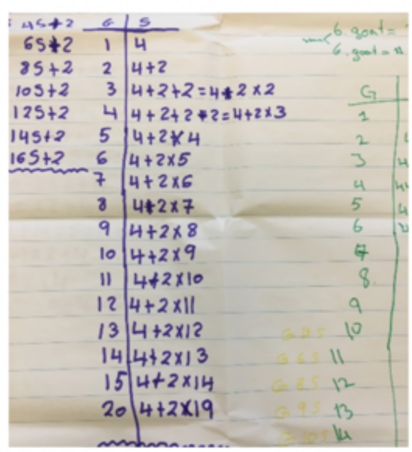
Discussion and Implications

Although the lesson described in this paper implemented a 5-Act Task in view of modeling as vehicle, the 5 Act Task Framework can be used for teaching modeling as content. Checking and supporting students' understanding in mathematics and language (Act 0) and communicating about solutions (Act 4) are essential for EBs in any type of modeling.

Figure 3

EB's modeling after teachers' guidance





Through this study, we found several keys to implementing modeling as vehicle to teach EBs challenging mathematics with space for language development. First, it is crucial to spend enough time on assessing and building students' understanding of the problem statement and situations by developing shared vocabulary, prior knowledge, and contextual understanding. Act 0 enhances every other Act and the modeling process because whenever teachers see students struggle, they can refer to Act 0 to redirect and remind students of the contextual clues they understood in Act 0. Since EBs have different cultural experiences from non-EBs, integrating contexts that EBs are familiar with into mathematics lessons is crucial when applying modeling for EBs. In this lesson, we chose the context of goats because goats are the most common domestic animal in the countries the EBs came from. Moreover, by contributing their own story and language, each EB had an opportunity to act as a valuable member of the class. Finally, the reflection process (Act 4) is often ignored when working with EBs due to time restrictions and EBs' difficulty in verbal presentations, but sharing what students found and learned is an important component of learning and language development.

The 5-Act Task Framework enables teachers to empower EBs in experiencing meaningful communication and deepening mathematical understanding through the modeling process. With this Framework, teachers can use modeling in multiple ways to effectively teach EBs while using rich cognitively-demanding tasks.

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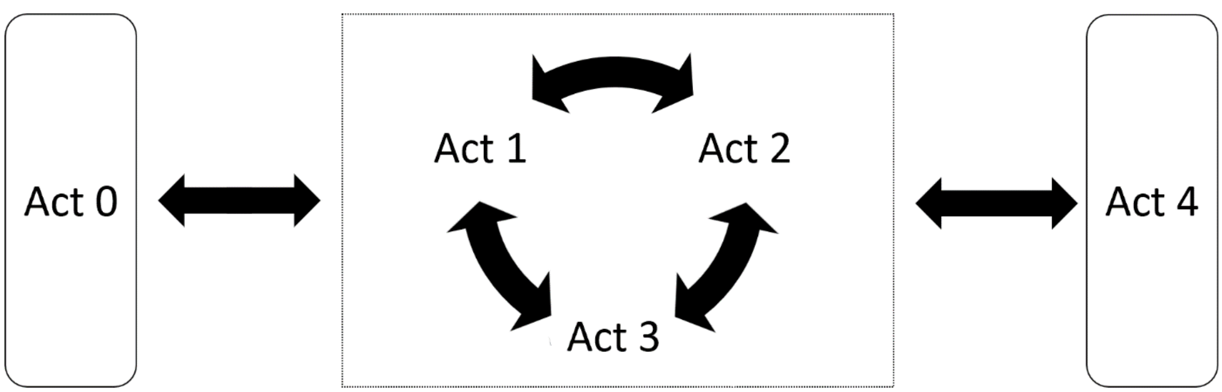
Appendix

5-Act Task modified for engaging EBs in mathematical modeling

	Description	Purpose
Act 0	Assessing/scaffolding before Act 1	To support EBs by providing background knowledge of the problem's language, context, and mathematics.
Act 1	A story with conflict via multimedia/visuals/physical movements and problem posing based on the story	To engage EBs in a relevant real-life story towards an authentic understanding of the mathematical situation and to empower EBs by having them pose their own problem(s).
Act 2	Student-driven collection of information to solve the problem they posed	To give EBs agency to reflect on the story and develop their own assumptions to solve them.
Act 3	Model construction with solution(s)	To provide EBs opportunity to visualize/model their mathematical process to analyze and improve their decisions in the real world.
Act 4	Collective communication of solutions/thinking process	To support EBs as they develop English proficiency while also building mathematical competency.



Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Consider a moment when you noticed your students struggled understanding a word problem due to language or culture differences. How did you respond in that situation? How do you think you would respond in the future now that you have read this paper?

2. What are additional ways to support EBs' sharing their mathematical ideas and solutions of mathematical modeling tasks?

3. What are some practices that help create an environment where EBs can feel safe to participate in the mathematical modeling process?

4. In what ways, if any, do you think the teacher's approach may have (or should have) differed if she were teaching a class that was 50% EBs and non-EBs rather than a 100% EB class?

5. How do you think the 5-Act Task Framework would work for other students who are not Emergent Bilinguals?

Exploring the Mathematical Agency of a Multilingual Child With an Identified Learning Disability

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Abstract

In this article, I illustrate how one student, Carlos, who is an emergent bilingual with a learning disability, expresses his mathematical agency dynamically and fluidly in multiple languages throughout 12 teaching sessions centered on mathematical discussions. The findings of this study show how Carlos made sense of fraction word problems, felt empowered to engage in conversations with peers about his thinking, and took ownership of his strategies. Implications are offered for the math instruction of bilingual children identified with a learning disability.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. In what ways would you adapt instruction to meet the needs of multilingual learners with identified learning disabilities?
2. How would you provide multilingual learners opportunities to take ownership of their mathematical ideas in your classroom?

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Exploring the Mathematical Agency of a Multilingual Child With an Identified Learning Disability

Juanita Silva

Carlos sat in the back of the room at a table in the corner six feet away from the rest of his peers during his fourth-grade math class. He faced his classmates and not the front of the room where the whiteboard was. Most of his peers were sitting in groups of two or three and were facing the board. At his table, he had a worksheet with mathematics problems. His worksheet had additional examples with solutions to some of the problems. As the teacher began to introduce the problems, Carlos began fidgeting with his pencil and then scribbling his answers without showing his work because he could solve these problems in his head.. The teacher then began showing the class how to solve for x, a missing supplementary angle. As the teacher demonstrated his [the teacher's] knowledge to his students, Carlos shouted the solution, "64." His peers and the teacher did not acknowledge his response. Soon Carlos stood up and went to sharpen his pencil even though his pencil seemed to have a sharp end.

Carlos was a Latinx student who had been identified with a learning disability (LD) and was classified as an English language learner (ELL) by the school district. In the vignette, I showcase an observation of Carlos's intentions to participate and demonstrate his abilities in a math lesson prior to this study. Carlos attempted to exert his agency by calling out a solution during the middle of instruction, his teacher and peers ignored him because he did not raise his hand to ask for permission to speak.

Research on mathematical agency with Latinx children has been a focus for many scholars (e.g., Gutstein, 2007; Martin, 2000, Sanchez-Suzuki Colegrove & Adair, 2014; Turner, Dominguez, Maldonado, & Empson, 2013). This literature explains the importance of children's agency and how it can greatly affect their mathematical learning (Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009). In contrast, much of the mathematics research on children with identified LD labels is usually related to student achievement (Lambert & Tan, 2017; Tan, Lambert, Padilla, & Wieman, 2018) and supports the use of direct instruction (Woodward, 2004). In intersecting the research of ELLs with identified LD labels, we find that there is an overrepresentation of Latinx children in LD in individual districts and states among the approximatly 9% of ELL students labeled with a disability in the United States (Artiles, 2013; Sullivan, 2011). Thus, it becomes critical to examine the experiences of Latinx dual-labeled children in special education. A few scholars like Lambert (2015) are beginning to focus on documenting the experiences of children with LD labels. In this work, she documented the mathematical experiences of two students with identified LD and found that when instruction focused on high stakes testing, children were "denied opportunities because of assumptions built on their labels" as needing "extra support" (Lambert, 2015, p. 15). Aligned with Lambert's work, this study offers an alternative to much of the LD mathematics research by focusing on the experiences of children with dual labels. I aim to address the gap in the literature by documenting the mathematical agency of Carlos when exposed to bilingual problem-solving discussions. I will unpack the following research question: In what ways does Carlos, a child with an identified learning disability, express mathematical agency when engaged in bilingual mathematical discussions?

Promoting Agency for a Marginalized Population

In general, the construct of *agency* is defined as the capacity to take action onto the world (Holland, Lachiotte, Skinner, & Cain, 1998). It is important to draw your attention to the idea that in every context, including mathematics education spaces, children are always exercising agency, taking or not taking action. In this study, I utilize Turner's (2012) construct of *critical mathematical agency*, where she defines as:

> student's capacity to (a) understand mathematics, (b) identify themselves as powerful mathematical thinkers, and (c) construct and use mathematics in personally and socially meaningful ways. (p. 55)

Specifically, I use this construct to identify the mathematical agency and agentic roles (Turner et al., 2013) exhibited by students with dual labels and to illustrate the ways in which they take actions during math discussions (Gresalfi, Martin, Hand, & Greeno, 2008). Particularly, I was interested in looking at instances where Carlos demonstrated agentive roles, such as seeing himself as a mathematician who had expertise, who could critique others' ideas, or could explain or justify his mathematical thinking.

Context

This study is a subset of a larger study, which involved Carlos and two other upper grades Latinx bilingual children, from an elementary school located in an urban city in the southern United States. Carlos was selected based on his identification as a Latino, who had ELL with LD labels, and who also had prior exposure to an *I do, we do, you do* mathematics instruction model (Harry & Klingner, 2014). Carlos was a charismatic child who was not afraid to be honest. I selected Carlos among the three participating children in the study due to his mathematical identity transformation during the context of bilingual problem-solving discussions. He shifted his view of himself as someone who was not a math person to someone who was capable of understanding math.

Data collection occurred during 12 50-minute instructional sessions over seven consecutive weeks. Sessions occurred during additional math instruction in a conference room equipped with manipulatives such as Unifix® cubes. The sessions were planned by the author and a research assistant using a teaching experiment methodology (Cobb, 2000). Base-ten and fractional tasks (Carpenter, Fennema, Franke, Levi & Empson, 2015) were used to help children make sense of their thinking.

Children's roles, including Carlos's role, were to attempt to solve problems in whatever ways made sense to them, to communicate verbally in any language they were comfortable, and to ask questions. As the teacher-researcher, I presented children with story problems, encouraged the use of prior knowledge and children's cultural background to facilitate discussions that promoted agency. I wrote word problems in English and Spanish, and the children and I used language flexibly (Garcia & Kleifgen, 2010; Planas & Civil, 2013) to help in making sense of and communicating ideas. I designed teaching to ensure that children were making sense of the word problems (e.g., Martin, what does the 11 represent in this story problem?; Jacobs and Empson, 2016). When appropriate, I also assigned competence to children's ideas by allowing children to share complete and incomplete ideas in multiple ways (Gresalfi, Martin, Hand, & Greeno, 2009). For example, I encouraged children to share or critique the ideas of others and to communicate these ideas using gestures, drawings, objects, or words. In facilitating children's power to enact choice and take actions that encouraged ownership and competence, I made several attempts to position children as *experts* (e.g., Jorge, can you tell us why María's strategy is valid?) and as *evaluators* (e.g., Martin, so you are saying you disagree with Jorge's ideas? Could you explain why you disagree?) of mathematical ideas (Turner et al., 2013).

A research assistant and I conducted ongoing analysis before and after each session and uncovered ways in which Carlos exhibited agency and shared his math thinking (Simon et al., 2010). Using the MAXQDA software, we identified 64 video episodes as coherent interactions around a single math strategy. Video episodes were transcribed and then analyzed for patterns of Carlos's mathematical agency, drawing on the literature on children's agentic roles (Turner et al., 2013). Video episodes were categorized by agentic roles, which included children seeing themselves as mathematicians who had expertise, who could critique others' ideas, or who engaged in actions to explain or justify their mathematical thinking. I selected examples of mathematical agency that showed gradual changes in degrees of sense making and ownership that allowed Carlos to identify as a mathematical thinker who had contributing mathematical ideas.

Carlos's Mathematical Agency

In this section, I illustrate three examples that depict Carlos's mathematical agency. Each expresses his degrees of ownership of his ideas, his convincing others of his reasoning, and his risk-taking to share newly developed mathematical strategies.

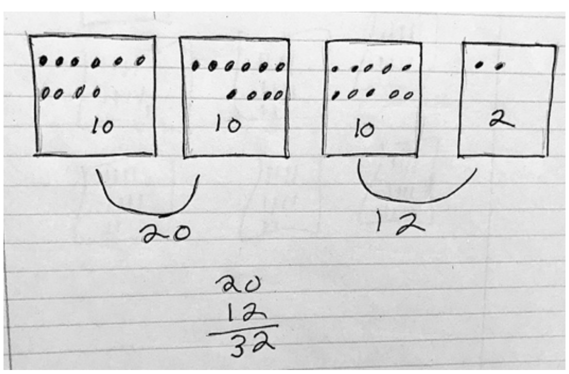
Carlos's Initial Math Agency

In initial sessions, Carlos would solve problems with standard algorithms, with no connections to the problem's story context. Carlos would say statements like, "I don't know," when asked to explain his thinking. At times, I would press for explanations (e.g., Why did you add 12 and 4 together?), and he would simply shrug his shoulders. Carlos resisted explaining his solutions and took little ownership of his ideas.

Changes in Carlos's sense-making began to occur in the third session when he began to solve problems using robust strategies. During this session, I provided a measurement division problem, where the total consisted of 32 toys, and the groups consisted of 10-toy bins. I wanted to know how many bins he could fill. In this interaction, Carlos began to make sense of his strategies, but Carlos hesitated in sharing his thinking. I considered that Carlos maybe needed some form of encouragement to share his thinking. Figure 1 shows how Carlos began to share his strategy.

Figure 1

Carlos's Strategy for 32 Toys in 10-Toy Bins



In addition to Figure 1, the following exchange occurred in English and Spanish, with language transitions between Spanish and English occurring flexibly in the same phrase or sentence. Spanish sections are translated into English and appear in parentheses.

> **Teacher:** Ooh this is interesting! ¿Este es un poco diferente a la estrategia de el? (*This is different than his strategy?*) [Teacher points to Martin's strategy.] ¿Me puedes decir como hiciste tu estrategia? (*Could you explain how you did your strategy?*)
> **Carlos**: I did, ten, ten, ten, two. I added twenty and twelve and then I added them up and got thirty-two.
> **Teacher**: You did ten, ten, ten, and two. And what does the twenty and the twelve represent?
> **Carlos**: Juguetes. (*Toys*.)
> **Teacher**: Juguetes, ¿y porque hiciste el veinte y el doce juntos? (*Toys, and why did you do twenty and twelve together?*)
> **Carlos**: I added them up.

Carlos had clearly made sense of the story problem by making groups of 10 dots inside each of his bins, and he then added two groups of 10 and one group of 10 with two to check that he had a total of 32 toys. However, he struggled in explaining his thinking. It was not until I stated in Spanish "This is different than [Martin's] strategy. Could you explain how you did your strategy?" that Carlos began to share the details of his strategy. In this instance, and many others, I made the decision to use English and Spanish flexibly, sometimes repeating what children said in Spanish or posing new questions in English to provide Carlos and his peers with opportunities to express their ideas in any way.

This excerpt described how Carlos took an agentic role as an "explainer" of thinking, as he began to explain the mathematical reasoning in his solution. Carlos was beginning to take ownership of mathematical ideas. In this example, Carlos began to see his ideas as valid mathematical strategies.

Carlos's Final Math Agency: Justifying and Taking Risks

As the sessions progressed, Carlos not only became comfortable sharing his mathematical thinking with others, but he also began to justify his thinking with others. For example, in session eight, Carlos began to engage in conversations with Martin, his peer, about his strategy when solving an equal sharing problem about eight chocolate bars and three children. In this exchange of ideas between the boys, Carlos tried to convince Martin of his strategy to cut the leftovers into three parts:

> **Carlos:** No es dos. (It is not going to be two.) [Carlos shakes his head and points to Martin's two leftovers on his page.]
> **Carlos:** [Martin looks at Carlos.] They all have to be in half. [Carlos is referring to the leftovers needing to be partitioned.]

Martin: Two are going to be left over.
Carlos: Yeah two leftovers, the two left over you can do, three, one, [Carlos points in the air and makes imaginary partition lines] and the other one too… Mira (look). [Carlos shows Martin his notebook where he has drawn two leftovers partitioned into three parts.]
Martin: [Smiles and nods.] Oh. [Martin then begins to make three partitions in his two leftovers.]

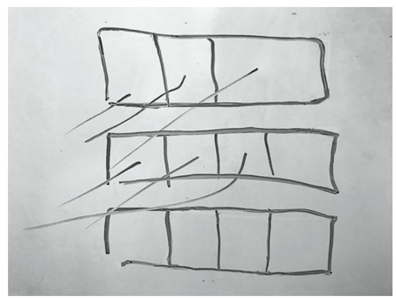
Carlos first initiated the conversation by questioning Martin's final answer of six chocolate bars and two leftovers. At first, Martin seemed confident about his answer of six chocolate bars with two leftovers. Carlos's questioning did not seem to discourage Martin from keeping his solution, so Carlos took it upon himself to justify the details of his solution to Martin, where he began to show the two leftovers cut into three pieces. After noticing the threes inside the leftover chocolate bars, Martin began to draw the leftovers and cut each into three parts as Carlos had done.

In this episode, Carlos believed that he had a valid strategy. Carlos justified his solution to Martin by gesturing the partitions in each of the leftovers and used language flexibly, such as using "Mira," meaning "look," to illustrate his thinking. Carlos's agentive role as a "convincer" was evident here, as he took complete ownership of his ideas in that he believed he was right. He then succeeded in helping convince Martin that his ideas were correct. Carlos exhibited mathematical agency as he actively persisted in convincing his peer, despite his peer's initial disagreement with his solution. He viewed his mathematical ideas as competent.

Towards the end of the sessions, Carlos had many opportunities to defend his reasoning and critically analyze his peers' reasoning. But in session 11, Carlos made a significant change in the way he expressed agency. In this example, Carlos, decided for the first time to solve a new problem in front of his peers without first solving it on his own (see Figure 2). The following excerpt describes a moment in which I decided to introduce this new problem (three candy bars shared among 4 people) to all three children and, to my surprise, Carlos took the lead.

Teacher: Si tengo tres barras de dulce [Teacher draws three rectangles on the board] y tengo cuatro personas. (If I have three candy bars and I have four people.) [Teacher draws four smiley faces.]
Carlos: Naah eso es fácil! (That is easy!)
Teacher: ¿Como los vas a compartir? (How would you share those candy bars?)
Carlos: [Carlos gets up from his seat, approaches the board, and grabs the marker from teacher.] Like this, look… [Carlos begins partitioning each of the three bars in half. Then he tries to make four pieces in each bar but forgets to partition one of the bars into four pieces, does three instead.]
Teacher: What did Carlos do? [Teacher directs question to the group.]
Carlos: I split them up into four.

Figure 2
Carlos's Strategy of Three Candy Bars Shared with Four People



Carlos began to take risks sharing his ideas about his peers' strategies and posing mathematical arguments as the sessions progressed.

Overall, Carlos gained a sense of pride in his ideas and his methods for solving problems. He no longer relied on the teacher to explain his strategies, so much that he took a risk in explaining his unfinished strategies to the group. In this instance, he showed his peers he was not afraid to tackle a new problem and solve it in front of them despite making small mistakes in the process. Carlos gained ownership of his mathematical thinking and showed how his mathematical agency transformed from resistance to participation in moving from explaining his thinking to justifying and convincing his peers of his ideas, to taking risks in exposing his initial unfinished mathematical strategies.

Implications and Final Thoughts

I sought to investigate the mathematical agency Carlos exhibited when instruction focused on the use of teaching moves that allowed choice and placed competence onto a child's thinking (Turner et al., 2013; Gresalfi, Martin, Hand, & Greeno, 2009). The findings of this study revealed that Carlos made sense of word problems, took ownership, engaged in conversations about his mathematical thinking, and, ultimately, felt empowered to co-construct mathematical ideas with peers. Carlos shifted from non-explanations of solutions to taking risks in sharing unfinished mathematical ideas. The flexibility to use language dynamically in the environment provided opportunities for Carlos to voice his thinking and engage in conversations with his peers that allowed him to engage in mathematical sense making and move towards higher degrees of mathematical agency.

This study offers an alternative to most of the research in special education that supports direct instruction (Woodward, 2004). This study focuses on instruction that gives a marginalized group of children opportunities to share, explain, and solve problems in English or Spanish. I believe some of the examples provided here give monolingual teachers a few ideas about how to promote the agency of a child with identified labels. For example, teachers could position children as experts or evaluators despite not knowing the native language of the child. Monolingual teachers could encourage multilingual children to discuss, critique, and justify their mathematical ideas with others in the classroom.

If the aim of research and practice is to provide equity for all, future studies in special education and mathematics should begin to integrate mathematical practices that promote choices for dual labeled multilingual children to use their native language flexibly whether teaching is by monolingual or bilingual teachers who do not know the language of the child, to work in small groups or individually, and to have opportunities to share their thinking with peers during instruction. Teachers could begin exploring different resources like Google Translate or someone who does know the langunge to support the multilingual child during mathematics instruction. Also, future studies should explore how different forms of teacher moves could encourage children to participate and share their thinking with peers who do not speak the same language.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. How would you use Carlos' story to reflect upon and analyze your teaching practice for multilingual learners with learning disability labels? Is it any different from your current teaching practice for all learners? If so why?

2. In what additional ways can you reflect on your teaching practice to leverage multilingual learners' with identified learning disability labels ways of expressing mathematical agency?

3. How else could learners express mathematical agency?

"DARE to Reach ALL Students!"

TODOS Mathematics for ALL

Learning to Leverage Obstacles, Resources, and Strategies in Math Classes With Multilingual Learners

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Abstract

Mathematics teachers must be ready for diverse classrooms, where students who are multilingual learners (MLs) bring new dimensions to the teaching and learning. While MLs face obstacles to learning particular to their linguistic and cultural background, they also bring resources and strengths to bear that can be applied to teaching and learning. We have developed a Challenge-Based Instructional activity to help teachers leverage their experiences of teaching math to ML students, to better understand the obstacles and resources, and to select more effective pedagogical strategies particular to this context. This paper reports the benefits teachers gain in implementing this activity.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What obstacles do multilingual learners (MLs) face in the English-taught mathematics classroom?

2. How can MLs' linguistic and cultural knowledge serve as resources for their mathematics learning during mathematics instruction?

3. How does teachers' knowledge of MLs' obstacles and resources affect their selection of teaching strategies?

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Learning to Leverage Obstacles, Resources, and Strategies in Math Classes With Multilingual Learners

Aaron T. Wilson, Hyung Kim, Mayra L. Ortiz, and Josef A. Sifuentes

Mathematics teachers in the U.S. have an increasing wealth of cultures and languages at hand in their classrooms. Although most teachers in the U.S. use only English to teach mathematics, children who speak a language other than English will often be more comfortable using a language other than English to communicate in the mathematics classrooms (Chval & Chavez, 2011; Gutstein et al., 1997). Consequently, teachers must have a deep understanding of multilingual learners (ML[1]) and learn how to teach mathematics within linguistically complex situations. Our research and observations with middle school teachers indicate that teachers can achieve this learning by envisioning and discussing specific teaching instances involving MLs. To help teachers develop a deeper understanding of the complexities of culture and language encountered when teaching specific mathematics content in the multilingual classroom, we created a unique professional development activity, called the "Teaching Multilingual Learners (TML) Project." In this paper, we present how we implemented the TML project and share findings from it.

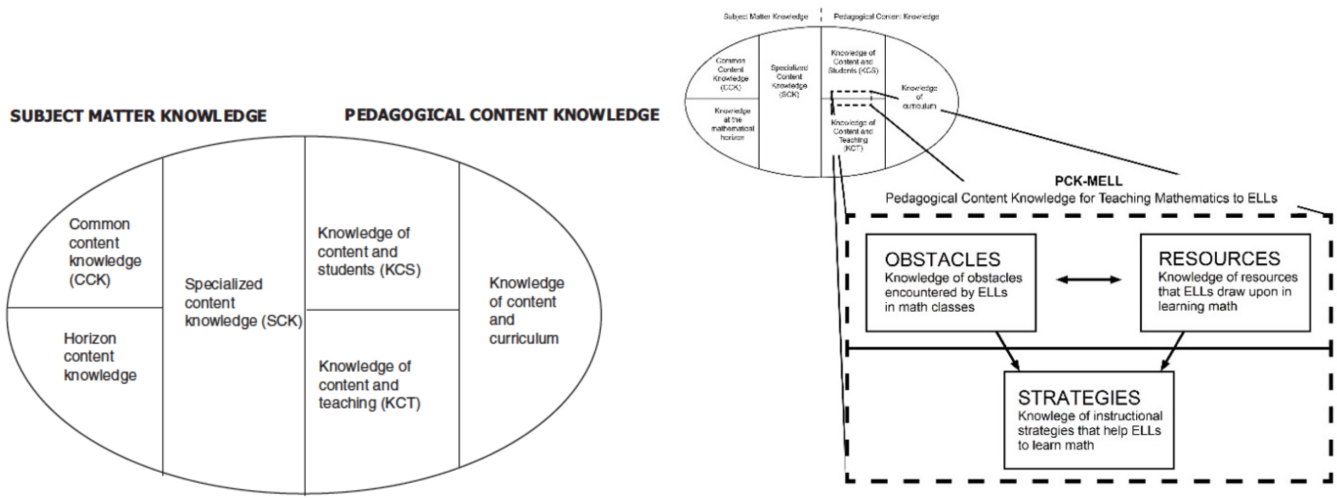
Theoretical Background and Overview of the TML Project

We designed the TML Project to develop pedagogical content knowledge (PCK) for teaching mathematics in diverse classrooms, specifically when MLs are present.

[1] Drawing from John (2019), we use the term *multilingual learner* in this paper, instead of the previously used terms *English language learner* or *emergent bilingual*, broadly to refer to a student who "can be an immigrant, a child of an immigrant who is bilingual, a permanent resident, a naturalized citizen, or an international student, who attends the academic English classroom today" and may come with "differing language abilities, learning styles, learning attitudes, and opportunities to communicate in English" along with "varying levels of inhibition, risk, and self-confidence" and "countries, cultures, sociocultural, sociopolitical, and socioeconomic backgrounds, all of which impact their learning significantly" (p. 41).

Figure 1

MKT (Ball, Thames, & Phelps, 2008; left) and Pedagogical Content Knowledge for teaching mathematics to ELLs (Wilson, 2016) framed within MKT (right) used by permission.



To do this we adopted Wilson's (2016) model of *pedagogical content knowledge for teaching mathematics to English Language Learners* (PCK-MELL) as a framework. The PCK-MELL model (Figure 1) draws on Shulman's seminal theorization of PCK (1986) and locates this knowledge within Ball, Thames, and Phelps's (2008) familiar model of *mathematical knowledge for teaching* (MKT). According to the PCK-MELL model, effective mathematics teachers of multilingual students draw upon three special aspects of knowledge related to their knowledge of content, students, and teaching, including knowledge of: A) Obstacles encountered by MLs in mathematics classes that are taught in English; B) Resources that MLs draw upon both to do and to communicate mathematics in these classes; and C) Instructional strategies that teachers may use to support MLs in mathematics, which is informed by teachers' knowledge of obstacles and resources. Wilson (2016) elaborates these domains of PCK-MELL in detail. For instance, *Obstacles* that ELLs face may include high-level speech formats of teaching (i.e., lecturing) and word problems that are linguistically complex by the multiplicity or multiple meanings (polysemy) of words. In contrast, *Resources* that ELLs draw upon in mathematics classrooms may include fluency in their non-English language to grasp and express concepts taught in English (by way of cognates, for instance), gesturing, and prior mathematical knowledge. Examples of *Strategies* include: teachers' usage of students' prior (mathematical, but also social and cultural) knowledge for teaching, using visual supports and displays (gestures, pictures, objects, word walls, etc.) and using students' own in-class writings and speech for teaching (Chval & Chávez, 2011).

A goal of the TML project was for teachers also to perceive MLs through an affordance lens: to see *difference not deficit* (Lewis, 2014) in MLs. The project was designed to help teachers to grasp the importance of the MLs' bilingualism and their diverse cultural backgrounds as resources in learning and communicating mathematics.

Phases of the TML Project

To accomplish these goals, we adapted a discovery learning method that has been used effectively in engineering education: *challenge-based instruction* (CBI). CBI uses a series of six Phases called the *Legacy Cycle* (Crown, Fuentes, & Freeman, 2012; Schwartz, Lin, Brophy, & Bransford, 1999), in which students are first given a *challenging problem*; they then *generate ideas* about the problem; they consider the challenge from *multiple perspectives*; they *research and revise* a plan for solving the challenge; they *test their mettle* by trying out their ideas and solutions; and finally, they *go public* by displaying their findings. We explain below the phases of the project as they were designed to be carried out in the TML project.

Phase 1: The Challenge. At the outset of the TML project, the teachers were formed into teams of two or three "assessment item-writers" and given a version of the following challenge:

> *You and your colleagues in the Psychometrics Department of the State Education Agency (SEA) have been tasked with drafting the next generation of teacher licensure exams. Policymakers are calling for exams that are more reflective of the actual content and actual tasks of teaching mathematics. Your group will work closely with the Equity Taskforce of the SEA to ensure that the exam items respond to a particular need: teaching mathematics to multilingual learners. This project will require your team to create three multiple-choice assessment items designed to measure whether prospective teachers who are about to receive their teaching licenses know how to teach specific mathematical concepts from school mathematics to students who are multilingual learners.*

The item-writing task given above played the central cognitive challenge in the TML project. It elicited interest in the topic and motivated effort toward the following tasks.

Phases 2 and 3: Generate Ideas and Multiple Perspectives. Per the CBI Legacy Cycle, completing the challenge required that teachers first generate ideas about teaching mathematics to MLs. Without precisely defining them or offering examples, we required that teachers initially thought about (brainstorm) obstacles faced by MLs and resources possessed and used by MLs in math classes. This was so that teachers could begin to envision the work of teaching MLs and discovering what knowledge would be needed. After developing brainstormed lists of obstacles, resources and strategies, teachers were instructed to consult with additional, text-based and other, sources of ideas about teaching mathematics to MLs. Furthermore, we required that teachers contextualize the teaching of MLs in a specific school mathematics topic.

Phase 4: Research and Revise. In this stage, the teachers wrote their items, submitted them to their group members and often (but not necessarily) to the professor for comment and criticism, and then revised their assessment items. Furthermore, to facilitate their careful consideration of more and less formidable obstacles for MLs, and of more and less powerful resources within these MLs' grasp or available teaching strategies, teams of item-writers were asked to justify the correctness of the "correct" answer options for their multiple-choice items and to explain the incorrectness of all "distractor" options in detail.

Phases 5 and 6: Test Your Mettle and Go Public. In this stage, teachers submitted their licensure items for discussion in the larger group. Discussing items with peers, by justifying their "correct" or "incorrect" answers, was an important procedural part of the TML project. When teachers were asked to "Test their Mettle" and "Go Public" with their assessment items, they were encouraged to articulate their thinking about teaching mathematics to MLs, and they responded to the myriad "What if?" questions their peers raised that challenged their thinking or altered the "correct answer" by modifications to the variables in their items.

Implementing the TML Project

This section illustrates our actual implementation of the TML project with participants.

Participants and context. While we originally planned to use the TML project with pre-service teachers, this paper's implementation context is from a week-long summer mathematics professional development institute for 35 in-service middle school math teachers from different U.S. school districts along the U.S.-Mexico border.

Teachers were predominantly from cities with large percentages of people who are of Latino/as or of Hispanic heritage, and who worked at public schools that have large numbers of MLs from Spanish-speaking backgrounds. Hence, it was imperative to us that teachers gained a deeper-than-surface familiarity with complexities that would arise when teaching in linguistically and culturally diverse classrooms. Many of the teacher participants had novice to expert fluency in the Spanish language, and some also reported to have been MLs during their primary or secondary schooling. Consequently, the outcomes of this study mainly concerned MLs from Spanish-speaking traditions. However, we believe that the core ideas and findings from

TML project could be adapted for usage in other linguistic or ethnic contexts.

Modifications of TML Project. Our actual implementations of the phases of the TML project have varied slightly depending on the professional development context and whether in- or pre-service teachers participated. During this summer implementation, the TML project was introduced and completed entirely on one day of a week-long mathematics professional-development institute. Completing the project in one day required modifications to the phases of the project (explained further below) and may have limited the breadth and depth to which teachers could develop their thinking. For example, the generating ideas, researching and revising phases were essentially seamless. Spreading this work over two or three days would have allowed for more research and collaboration. Yet, completing the TML project over one day seemed to provide a self-contained and focused look at teaching mathematics with MLs.

Outcomes of the TML Project

In this section, we illustrate in-service teachers' thinking about the *Obstacles*, *Resources*, and *Strategies* involved with MLs in mathematics. We first explain the item-writing tasks that we used to prepare teachers for the main work of the TML project, both to contextualize the outcomes and to help readers consider implementing the project.

At the start of the day, we began Phase 1 by setting up the challenge, informing teachers that they would be working in groups of four people as "educational licensure test-developers." In our prior work on the TML project with PSTs, we had observed that participants can struggle with the abstract thinking required for writing items intended to measure teachers' knowledge about teaching mathematics. Hence, to introduce the kind of thinking that they would be doing as well as to bring MLs into the focus, we discussed in whole-group an example test item from a teacher licensure exam, an item concerned with MLs. By analyzing such a test item, teachers could deduce the kind of teacher knowledge that the item was intended to measure, and how the distractor options failed to capture that knowledge. But an important result of looking at the selected test item was also in demonstrating the lack of relevancy of some such items to the actual work of teaching; teachers commented that the item seemed to come from a textbook and not from the classroom. This observation motivated them to try to write more authentic items!

Furthermore, we introduced teachers to the two, seemingly opposing, PCK-MELL domains of *Obstacles* and *Resources* by having different groups read and think about teaching vignettes found in the literature concerning MLs: a vignette concerning *Obstacles* that MLs face in math classes (many examples exist) and a vignette concerning MLs' usage of language as a *Resource* (a nice example is given in Moschkovich, 2005, p. 133).

Teachers' Knowledge of Obstacles, Resources, and Strategies

Phases 2 and 3 involved having individual groups brainstorm lists of either *Obstacles* or *Resources* in place with MLs, or *Strategies* for teaching MLs mathematics that were then shared with the whole group for discussion and revision. The outcomes of this initial brainstorming task were insightful in several ways. To begin with, compared to similar lists evinced by PSTs with whom we have completed the TML project, the number of items in each PCK-MELL category that the in-service teachers were able to identify were much more extensive, an indication of the in-service teachers' more advanced teaching and learning experiences.

With respect to obstacles faced by MLs, teachers identified different ways in which language was a problem ("academic vocabulary vs. social language"). They also elaborated the extent to which language was seen within the instruction phase of teaching, as well as throughout the assessment phase by their own statement that, "Everything is verbal." Teachers identified problematic linguistic structures that cause students difficulty (see Martiniello, 2009): homonyms, homographs. But they also named several non-linguistic obstacles to MLs' learning of mathematics (e.g., self-confidence), as well as obstacles found in students' school and home environments (e.g., the threats of bullying, or economic hardship). For example, teachers recognized that some MLs faced bullying at school or faced instability and poverty at home, which could be formidable obstacles to learning.

With respect to resources that their MLs drew upon, the number of resources teachers articulatedexceeded the number of obstacles, which had not previously happened in our work with PSTs. Examples of resources named included: prior knowledge, cognates, peer tutoring, visuals, word walls, gesturing, and native language use. This may be a positive indication that participating teachers in this group were better able to see the unique assets that MLs possess, beyond their challenges alone. But it was also evident that the resources and strategies lists were not disjoint; several resources were also listed as strategies, and this overlap was addressed during whole-group discussion. As teachers explained it, *when a teaching strategy has been implemented, that strategy often results in a resource that MLs then utilize for their own learning*. Word walls and peer tutoring were examples: Teachers used them as strategies and MLs used them as resources. Indeed, peers who also speak the same languages can be among the greatest resources for MLs in mathematics classes, where translating concepts into a different language benefits both the ML and the bilingual peer.

Operationalizing Obstacles, Resources, and Strategies: Writing Test Items

At this point (and after a good lunch!) teachers were ready for Phases 4 – 6 of the TML project: writing test items in groups and sharing with others. Three selected assessment items are presented from teacher groups that focused, respectively, on Obstacles, Resources, and Strategies.

TML Example 1: Obstacles That MLs Face in Mathematics Classes

Teacher group #7 focused their work on obstacles MLs face and they created the following situation involving an ML in the classroom of a teacher who used "long lectures:"

> *Mrs. Yanez, known for her long lectures, has a new ELL [English language learner] student in her class. Chum Lee comes with a strong concept of math, but very limited in the English Language. Which obstacle would affect this student the most?*
>
> A. *Everything is verbal*
> B. *The confidence level*
> C. *Academic Langauge*
> D. *Comprehension*

This simply stated item presents a complex situation and decision for the respondent. The teachers who wrote the item intended to present a situation in which the ML was of a different linguistic and cultural background than the instructor: Mrs. Yanez is evidently Latinx, while Chum Lee is Asian. Furthermore, Chum Lee has limited knowledge of the English language, but is perceived as being strong in math.

The potential obstacles that would affect Chum Lee illustrate the different perspectives the teachers who wrote the item took, and they are also reflective of obstacles discussed in the earlier brainstorming session of Phase 2. Some obstacles here seem to be related to Chum Lee's external classroom environment and some to his own prior experience. For instance, "Everything is Verbal" may refer to the nature of Mrs. Yanez's instruction or to the emphasis in standardized state assessments, and hence in Mrs. Yanez's class, on solving word problems. Contrarily, "Confidence Level," "Academic Vocabulary" and "Comprehension" seem to imply limitations found within Chum Lee's prior experiences that the teachers envisioned as obstacles for this student. As teachers discussed this item, the relative difficulty posed to Chum Lee by any of the four given options was deliberated by all: each item could be a real obstacle. Some teachers found it difficult to choose the greatest obstacle. To others the choice was clear.

TML Example 2: Resources That Multilingual Learners Call Upon in Learning Mathematics

Teacher group #3 focused their work on resources that MLs employ in their classroom involving mathematical and grammatical complexity perhaps familiar to teachers of algebra:

> *Mr. Zamora is teaching 7th grade students sequences. He knows the students will have difficulty comprehending why <u>3n</u> means <u>3 times n</u>. Which of the following resources will help address the students' difficulty?*
>
> A. *Prior knowledge*
> B. *Peer tutoring*
> C. *Vocabulary*
> D. *Visual Model*

The item describes a challenging mathematical situation for both MLs and students who speak English as their first language. This is due to a symbolic representation of a concept that students may not be familiar with regardless of their language background. However, the teachers who developed this item considered that MLs may find an even greater challenge than those students who speak English as their first language, because the explanation of "3*n*" could heavily depend on a verbal description. The answer options presented as potential resources for MLs showed different perspectives taken by participating teachers, as in Example 1. The choices also reflected earlier brainstorming (as described in Phase 2), where teachers engaged in potential resources possessed uniquely by MLs.

Furthermore, from the perspective of PCK-MELL, we noted how closely teachers' knowledge of obstacles students faced was linked with the ways that teachers expressed knowledge of resources that MLs could draw on to learn mathematics. The situation begins with an anticipated "difficulty" to which a list of optional "resources" could be applied as solutions. We also noticed that, among the resources offered, two of them could be advantages already in MLs' possession (prior knowledge and vocabulary), and one was a resource that teacher could offered (peer tutoring), the fourth option (visual model) could originate with the ML or the teacher could provide. These options also reflected the apparent intersection of MLs' resources and teachers' strategies discussed above. But, the authors of this item offered us a real challenge: selecting which of these resources would best help the ML to arrive at an understanding of the targeted symbol is not easy. In group discussion of this item, it was noted that, "one size does not fit all," and different resources would vary in value to different MLs.

TML Example 3: Strategies for teaching MLs mathematics

Teacher group #5 focused their work on the strategies for teaching MLs. The following item presented the integration of four instructional strategies suggested for the benefit of an ML student learning about unit rate and slope:

Which of the following strategies would benefit an ELL [English language learner] student learning about unit rate/slope?

A. *The teacher will model an example for the student to implement on their own*
B. *Provide students with manipulatives to reinforce the concept*
C. *Provide a word wall as a reference*
D. *Small group instruction to provide guided practice on the concept*

This item demonstrates the value of the Phase 2 brainstorming to the TML project: each answer option had been specifically suggested during the brainstorming phase. The four answer options portrayed diverse instructional strategies to facilitate the learning process for MLs, which included both teacher- and peer-led components, as well as visual supports and displays. Finally, it became evident in whole-group discussion that all students, and not only MLs, would benefit from teachers' usage of these strategies. Teachers acknowledge that almost all students are supported in mathematics through the strategies set forward. Therefore, discussion of items like this one focused on carefully selecting the strategy that, beyond being helpful for all students, was singularly essential for MLs.

Findings of the TML Project

Based upon our implementations of the TML project, we made two observations relevant to teachers' gaining and sharing of knowledge about MLs in the mathematics classroom. First, the TML project has the potential to highlight multilingualism as a resource for students. In-service teacher participants developed rich lists of resources associated with MLs. In our work with PSTs, the process of finding and generating ideas about MLs' resources in specific mathematics contexts has been more challenging for them than producing ideas about MLs' obstacles or even strategies for teaching them. This may be related to a tendency to perceive nontraditional students, such as MLs, through a deficit lens rather than an asset lens (Gutiérrez, 2008; Lewis, 2014). The TML project enables us to address this imbalanced perspective and to bring into teachers' sight the particular advantages and resources that MLs have in learning mathematics.

Secondly, teachers thought about MLs strategically. Multiple-choice items written by in-service teachers tended to require a decision between better and worse instructional strategies or teaching decisions. Even items that were intended to indicate knowledge of obstacles or resources that MLs encounter were frequently framed as decisions about what the teacher should or should not do in the circumstances. Compared to PSTs we have worked with, it seemed that in-service teachers showed strength in translating knowledge about obstacles and resources into actionable strategies, which is also implied by the two downward arrows from obstacles and resources to strategies (Figure 1). Furthermore, most teacher items presented an obstacle (not resource) followed by multiple choices between strategic decisions. Hence, we observed that the action from obstacles to strategies seemed to be stronger than from resources to strategies. This observation may be related to the aforementioned challenge that some participants have had in identifying MLs' resources. But this difference may also be explained by observing that teachers' possession of actual classroom experience may equip them with more strategic perspectives oriented toward handling challenging situations in the classroom.

Suggestions for Using the TML Project

The major goal of the study was to understand how the implementation of the TML project might assist teachers to develop a deeper understanding of the complexities of culture and language commonly encountered when teaching mathematics in classrooms with MLs. The discussions that occurred between in-service teacher participants in our implementations of the TML project were very lively. Framing issues as multiple-choice questions that have only one correct answer elevates, perhaps artificially, but we believe beneficially, the stakes. Participants really want to know, "What is the right answer?" Nevertheless, we suggest some cautions and areas for improvement of the TML project. First, the facilitator should give careful attention during the activity at the outset of the project in framing and preparing teachers for the item-writing task, which can be challenging for participants at a meta-cognitive level. As mentioned earlier, participants can have initial difficulty in understanding that they are instructed to write multiple-choice questions about teaching specific mathematics topics and not multiple-choice questions that are merely mathematics problems. PSTs and in-service teachers completing this project may need more time and more examples.

Second, the items that teachers in our sample produced, which were often classroom problem situations, possibly including hypothetical student work or MLs' spoken responses, were primarily derived from teachers' subjective personal experiences. This was expected and intentional, given the one day of the professional development that was allotted for the activity. Our results indicate that the teachers understood the needs of the MLs, the obstacles MLs encountered, and how the resources and strategies teachers identified and used can impact the whole learning experience. A more prolonged activity would allow for a deeper Research and Revise phase of the project in which teachers could compare their own experiences against relevant results found in the research and practitioner literature. After completing the TML project, teachers may even benefit from considering other aspects of their work, such as lesson-planning, through the lens offered by the PCK-MELL framework.

Conclusion

We find that when teachers write and discuss multiple choice items in the TML project, implicit beliefs about what they should know and do with MLs are brought forward for detailed consideration and discussion. The focus may be expanded to what teachers of other particular groups, traditionally marginalized groups for instance, need to know and do to be effective teachers. There are many ways to frame the challenge, and the results should produce lively and fruitful discussion.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What obstacles do multilingual learners (MLs) face?

2. What linguistic and cultural resources do MLs possess and access, and how can teachers leverage these factors to make strategic instructional decisions that positively impact mathematics learning?

3. How would you modify the TML project to address a different aspect of mathematics teacher knowledge? For instance, how could this project assess knowledge for equitable mathematics teaching more generally or mathematical knowledge for social justice teaching specifically?

4. How would you modify the TML project to uncover obstracles, resources and strategies related to different special student populations, such as MLs from non-Spanish-speaking backgrounds, hearing-impaired students, or gifted students?

Preparing Bilingual Pre-Service Teachers to Foster Equitable and Open Communication With Latinx Immigrant Parents *en la Enseñanza de Matemáticas*

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Abstract

We examine how bilingual pre-service teachers developed a practice of communicating to parents their children's mathematical thinking and how the teachers invited parents to participate in instructional practices in the mathematics classroom. We argue that these practices are knowledge-intensive, in that bilingual pre-service teachers draw on both their knowledge of children's mathematical thinking and their own experiences as bilingual students, and that communicating this to parents reflects this knowledge. We conceptualize this knowledge as situated in, and integrated with, the practice of teaching. We therefore consider it necessary to support the development of this knowledge early in pre-service teacher education.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. As a mathematics teacher educator, how do you incorporate the lived experiences of bilingual pre-service teachers (BPSTs) into their preparation as mathematics teachers?

2. In your experience preparing future teachers, what are specific assignments or activities you use to teach future teachers to communicate with parents and involve them in the teaching and learning of mathematics?

3. As a mathematics teacher educator, how do you prepare BPSTs to notice and interpret children's mathematical thinking to inform their instructional decisions when teaching mathematics to bilingual elementary school students?

4. How are you, as a teacher educator, preparing BPSTs to communicate knowledge of children's mathematical thinking effectively to parents?

5. In your own experience as a teacher, what have you identified as the most effective way to communicate with parents and involve them in their children's mathematics learning?

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Gladys H. Krause and Kiyomi Sánchez-Suzuki Colegrove

Introduction

This paper builds on previous work that proposed the creation of a bridge by which Latinx immigrant parents and schools could engage in a dialogue that establishes equitable instructional practices of open and dynamic communication between both parties (Colegrove & Krause, 2017). The communication we refer to in this paper is framed within the mathematics curriculum following the findings from our previous work: (1) Latinx immigrant parents expressed feeling tremendous responsibility to support their children's schoolwork at home, (2) Latinx immigrant parents made clear their desire for open communication and teamwork between the school and home emphasizing that this communication includes curricula, (3) the ways in which Latinx immigrant parents' navigated the differences in mathematics teaching methodologies in the United States and their countries of origin. In our previous work, we proposed that teachers must engage in dialogue with families and that such dialogue needs to be initiated by the teachers at schools. Starting a dialogue might seem simplistic, but is, in fact, complicated. It engenders a natural series of questions:

- What should this dialogue entail?
- Where and how should it start?
- What aspects of teaching and learning should be included in this dialogue?

In this paper, we share how our previous work led to practical changes in the classroom. The first author modified assignments and developed a Problem Solving unit in her bilingual mathematics methods courses to start building a bridge for meaningful and reciprocal communication between pre-service teachers and Latinx immigrant parents. The work we present in this paper is just one step forward after our initial work with Latinx parents. We do not intend to present a "solution" for a problem that has deep historical, political, social and economic roots. Neither does this work present answers to the questions above, nor do we think these questions can be answered by implementing an assignment in a methods class. However, our work here foregrounds instructional practices that can be used in bilingual mathematics methods courses to better understand bilingual pre-service teachers' (BPSTs') perspectives on teaching and learning mathematics. This understanding, in turn, contributes to the development of a teaching practice that is culturally inclusive.

Connection to Literature

In most mathematics education courses pre-service teachers (PSTs) learn how to make connections with out-of-school experiences through coursework and assignments that allow for these opportunities. For example, TEACH Math (Drake et al., 2015) has a complete module, Community Math Exploration, consisting of three activities: a community walk, a mathematics lesson, and a final writeup and reflection where PSTs typically work in groups to design a problem-based task or lesson that connects to the communities where they would teach (Turner et al., 2015). These modules have been used and adapted in many mathematics methods courses around the country (Krause & Maldonado, 2019).

Research exists on how PSTs learn to make mathematics connections to children's home and community (Foote et al., 2013; Turner et al., 2012). Turner et al. (2012) make the point that despite this common practice, PSTs nevertheless spent limited time in their classrooms learning about how to make these connections and had limited exposure to the communities where they would teach. Moreover, even after identifying mathematical practices in students' homes, research has found it difficult for in-service teachers to link those practices directly to school mathematics (Civil et al., 2005a). From our own experiences as teacher educators and researchers, we agree with Turner et al. (2012) that teacher preparation programs devote too little class time to making connections between parents and school and to preparing teachers to work in meaningful ways with parents. The dilemma, then, is how to incorporate mathematical practices into teacher preparation programs in ways that are meaningful to students in school classrooms and which do not trivialize the mathematical ideas inherent in those practices. This issue remains a significant one for mathematics education research (Wager, 2012).

The research cited above refers to PSTs, not BPSTs. This is an important distinction. Research shows that bilingual teachers are more prepared to support the learning of students whose first language is not English, not only because they can make connections with the cultural resources in themselves and their students, but also because – to a greater degree than non-bilingual teachers – they position bilingual learners as contributors of knowledge and ideas in the classroom, valuing their expertise and ways of knowing and learning (Celedón-Pattichis et al., 2010; Musanti et al., 2009; Sleeter et al., 2015; Turner et al., 2013). Unfortunately, while this list of empirical work is significant, the research community has not yet focused on the experiences of BPSTs learning to teach mathematics.

Krause and Maldonado (2019) presented findings from their study of 28 BPSTs, in which they identified common experiences within this group of BPSTs around growing up and learning mathematics. The stories shared helped to illuminate how these experiences shaped the BPSTs' identities as mathematics teachers, and the depth of their educational insights. Their findings, though specific to the situation in which the BPSTs' linguistic and cultural backgrounds parallel those of their bilingual students, align with what a large body of research has also found about bilingual teachers. Based on the literature, the fact that BPSTs share the experience of moving between cultures, whether they be the same two cultures as their students, allows them to better imagine themselves in their students' shoes. In the specific case of Spanish-English BPSTs, which our study treats specifically, they often enter and remain in teaching, driven by a commitment to give back to their own communities and to challenge the inequities and injustices they themselves experienced in schools for students who share their cultural backgrounds (Achinstein & Aguirre, 2008; Achinstein & Ogawa, 2011; Krause, 2014). This might suggest that BPSTs who share the linguistic and cultural backgrounds of their students find themselves in a more advantageous situation for teaching than PSTs who do not share this linguistic and cultural background.

In this article, we build on the BPSTs own experiences learning how to teach mathematics learners. We describe the design of a Problem Solving unit that was implemented in the first author's bilingual mathematics methods courses, and what we learned from it in order to support BPSTs to start building a bridge of reciprocal communication with Latinx parents.

The BPSTs in our Mathematics Methods Course

Participants in the analysis we present here are 22 BPSTs enrolled in two bilingual mathematics methods courses in the third year of their teaching preparation program at a university in the southern United States. All BPSTs were English-Spanish bilinguals receiving instruction in Spanish.

The Assignment: Problem Solving Unit

Throughout the semester in this mathematics methods course BPSTs worked on a series of assignments designed to assess their learning experiences in the classroom and their field experiences. The textbook required for the course was *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2014). We also used excerpts from *The Impact of Identity in K-8 Mathematics* (Aguirre et al., 2013), *Teaching Developmentally* (Van de Walle et al., 2013), *Beyond Good Teaching* (Celedón-Pattichis & Ramirez, 2012), and articles from different authors (e.g., Dominguez & Adams, 2013; Karp et al., 2014; Martínez & Ramírez, 2018; Torres-Velasquez & Lobo, 2004). The course placed a strong emphasis on parents and communities. Our approach to mathematics education is grounded on the concept of parents as intellectual resources (Civil & Andrade, 2003) and parents were positioned as contributors and we sought to learn from them. Through this lens, we worked to challenge the rhetoric of lack of parent involvement (Delgado-Gaitan, 2001). We purposefully focused on the strengths and assets of the families and communities with which the BPSTs in our class worked in order to change the focus from *needs of* the communities to the *possibilities presented within* the communities (Guajardo & Guajardo, 2002).

In this paper, we are only presenting and describing the Problem Solving unit we developed. In preparation for developing the unit, BPSTs viewed video examples of clinical interviews, read the Ginsburg (1997) chapter on conducting clinical interviews, and had substantial conversations about mathematical problem types, students' mathematical strategies, and how teachers can elicit mathematical thinking through questioning.

The Problem Solving unit consisted of three parts: the first derived from an instructional task within Teach Math (Turner et al., 2015), the second a modification of the existing Teach Math protocol, and the third an addition created for the specific contexts of the first author's mathematics methods course. (1) A Getting to Know You Interview. Here BPSTs choose a focal student from their placement who was different from them in one or more socio-cultural aspects (e.g., gender, race, home language). (2) A second interview for which the BPSTs were required to design their own word problem for their focal student. They were asked to explain why they chose the problem type for the student, the context, and the problem's numbers. In addition, BPSTs were asked to write the problem in English and Spanish and choose their language of preference to conduct the interview. The purpose of this interview was for the BPSTs to assess how a child solved mathematics problems that included addition, subtraction, multiplication, division, or base-ten concepts and to examine their role in supporting and extending their focal student's mathematical thinking. For this part of the unit, the BPSTs were also asked to write a follow-up problem informed by what they learned on the two interviews, and conduct a third interview. (3) The last part of the unit consisted of BPSTs examining and assessing what they learned from their student's mathematics understanding and sharing their documentation and observations with the student's parents. For this part of the unit, BPSTs were free to select the format of the information compiled for the parents. However, they needed to ensure the content addressed the following: (a) What their immediate goal was in giving the student the specific problems selected; (b) What the student knew; and (c) What their overarching goal was for their focal student's mathematical development.

Findings

The BPSTs in our study communicated their knowledge of children's mathematical ideas to parents in different ways. BPSTs often focused on communicating aspects of mathematics instruction and curriculum, which is not surprising given the emphasis we placed on children's mathematical thinking over the course of the semester. BPSTs also foregrounded two other aspects in their communication with parents: bilingualism and culture, and parent involvement in supporting children's work on mathematics. We report below on BPSTs' attention to the three aspects they focused on in their communication to parents. All names used in reporting the findings are pseudonyms.

Focus on Content: Mathematics and Children's Mathematical Thinking

Most BPSTs chose the format of a letter to communicate with parents. Regardless of format, in all assignments

BPSTs started by focusing on positive aspects of the students, and to begin the conversation with parents they overwhelmingly focused on the mathematics that the students knew and understood. Benito, a BPST placed in a fourth-grade classroom, wrote:

> Throughout this semester I have been working on math with Elena, specifically division problems. I took an interest in her division because she expressed to me that it gave her the most trouble. I know that Elena is very smart and she is very capable of solving division problems, so I sat down with her to see how she practiced solving these types of problems. I designed a math problem to see how she divides different combination of numbers. I made sure the context of the problem was relevant to her.

Above we notice how Benito describes his interest in what Elena can show him about her understanding of division before he makes any statement regarding whether she understands division or not. Benito wrote a partitive division problem (Empson & Levi, 2011) (Problem 1 in Figure 1), a type of problem we introduced in our course while learning about the different strategies children typically use to solve them.

Figure 1

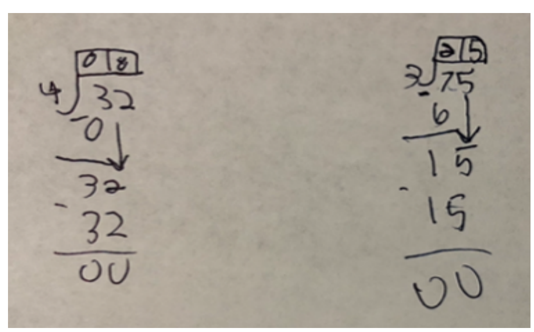
Partitive division problems Benito wrote

Problem 1	Elena just received a letter from her aunt in Germany! She was so excited about receiving a letter that she decided to write back to her aunt and to all her family members still living in Germany. After writing all the letters, Elena also decided to include photos in them. Elena wants to send ____ letters and include the same number of photos in each letter. She has a total of _____ photos that she wants to mail. How many photos are in each letter? (4, 32) (3, 75)
Problem 2	Elena enjoyed mailing her family in Germany very much. She thinks it will be really fun to send her friends pictures of what she did over spring break. Elena wants to mail her friends ____ pictures. How many pictures will each friend get if Elena is going to send ___ friends mail? (28, 6) (30, 11) (344, 8)

Benito provided two number choices for the problem. These number choices were also part of the Problem Solving interview assignment: BPSTs are asked to make instructional decisions and justify why they selected the numbers for the specific problem they gave to the student (Krause et al., 2017). Benito continued his letter, saying: "I asked her to solve this problem using the following number pairs: (4, 32) and (3, 75). She was able to solve the problem without any problems. So I wanted to push her some more to get to the root of why she thought she struggled with division." (Figure 2)

Figure 2

Elena's division strategies



Benito continues to explain his instructional decisions to the parents, showing them how he changed the problem and number choices for Elena (Problem 2 in Figure 1).

> I altered the original problem and number options to push Elena to show me more of what she knows. The context of the question remained the same, but the order of the wording was changed to get her to think of the problem in a different way. This time, Elena had some trouble solving the problem using the number set (30, 11). She tried but could not solve it using the standard algorithm. I asked her to try it one more time and she said she could solve it using the spider method. The spider method involves placing the number that is being divided in the center, and then draw legs equal to the number that it is being divided by. She then used the spider method to solve the problem correctly.

Benito's instructional decisions were informed by Elena's own thinking, and he was able to identify the root of her struggle with division: when she tried using the standard algorithm for division with two-digit numbers (traditional "long division" in the United States system), it did not work for her. However, Benito let her use a method that worked for her, and she solved the problem. It appears that Elena's struggle was not with the concept of division, but with the method employed to solve the problem.

It is also important to highlight that BPSTs used positive language and identified areas of strength, instead of using deficit language or focusing on what students did not understand or lacked.

Focus on Language: Bilingualism and Culture

Olivia, a BPST placed in a second-grade classroom, shared this reflection (in Spanish) on how she decided to approach the use of language while working with her student:

> *Cuando le dí el primer problema, hablamos en "Spanglish" porque así es como habla él en el salón. Quise que él se sintiera cómodo y que el lenguaje no fuera una barrera para resolver el problema. El problema estaba escrito en Inglés y Español.*
> [When I gave him the first problem, we spoke in "Spanglish", because that is how he speaks in the classroom. I wanted to make sure he felt comfortable and that the language was not going to be an impediment to solving the problem. The problem was written in English and Spanish.]

Gabby, another BPST also placed in a second-grade classroom, shared this reflection with a parent:

> *A diferencia de Misha, yo crecí en una familia que sólo hablaba Inglés, mientras que Misha tiene muchas exposiciones de idiomas en casa, lo que beneficiará su trabajo escolar. Me ha comentado que sus abuelos son de la República Checa y esto le hace sentir orgulloso. Él me ha dicho que usted le habla tanto en Inglés como en Español en casa.*
> [Unlike Misha, I grew up in a family that only spoke English, while Misha has exposure to many languages at home, which will benefit his school work. He has mentioned that his grandparents are from the Czech Republic and that makes him proud. He told me that you speak to him as much in English as in Spanish at home.]

In both cases, the BPSTs refer to the use of language. Olivia references the use of *Spanglish*, recognizing this linguistic practice as important for having access to her students' mathematical ideas. Gabby acknowledges the diverse use of languages and recognizes the advantage Misha has compared to her own experience growing up in a monolingual home. Both BPSTs wrote the letter in Spanish, favoring and acknowledging the language used by the parents at home.

Focus on Parents: Welcoming Parents to the Classroom

We found a few instances in which BPSTs directly invited the parents into the classroom, and we believe the explicit invitations are necessary. But most importantly, teachers should maintain a welcoming environment, fostering open and clear communication between home and school. Our aim with this assignment was for the BPSTs to understand that this was the type of relationship that we were hoping for them to create in their classrooms.

Gabby explicitly invited Misha's parents:

> *Sería fantástico, por ejemplo, si usted o el padre de Misha puedan venir a la clase y hacer un problema de matemáticas enfocándose en tu trabajo de enfermería o en su compañía de impresión de papel.*
> [It would be fantastic, for example, if you or Misha's dad could come to class and work on mathematics problems focusing on your work in nursing or on his printing company.]

In Gabby's letter, we see that she already knew what Misha's parents did for a living and invited them to share how their work could relate to mathematics. Another BPST, Charito, provided suggestions for the parents to work with their daughter, Sabrina, and encouraged the use of strategies that make sense to her instead of specific learned procedures.

> I will lend Sabrina her own baggie of base-ten blocks to be used as aids with her homework. Please encourage her to use them, or even come up with other strategies for addition. I will do this in the classroom as well. Please encourage Sabrina to use what works best for her. Thank you so much for your support as always.

Gabby offered an open invitation, while Charito welcomed parents to work with her in a way that made sense to Sabrina and provided familiar manipulatives to take home to support practice and learning with her parents.

Discussion

Our findings suggest that BPSTs can potentially create positive relationships with families in multiple ways. For example, they can establish a communication channel with parents on their children's mathematical understanding, on the curricular and pedagogical practices with which their children were presented during part of their mathematics instruction, and on what parents can do to support their children in learning mathematics. These findings also speak to the important role children's mathematical thinking has in starting an open and equitable dialogue between parents and school about the learning and teaching of mathematics (Civil et al., 2005b). At the end of the day, if a teacher does not know what the student knows or understands, or does not recognize the students' ideas, the teacher will not be able to communicate those understandings or ideas to the student's parents. Access to children's mathematical ideas requires skills in asking questions and in noticing, interpreting, and making sense of students' mathematical ideas (Jacobs & Empson, 2016). In the bilingual classroom, it is also crucial for BPSTs not only to develop these skills but also to recognize the complex dynamics of teaching in a bilingual setting (Barwell et al., 2016; Moschkovich, 2015). The present work revealed the complexity and challenges of the bilingual teaching practice, and therefore the concomitant complexity of preparing BPSTs for this practice. For instance, work with BPSTs should support their development of a teaching practice that permits students to talk, engage in meaningful conversation, and share their mathematical ideas. Second, support of BPSTs should help them add tools for recognizing what these ideas are, and for building on them in their instructional decisions. And third, BPST preparation should include assignments that provide practice assembling all these ideas and communicating them to parents. The recognition of the knowledge-intensive nature of the mathematics teaching practice provides new insight into how to support future teachers, in particular BPSTs who are often particularly well equipped to work with culturally and linguistically diverse students and their families.

Our work presented in this paper focuses on BPSTs and teacher preparation programs. But the findings we describe might also contribute to engaging in-service teachers in a dialogue with immigrant families. Skillful communication can allow parents to become a valuable resource in supporting teachers providing meaningful mathematics instruction. Creating a partnership with the families can broaden the cultural understanding of the teacher and the class.

A final contribution of the present work is the design of the Problem Solving unit, an instructional activity for the mathematics methods course, which further develops the instructional practice of noticing, attending to, and interpreting children's mathematical thinking. More specifically, the Problem Solving unit's emphasis on interviewing as an instructional practice has the potential to support BPSTs' learning by serving as what Grossman et al. (2009) called approximations of practice. The more congruent to and integrated with the actual work of teaching these approximations are, the more authentic they become, and the more likely they are to help BPSTs develop skills useful for their instructional practice. Our findings support the idea that these interviews can serve as a catalyst for classroom dialogue and help develop the expertise needed for responding to children's mathematical ideas.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. As an educator, how do your assignments reflect your commitment to providing equitable mathematics learning opportunities?

2. As a teacher educator, how do your assignments reflect BPSTs' experiences in learning to notice, elicit and interpret children's mathematical thinking?

3. How do you include spaces in your mathematics methods courses to foster real and meaningful connections with bilingual parents?

4. As a teacher educator, do you have opportunities to work with BPSTs? If so, what opportunities do you provide for BPSTs to enact their agency, and to encourage that they make instructional decisions that support the learning of mathematics in a bilingual setting?

5. How can you enhance and develop the ability of BPSTs to communicate the mathematics knowledge of elementary students with the families?

2020-21
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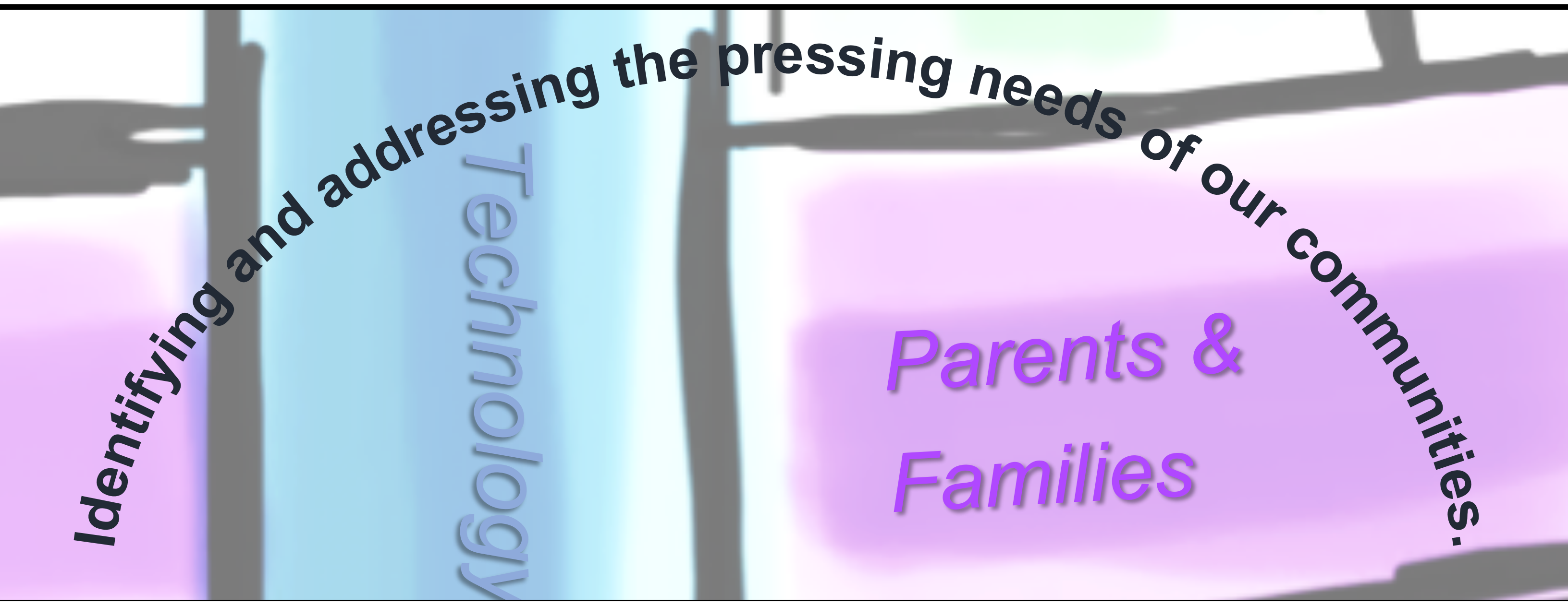

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