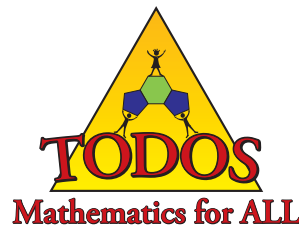
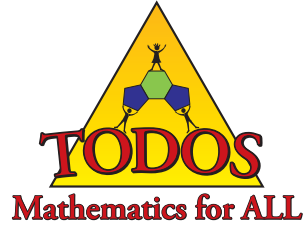


TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

*Special Issue on Multilingual Learners:
Translanguaging*





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Translanguaging*

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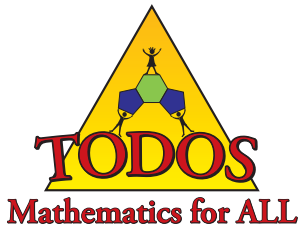
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TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

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**From the Editors of *TEEM* Special Issue
Multilingual Learners: Translanguaging**

This special issue of *TEEM* on teaching multilingual learners was created with a call for papers showing how mathematics teachers of multilingual learners can implement equitable, rigorous, and coherent mathematics instruction (TEEM, 2019). Though many teachers are aware of the need to implement such instruction, they are unsure of how to *act* on this awareness, as evidenced by their lack of confidence in their ability to teach diverse groups of students (Banilower et al., 2018). *TEEM* solicited manuscripts from classroom teachers and/or teacher educators that provided evidence-based examples of how to enact effective instructional strategies with multilingual students. The call specifically focused on highlighting assets-based and translanguaging approaches to teaching multilingual learners.

This is usually the point where the authors of the introduction frame the theoretical perspectives that motivated and undergirded the special issue. Yet, as we write this introduction, mass protests against racial injustice are ongoing, and we would be remiss not to acknowledge this moment here and now. The people of the United States—and the world—are witnessing and participating in uprisings against racism. These uprisings were sparked by state-sponsored violence against Black people, including George Floyd and Breonna Taylor, who were killed by police officers in summer 2020. The murders of Floyd and Taylor followed the murders of hundreds of other Black people at the hands of police or vigilantes: Ahmaud Arbery, Trayvon Martin, Michael Brown, Eric Garner, Natasha McKenna, Tamir Rice, and, unacceptably, too many others to name. It is sad that this uprising is necessary, and it is heartening to see so many people joining the effort to dismantle racism and racist systems. Of course, it is critical to note that these uprisings did not spring from thin air. Organizers have been laying the foundations for this moment for years, and the work of dismantling racism and racist structures will continue long after the summer of 2020 is over. It is equally important to remind ourselves that the aforementioned systems that have been built upon pernicious racism and white supremacy will not be overcome without persistent vigilance, education, interrogation of complicity, and, in the words of Dr. Martin Luther King, Jr, maladjustment (King, 1963).

In light of this moment, we reflect on the work of Black scholars such as Danny Martin, who have challenged fellow mathematics educators to consider how—and whether—reforms of mathematics teaching and learning can dismantle racism and racist structures (e.g., Martin, 2011). Additionally, scholars such as Nicole Joseph have presented case studies illustrating how teachers can create more humanizing mathematics classrooms for their Black and Brown students (e.g., Joseph, 2016). The works of these scholars, and that of many others, inspires us as we consider this special issue in light of this current moment. While this collection of papers is a small contribution, we hope it is part of the foundation that can build up a more equitable mathematics education enterprise.

In this issue, we present papers that evidence how mathematics teachers of multilingual learners can engage in equitable, rigorous, and coherent mathematics instruction. Many learners are excluded from mathematics learning environments on the basis of their identities, including students’ language(s) spoken, race, ethnicity, nationality, disabilities, gender, and sexuality. At a minimum, disrupting these patterns of exclusion, and building a more inclusive mathematics education requires that teachers *purposefully* include students from non-dominant communities in their classrooms. By creating linguistically inclusive mathematics classrooms, teachers can address one of the ways in which non-dominant students are excluded from mathematics learning environments.

Our call for papers resulted in many submissions—an encouraging sign! In response to the large number of submissions we received, we have created two issues of *TEEM* from this solicitation. In this issue, we focus on papers that that examined the use of language(s) while teaching mathematics using a translanguaging framing. In the other issue, the articles focused on teaching mathematics with multilingual students. One of the uniting features of the papers in this issue is that each paper

included special discussion about multilingual mathematics classrooms that have been overlooked in prior research and guidance for teachers.

In the present issue, we begin with an article by the Translanguaging Study Group. This group included scholars from across the country who came together to read, discuss, and consider what work from a translanguaging lens might look like in terms of learning, teaching, and researching mathematics. This group provides an overview of translanguaging for those unfamiliar with this theory and pedagogy. This article is a culmination of the Translanguaging Study Group's learning process and shares the group's tensions and assumptions that stem from adopting a translanguaging stance on language.

Maldonado Rodríguez and colleagues take a deep look at translanguaging in multilingual mathematics classrooms. Through an examination of one teacher's classroom, the authors evidence the "translanguaging *corriente*," the current of language practices. They beautifully illustrate how rethinking language separation can support students' full language practices and also provide a foundation for deeper mathematical understandings.

DiNapoli and Morales explored four, Latinx bilingual students as they drew on their bilingualism to make sense of a particular task. They include a vignette that illustrates students' language practices as they flow through language(s). In unpacking the vignette, the authors highlight a monolingual English-speaking teacher's pedagogical moves that worked to support students' translanguaging practices. This article evidences the power of drawing on students' full linguistic repertoire as a means of supporting mathematical understandings.

Kim and Suh also evidence practices in a multilingual mathematics classroom. The authors propose transmodalising as a practice that builds from and with translanguaging as a means of supporting and encouraging the use of multiple modalities (e.g., linguistic, audiovisual, gestural). Kim and Suh's case study of Lydia, a first-grade teacher, evidences how she was able to enact instructional practices aligned with a view of language as a resource. Cases such as this are necessary to illustrate the ways in which translanguaging and transmodalising are enacted in practice.

Finally, Gomez and colleagues close out this issue with an article based on their conversations with 46 upper-elementary Latinx students at predominantly white schools. The authors share the children's *testimonios* in order to understand students' experiences. This powerful piece evidences the need to engage students in dialogue as humans, who have insights into their own needs and experiences.

In sum, the papers in this special issue provide ideas for considering how teachers and teacher educators can transform mathematics education to create a more just and equitable future, especially for multilingual learners who have endured over a century of marginalization and discrimination (Donato et al., 2017). In addition to the articles in this issue, we hope readers will continue to learn about and advocate for much-needed changes to the ongoing immigration crisis in the United States that has resulted in the unjust and inhumane treatment of people seeking to enter this country. We note that immigration issues greatly impact multilingual students and families, and, thus, supporting students requires us to learn about and address these issues. While this scholarly work feels removed from the direct action we are seeing on the streets and in the halls of government in response to state-sanctioned violence against Black people, we hope that this work will contribute to ongoing efforts to create a more humane mathematics education enterprise, and, ultimately, a more equitable and just world.

Zandra de Araujo, Sarah A. Roberts, Craig Willey, and William Zahner

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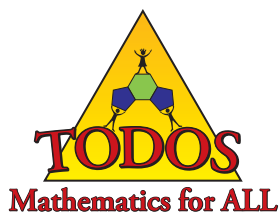
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Translanguaging and the Mathematics Classroom

The Translanguaging Study Group

Abstract

Understanding how students use language is important for mathematics teachers, particularly when considering how best to teach mathematics with bilingual students. Translanguaging is a theory that provides a useful lens for understanding the language use of bilingual students. In this article, we share our perspectives on translanguaging and how this perspective might impact our instruction with bilingual students.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What is language?
2. What was your first experience thinking about language or what it means to know a language (or multiple languages)?
3. Some linguists have described a common understanding of multilingualism as a collection of individual languages. How does this align or differ from your conception of language?
4. How might your conception of language influence your mathematics pedagogy with bilingual students?
5. How does a mathematics classroom that encourages students to utilize all of their linguistic resources look and sound?

The Translanguaging Study Group is a group of scholars who came together to study translanguaging. This paper was collaboratively conceptualized through conversations with the Translanguaging Study Group (TSG). The TSG members who participated in the writing of this article included (in alphabetical order): Zandra de Araujo, Carlos Nicolas Gomez, Ji Yeong I, Elyssa Miller, Hector Morales, Sarah A. Roberts, Erin Smith, Miwa Takeuchi, Mary Truxaw, and Craig Willey. Correspondence can be addressed to Zandra de Araujo, dearaujoz@missouri.edu.

Translanguaging and the Mathematics Classroom

Mr. García teaches mathematics in a classroom of newcomers in an urban school serving almost entirely students of color. The students range in ages from 11-14, come mostly from Central American or Middle Eastern countries and have a variety of experiences with formal or interrupted education. Mr. García is a Latinx, Spanish-speaker with six years of experience teaching bilingual students. He favors the use of technology (e.g., Google Maps, digital cameras) to build a sense of community, which he views as a prerequisite to supporting students' mathematics learning. He frequently exposes the historical roots of mathematical concepts and applications to help students understand the value of cultural heritage and disrupt prevalent notions of who does mathematics. He moves deliberately through the curriculum, emphasizing problem solving, conceptual understanding, and mathematical reasoning.

Mrs. Ellis's school is in the suburbs of a major metropolitan city. The majority of students attending this school are from middle-income families, but there are also students from working class background and affluent families. Most of the students are white, but there is a small population of children (about 9%) whose first language is either Spanish or Hakha Chin. Mrs. Ellis, a fourth grade teacher, is in her third year of teaching and has received a minimal amount of professional development (PD) around teaching English learners (ELs), as they are commonly referred to in this school setting, from both her university-based teacher preparation program as well as on-site PD provided by the school's EL coordinator. She believes in the value of native language use to support a child's sense-making in mathematics but struggles to maximize this resource in both interpersonal interactions with the child and assessments. Mrs. Ellis also is cognizant of her responsibility to help provide access to the mathematics content for ELs, which is typically demonstrated by her focusing on certain nouns and verbs she perceives to be unfamiliar to the students, and is supported by showing pictures or drawing diagrams. Beyond these two key strategies, Mrs. Ellis admittedly struggles to support the students.

While these are very different classrooms, they represent two of the many classrooms that serve—or struggle to serve—bilingual learners in mathematics. Still, the two teachers share a desire to help all children learn, as well as an understanding that language and language development play key roles in these learners' mathematical success.

Teachers are increasingly aware of our historical failures to adequately support bilingual learners and have been exposed to various protocols (e.g., SIOP; Echevarria, Vogt, & Short, 2007) and strategies to increase access and engagement with mathematics. Yet, scholars have long warned that strategies, by themselves, are insufficient and the inequitable results on mathematics assessments among bilingual students show we are slow to make significant progress at a time when bilingual students constitute about 10% of K-12 enrollment in U.S. schools (National Center for Education Statistics, 2019).

In this article, we consider translanguaging as a means of reconceptualizing language in the hopes it will provide a useful lens to more effectively teach mathematics with bilingual students. Translanguaging is a theory of language that shifts us ideologically from perspectives commonly associated with language separation to one that values the complex and interrelated communicative practice that makes up bilingual students' linguistic repertoire (Cenoz, 2017). We use translanguaging (1) to focus on underdeveloped conceptions of language that undergird or inhibit more effective engagement of bilingual learners, and (2) to sketch a way forward, recognizing that aspects of translanguaging pedagogy have been used for years and that there is not one "best approach" to enact translanguaging across all contexts. In the following sections we present our current assumptions that have grown alongside our continued study of translanguaging.

1. Be cognizant of the power of labels.

There is a growing acknowledgement for the need to be purposeful about the terminology that we are using when we talk about linguistically diverse students. We want to acknowledge that labels matter and have power. Although

we are still not quite sure what that terminology should be, we do have ideas about what it should not be.

In this paper, we, like García (2009), use the term *bilingual* to encompass bilingual students and ascribe to the definition that bilingual is “all language practices that include features beyond those described by linguists and educators as forming a single autonomous language” (p. 158). Under this definition, students who have access to or are in the process of acquiring access to multiple languages would be described as bilingual. Although the

(inter)national discussion around labels will continue, it’s important to recognize which labels currently exist and are used. In addition, we must consider the hidden assumptions and other considerations associated with these terms (Table 1). We feel it is important for researchers and teachers to be purposeful about the labels they use, think through these labels, continue to contribute to this conversation, and acknowledge the political and social nature of these conversations.

Table 1
Considerations for Particular Labels

Term	Considerations
Limited English Proficient	<ul style="list-style-type: none"> ● Highlights English dominance ● Highlights deficit perspective ● Often used as a local “measurement” of proficiency
English (Language) Learner	<ul style="list-style-type: none"> ● Highlights English dominance ● Highlights deficit perspective ● Often used as a local/federal “measurement” of proficiency ● Used regularly currently in the literature, practice, and in policy documents
Emergent Bilingual	<ul style="list-style-type: none"> ● Suggests individual is not already bilingual ● Assumption that individual speaks only two languages ● Assumption of separate languages
Bilingual	<ul style="list-style-type: none"> ● Assumption that individual speaks only two languages. ● Possible assumption of separate languages. ● Widely used in literature, practice, and policy
Multilingual	<ul style="list-style-type: none"> ● Suggests individual speaks multiple languages ● Possible assumption of separate languages ● Used in literature but not widely used in policy
Translanguager	<ul style="list-style-type: none"> ● Suggests individual translanguages ● Not widely used in the literature, practice, or in policy

2. Acknowledge fluid, dynamic language practice (translanguaging) as the bilingual norm.

García’s (2009) notion of translanguaging affirms fluid language practices that are the norm to bilinguals’ lives. Instead of seeing bilinguals as people having multiple independent and self-contained languages, García viewed one linguistic repertoire for bilinguals, that is,

translanguaging. García (2012) posits that “translanguaging takes as its starting point the *language practices of bilingual people as the norm*, and not the language of monolinguals, as described by traditional usage books and grammars” (p. 1, emphasis in original). This lens legitimizes fluid language practices as unique resources and practices that bilinguals can access, instead

of perceiving fluid language practices as a deficit or a sign of incompetence.

Translanguaging is an action. Maturana and Varela (1992) and García and Wei (2013) maintain that languaging is the continuous knowing and meaning-making with languages through ongoing interactions with others. Therefore, translanguaging continues to evolve as bilinguals engage in interactions and meaning-making through new encounters and new situations. This contrasts with historic notions that perceive languages as operating statically and independently within individuals. Consequently, García contends that we cannot accurately gauge bilinguals' authentic capabilities if we separate their language repertoires artificially. Such artificial separation will mask bilinguals' capabilities of generating new linguistic expressions that can bring forth new aspects of knowing. To illustrate what this dynamic language process looks like while learning mathematics, we present a vignette of a group's collaboration on a problem involving exponential functions.

Ines: ...empezamos de cuatro pies. (...we start at four feet.) Si toma si come un pedacito son ocho, si come un pedacito son dieciseis, el tercer pedazo dieciseis y dieciseis. Treintaidos ¿no? (*If she drinks, if she eats one piece it becomes eight, if she eats one piece it becomes sixteen, the third piece, sixteen and sixteen, thirty two, no?*)

Jessica: Pero, ¿cómo sacastes eso? (*But how did you get that?*)

Ines: Porque si empezamos con cuatro pies, como yo les digo, si come un pedacito y sale, aumenta de altura de doble. (*Because, if we start at four feet, like I'm telling you, if she eats one piece and it comes out to, her height grows double.*)

Jessica: Ohh, her height doubles.

Elena: You know it's the same thing. Mira, dos, you multiply one times two is two, two times four is eight, y si pones two times two is four, four times four is sixteen. (*You know it's the same thing look. Two, you multiply one times two is two, two times four is eight, and if you put two times two is four, four times four is sixteen.*)

Carina: In squared times 2 is equal to your out.

This above vignette illustrates how the dynamic and fluid movement between mathematical and everyday language in Spanish and English provides the group the necessary tools to clarify their mathematical thinking (Morales, 2004).

Language is living, emerging, and dynamic and does not construct walls between itself (García, 2012). Thus, bilinguals develop one holistic language repertoire that they are fluidly and tactically enacting and renewing. From the translanguaging lens, creating a classroom culture where bilinguals' fluid language practices are affirmed can bring forth mathematical understanding that maybe be otherwise masked or suppressed.

3. Construct mathematics classrooms rich in language across modalities.

Developing a classroom culture where fluid language practices are affirmed requires rich language practices across all modalities. In the context of learning, modalities are "socially and culturally shaped resource[s] for making meaning" (Bezemer & Kress, 2008, p. 171). Examples of modalities include traditional language practices such as speaking, listening, reading, and writing; they also include resources beyond words such as gestures, context, visuals, objects, artifacts, touch, tone, multiple languages, etc. (García & Wei, 2014). Because different modalities provide different affordances and constraints of meaning making, translanguaging recognizes the importance of "all meaning-making modes" (García & Wei, 2014, p. 29). We extend García's (2017) translanguaging framework to accommodate how bilingual students use mathematical discourse and other linguistic repertoires, such as the mathematical register (Avalos, Medina & Secada, 2018) to enhance our view of understanding language in relation to teaching and learning mathematics. This is in line with other scholars (e.g., O'Halloran, 2015) who call for greater attention to the ways students move between language, symbolic notation, and visual representations as they engage in mathematics.

We have somewhat struggled to identify how multimodal communication and learning may manifest in classrooms where translanguaging is embraced. We acknowledge the practicality of resources that suggest specific multimodal strategies and tools for use within bilingual and multilingual classrooms (e.g., Celedón-Pattichis & Ramirez, 2012; Echevarria, Vogt, & Short, 2007). For example, Shein (2012) discussed the value of teachers using gestures while using questioning with bilingual students in mathematics classrooms.

Alternatively, teachers may use an interactive word wall displaying key terms (with definitions, examples, and/or visuals) and revisit interactively during lessons. While appreciating the practicality of specific strategies and tools, our understanding of translanguaging suggests that we should strive for a more holistic view of students and interactions that support meaning-making rather than relying solely on individual tools and strategies. To clarify this distinction, García and Wei (2014) provide an example of the differences between using translation features on an iPhone with texting practices where users select from emoticons, photographs, multiple languages, and so on in order to communicate effectively. Thus, translanguaging suggests multimodal perspectives that are fluid and dynamic as opposed to disjointed and static.

4. Recognize the interplay between identity and language.

Our identity is bounded within language. We are dependent on our language and discourses to be able to describe who we are through the stories we tell (Kaasila, 2007). As we become socialized into communities and subcultures, we learn new discourses to describe our ideology, beliefs, and what it means to be in the world. The interplay between identity and language is significant as it influences one's linguistic repertoire. Consequently, one's languaging is impacted by the access one has to different identity discourses (this includes symbols). For example, Lewin (2015) discussed how individuals who choose to participate in a punk subculture learn to use members' discourse by participating in rituals (e.g., going to shows and "losing themselves"; p. 173). Our social identities (e.g., gender, socioeconomic status, race, ethnicity) also provide access to particular discourses about our ways of being. Participation in these discourses is not chosen. Demonstrating white cisgender male attributes socializes one to particular ways of being and languaging. One has to then opt-in to learning the languaging practices of others. Consequently, the discourses of our chosen membership in subcultures and our social identities become embedded in our linguistic repertoire.

The interplay between languages and identity can also be seen in how Latinx students describe the relationship between language and being a doer-of-

mathematics. For example, in the following transcript from an interview with Alondra, a 3rd grade Latinx student, she discussed how she felt math came easier to her because Hispanics do mathematics the same way as English people:

Interviewer: How do you think being Hispanic plays a role in how you learn math?

Alondra: Math is like kind of more easier for me than reading because like sometimes I don't know all of the words.

Interviewer: Ok and how does being Hispanic help you with that in the math?

Alondra: Because the numbers in Spanish are the same and some problems are the same like [how] Hispanics do the [math] problems. And the way they, the English people [sic] do it, I like—Hispanic they do it the same way.

Antonio, a third-grade Latinx student, discussed his languaging practices when doing mathematics:

Well sometimes when I'm trying to do math I, I sometimes do it in Spanish and then I try to say it in English. And once it's like wrong, I do it—the whole thing in Spanish. And then if it's wrong still, I do the whole thing in English. And it's right then, then um, I'll like feel like I got it right just with only one try but I only got it right in like three tries sometimes. (Antonio, 3rd grade)

Alondra and Antonio demonstrate how their use of Spanish is embedded within their enactment of mathematical tasks. It also reflects how their identity as doers-of-mathematics is linked to their linguistic repertoire. These translanguaging practices can be a transformational power in developing bilingual learners' academic identities (Cenoz, 2017).

5. Not equate English proficiency with mathematical proficiency.

Prior studies have found that teachers hold deficit views of bilingual learners (de Araujo, 2017; I, 2019) carrying a number of negative consequences for bilingual students. One such consequence is that bilingual students are assumed to have lower mathematical capabilities evident in the types of mathematical opportunities teachers afford bilingual students. For example, de Araujo (2017) found secondary mathematics teachers purposefully selected low cognitive demand tasks for their bilingual students.

The teachers did so, not only because the tasks contained fewer words, but also because they perceived the students as needing more procedure-based practice tasks. Curriculum materials also contribute to this conflation of English and mathematical proficiency as many resources marked for “English learners” are identical to those recommended for special education students (Smith, Dwiggin, & de Araujo, 2017).

The learning of mathematics is a discursive activity (Moschkovich, 2002). Therefore, we negotiate meanings and understandings of mathematical ideas. Moreover, we communicate those understandings to one another through language. If we perceive academic language in relation only to spoken and written English, then we will view bilingual students with language deficits. We might hear a student has difficulty articulating something aloud in English or notice they do not comprehend written English text. We then filter these student difficulties through the dominant views of bilingualism concluding that the student does not understand the mathematical ideas because they are not able to evidence their knowledge in English.

If we instead take a more expansive view of language, we would take the same student challenges and instead seek to support the student’s communication in multiple modalities; drawing on their full linguistic repertoire. We would seek to understand the extent of the student’s understanding by allowing and encouraging them to draw on all of their linguistic resources, not just those in English. We also would recognize *our* full repertoire of language which is rich in symbols and other modalities to help communicate. The emphasis throughout will be on understanding, not the limits of language. Teachers who embrace translanguaging as a theory of language would not provide easier mathematics tasks to bilingual students, they would seek to provide language rich experiences encouraging students to make sense of and communicate with their full linguistic repertoire (de Araujo et al., 2018). This would dethrone the supremacy of English as evidence for mathematical learning and instead aim to make sense of meaning through and with our full arsenal of linguistic resources.

Ongoing Questions and Discussion

In taking a translanguaging perspective, we can encourage bilingual students to embrace and utilize their full linguistic repertoire and agency as they develop and assert their identities as mathematics learners. We believe this perspective is necessary to rehumanize mathematics for bilingual students (Gutiérrez, 2017). A translanguaging perspective focuses on students’ assets and helps us to foster respect and dignity for their language practices.

We acknowledge that we are still working to understand how we might take up a translanguaging perspective in our work. Our goal for this article was to provide an overview of translanguaging and potential implications this perspective has for our practice. We also hope this article has sparked a desire for continued study and exploration for researchers and practitioners. In particular, we hope future work will help answer questions such as: (1) What instructional practices support bilingual students in the mathematics classroom when we take a translanguaging perspective? (2) How might we prepare preservice teachers to understand and enact practices consistent with this perspective? (3) How might mathematics education research be transformed with and through a translanguaging perspective? As we seek to answer these questions, others will surely arise. It is our stance that these continued explorations and innovations are necessary as we seek to improve mathematics education for bilingual students.

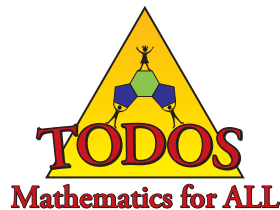
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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. In what ways do you identify with or feel dissonance with the classroom/teacher depictions in the opening?
2. Consider the opening classroom/teacher depictions. In what ways do you see the teachers' practices aligning with translanguaging (or not)?
3. In what ways do/can you plan for mathematics instruction to make room for students' use of their home languages and cultural understandings?
4. What are the challenges that translanguaging poses for you as a mathematics educator? Identify and talk through with other educators the challenges that are posed.
5. What ongoing questions do you have about translanguaging? How might you explore the topic further?



The mission of TODOS: Mathematics for ALL is to advocate for equity and high quality mathematics education for all students—in particular, Latina/o students.

Five goals define the activities and products of TODOS: Mathematics for ALL

1. To advance educators' knowledge and ability that lead to implementing an equitable, rigorous, and coherent mathematics program that incorporates the role language and culture play in teaching and learning mathematics.
2. To develop and support educational leaders who continue to carry out the mission of TODOS.
3. To generate and disseminate knowledge about equitable and high quality mathematics education.
4. To inform the public and influence educational policies in ways that enable students to become mathematically proficient in order to enhance college and career readiness.
5. To inform families about educational policies and learning strategies that will enable their children to become mathematically proficient.

Poetry Corner

UTEP's Lawrence Lesser turned to poetry to help navigate these pandemic times, a reflection on infection and inflection! First published in the June 2020 issue of *Radical Statistics*, this poem could actually be used with high school students learning about the inflection point of a (logistic or other) graph, and then going on to make corona-connections with the meaning of the word *inflection* in the registers of linguistics and communication.

The Point of Inflection by Lawrence Mark Lesser

The point of inflection
is where towering terror of
cumulative cases
finally
slows its rise.

The point of inflection
of that logistic curve
depends on logistics
of testing, tracing,
and distance.

The point of inflection
is to let our words show number, person, mood, tense:
distinguishing *case* and *cases*,
infect and *infected*,
dense and *denser*.

The point of inflection
is how the rise and fall of our voice
shows tone,
beyond what's typed
in chat windows.

The root of inflection
is the Latin word meaning to *bend*,
like a curve, a government,
arc of the moral universe,
or us.



**Flowing With the Translanguaging *Corriente*:
Juntos Engaging With and Making Sense of Mathematics**

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Abstract

The translanguaging *corriente*, or current of language practices, as described by García et al. (2017), is always flowing through your mathematics classroom, whether you realize it or not. The *corriente*, how multilinguals use all their languages to learn and engage with content in school and make sense of a complex world, requires educators to reconsider what is understood about language and mathematics. By rethinking how we view language separation in the multilingual mathematics classroom, we propose that teachers teach with a translanguaging stance in order to access multilingual students' full linguistic repertoires and to develop deep mathematical understanding.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What does it mean to “know” a language? Is there only one English language, one Spanish language, one Russian language, etc.?
2. Consider how you communicate with different groups of people in your life. Does the context make a difference in what you say and how you say it? How could we start to understand language as occurring in practice and in particular contexts and not having to do with the amount of words you know in a particular language?
3. What are your beliefs with regards to language of instruction in mathematics for multilingual students? For example, what are your thoughts about the language that should be used for instruction during math time? Where do these beliefs come from?
4. Have you heard the term translanguaging mentioned when talking about language acquisition? Consider what you know about language acquisition. Are your ideas of language acquisition built on a model of language separation, where specific languages get directed instruction time? What might it mean if language learning was always occurring regardless of the language of instruction?
5. What should be some goals for what it means to know and learn mathematics in multilingual classrooms?

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Flowing With the Translanguaging *Corriente*: Juntos Engaging With and Making Sense of Mathematics

Luz A. Maldonado Rodríguez, Gladys Helena Krause, and Melissa Adams-Corral

For many, the terms “bilingual student” or “multilingual learner” are synonymous with “English learner” and represent challenges for mainstream educators (Flores & Rosa, 2019). As Gutstein (2007) reminds us, “language is about power” (p. 245) and this has been proven through the history of subtractive bilingual programs in the U.S. and a monolingual bias towards English-speaking children that continues to position multilingual students’ languages as insufficient or lacking (Flores, 2016; Gándara & Hopkins, 2010; Langer-Osuna et al., 2016). Research has often focused on the mathematical vocabulary multilingual students lack and the difficulties they face in learning in a monolingual English mathematics classroom (Téllez, Moschkovich, & Civil, 2011). These students are assessed through a view of languages as separate entities to develop in isolation, and thus, the instructional focus is on the products that children produce, or do not produce, in English-speaking classrooms (Otheguy, 2016). But educators are being

asked to reconsider this inaccurate view of language acquisition and to instead consider a dynamic bilingualism that transcends beyond social borders of language (García & Kleyn, 2016b). We encourage the reader to refer to the work of García & Kleifgen (2010) and García & Kleyn (2016b) for more information on different models of bilingualism and language practices.

Multilingual children often face strict language separation in their schooling, through various bilingual education program models, including dual language, in which a “target language zone” is used for specific content learning in one language at a time, operating under the view that acquiring language depends on using only one language at a time (Palmer, Martínez, & Mateus, 2014). A renewed understanding of the dynamic language practices of multilinguals asks us to move beyond a narrow definition of language and use an understanding of *translanguaging*: “the deployment of a speaker’s full linguistic repertoire which does not in any way

correspond to the socially and politically defined boundaries of named languages” (García & Kleyn, 2016b, p. 14). In other words, educators must learn to view multilingual children as using all their languages all the time. So, what does that mean for the mathematics classroom?

Research in mathematics education advocates for practices that celebrate all students’ mathematical reasoning and ways of knowing (Turner & Drake, 2016). The translanguaging literature (García & Kleyn, 2016b) would call this a *stance* about mathematics teaching and learning and posit that teachers of multilingual students must also teach from the understanding that language is always a resource for the learning, thinking, and creating that occurs in a mathematics classroom. A *translanguaging stance* in a mathematics classroom means designing a space in which the teacher views and understands the complexity of multilingual students’ language practices, recognizing that they are a powerful resource to draw upon and connect to mathematical learning. García, Ibarra Johnson, and Seltzer (2017) further identified four translanguaging pedagogical strategies to enact with students: (1) supporting students as they engage with and comprehend complex content and texts; (2) providing opportunities for students to develop linguistic practices for academic contexts; (3) making space for students’ bilingualism and ways of knowing; and (4) supporting students’ bilingual identities and socioemotional development. We argue that these pedagogical strategies also have a place in the multilingual mathematics classroom.

In this article we propose a description of a translanguaging stance in multilingual mathematics learning environments. While examples of translanguaging have been made in other contexts, there remains a need to better understand translanguaging in mathematics contexts (Kleyn, 2016). In particular, we make the case for moving past traditional language separation practices and utilizing and understanding how language flows like a *corriente*, a river current, in the mathematics classroom to make deep mathematical connections (García et al., 2017). We provide an example from a second and third grade two-way dual language classroom to supplement our discussion of translanguaging mathematics classrooms.

Making the Case for Translanguaging in the Multilingual Mathematics Classroom

At the root of translanguaging theory are notions of social justice (García et al, 2017; García & Kleyn, 2016a), which occurs by working alongside, or *juntos* with, multilingual students and the empowerment that comes from the accepting and valuing of their multilingualism. Awareness of the history of how language is used to control, categorize, and racialize the language practices of multilingual students (Flores & Rosa, 2015) requires analyzing educational policies that continue to position these students as children in need or limited in language (García & Kleyn, 2016a). As hooks (1994) states “...it is not the English language that hurts me, but what the oppressors do with it, how they shape it to become a territory that limits and defines, how they make it a weapon that can shame, humiliate, colonize” (p. 168). García & Kleyn (2016b), whose translanguaging study in New York encompassed multilingual teachers and classrooms, often asked teachers to think of how Spanish, spoken by those from various economic and social backgrounds, comes in many acceptable forms in which people manage to communicate. They asked educators to make the connections to when Spanish was first “standardized” as a way to control the colonized people of Latin America in the 1400s and recognize that what we call Spanish, Russian, and English are merely social labels that do not really define language and how language is used in day-to-day contexts. They argued that remnants of this control are still exerted in schooling today through limited and narrow assessment of multilinguals’ language abilities. Translanguaging, then, also requires educators to understand the context of being multilingual in U.S. schools and further recognize how many of these students face a racialized experience of school and of mathematics learning (Adams, 2018; Martin, 2009; Flores & Rosa, 2015; Zavala, 2017) A translanguaging stance in the mathematics classroom means that attempts to improve multilingual students’ experiences with mathematics require an anti-racist stance which challenges traditional practices and reimagines what it means to be in math class. This further requires understanding how language has typically been constructed by educational policy and considering what it

means to disrupt traditional views of multilingual students' language repertoires (García & Kleyn, 2016a).

García and Kleifgen (2010) encourage us to move past traditional language instructional practices that separate languages (i.e., English time and Spanish time, either by day or content area) and instead consider the “dynamic bilingual practices” that multilinguals enact that are context- and person-specific and fully immersed in interactions with different speakers. The visual of the translanguaging *corriente*, a flowing river metaphor by García et al., (2017) encourage us to interpret the dynamic and constant flow of language features that multilingual students are tuned into in the classroom. They recommend educators take a step back and listen to the flow of dialogue and conversation that takes place in the classroom, school, and community, all without the need for language separation. For the mathematics classroom this *corriente* is present as mathematical ideas are shared and taken up by the members of the classroom, and instead of focusing on a concept of linguistic proficiency, the focus can be on linguistic performance.

In a translanguaging mathematics classroom, then, part of the *corriente* means allowing the flow of mathematical ideas to be shared without restriction towards form, especially for making conceptual connections (Maldonado, Krause & Adams, 2018). A translanguaging mathematics classroom includes teachers making deliberate choices to use children's thinking while engaging in mathematics instruction that develops knowledge, dispositions, and practices that not only support the development of children's mathematical thinking, but also build on students' cultural, linguistic and community-based knowledge (Turner et al., 2012). When teachers purposefully co-construct a translanguaging mathematics classroom *juntos* with multilingual students and flow with the translanguaging *corriente*, we posit that students expand their sense of what they can do mathematically and develop a sense of what mathematics can be.

We now provide an example from a second and third grade dual language classroom to reflect on the translanguaging pedagogies that García et al. (2017) explained are critical for supporting and valuing multilingual students: (1) supporting students as they engage with and comprehend complex content and texts; (2) providing opportunities for students to develop linguistic practices for academic contexts; (3) making

space for students' bilingualism and ways of knowing; and (4) supporting students' bilingual identities and socioemotional development.

Translanguaging in a Dual Language Mathematics Classroom

Ms. Acuña (a pseudonym) taught in a two-way dual language (TWDL) classroom at Granger Elementary in a large city in the Southwest U.S. She taught the same group of students for their second and third grade years. The district had imposed a form of TWDL, requiring language separation that occurred along the lines of content area, with mathematics to be taught in English. However, administrators at Granger granted its teachers some professional autonomy and allowed instructors to design their own instruction.

Out of the 23 students in Ms. Acuña's classroom, one student identified as both Black and White. while the other 22 identified as Latinx, with families from Mexico, El Salvador, Honduras, and Puerto Rico. All 23 students spoke Spanish and English, and one also spoke Otomí, an indigenous language from Mexico.

Ms. Acuña and her students were used to engaging with mathematics in multiple ways. There might be number talks to share mental computation strategies, word problems that were based on a book, or the class mathematizing something a specific student had shared during the morning community meeting, or a mini lesson based on a specific math objective. Ms. Acuña often taught integrated units based in children's literature that involved building students' critical consciousness of power dynamics and inequities in the world around them. As a result, math time was not limited solely to a particular time of the day, and students could often be heard making mathematics connections during “non-math” time.

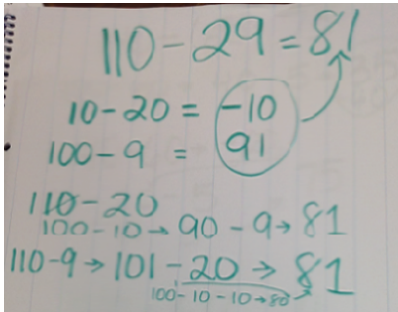
Ms. Acuña's students shared strategies for the following number talk in May of third grade:

$$110 - 29 = \square$$

Rubén shared his invented algorithm strategy orally, and Ms. Acuña scribed his idea. Next, Hernán shared his strategy, which Ms. Acuña recognized as an incremental strategy that was similar to his past mental strategies for subtraction problems. Ms. Acuña then used Hernan's oral strategy to write an equation that represented the strategy he was sharing (Figure 1).

Figure 1

Rubén's and Hernán's strategies



Ms. Acuña: *Rubén, ¿qué te salió?* (Ruben, what did you get?)

Rubén: *Eighty-one*

Ms. Acuña: *¿Cómo sacaste eighty-one?*

Rubén: *I took... ten... hice diez quitale veinte y luego, negative ten*

Ms. Acuña: [writes $10 - 20 = -10$] *¿Así?*

Rubén: *Sí. Y luego le quité nueve al cien* (Yes, and then I took 9 from 100.)

Ms. Acuña: *¿Y qué te salió?* (And what did you get?)

Rubén: *Me quedaron noventa y uno* (I was left with ninety one.)

Ms. Acuña: [Writes out $100 - 9 = 91$] *Noventa y uno* [circles 91 and -10].

Rubén: *... y al noventa y uno le quito un diez y me queda ochenta y uno.* (And to the ninety-one I took a ten and I was left with eighty-one.)

Ms. Acuña: *Ah. Entonces, noventa y uno con menos diez, te da ochenta y uno.*

Ms. Acuña: *Okay. ¿Alguién lo hizo de una manera diferente?* (Did someone do it a different way?)

Hernán: *Um, what I did is that I minused, twenty, twenty from the ten and I get ninety.*

Ms. Acuña: *Okay, so kind of like a hundred and ten quitale veinte, take away ten.*

Hernán: *Yeah, that equals ninety and then I did ninety minus nine.*

Ms. Acuña: *Y te salió ochenta y uno. Me acuerdo que tú has estado quebrando números pero manteniéndolos juntos, has estado compartiendo estrategias así.* (And you got eighty one. I remember that you have been breaking up numbers but keeping them together. You've been sharing those kinds of strategies.)

Throughout this interaction, we observe how Ms. Acuña facilitates an interaction that elicits mathematical ideas in both English and Spanish, thus translanguaging

was occurring both between two students and between students and the teacher (Kleyn & Yau, 2016). The switch from one language to another was instantaneous and needed no translation, the teacher and students were immersed in the translanguaging *corriente*. The students were already accustomed to an expectation on comprehending the strategies being shared. Again, a traditional view of language might view the “code-switching” between languages in one sentence, such as Rubén and the teacher did, as “incorrect language” or a lack of either English or Spanish proficiency. However, with a translanguaging lens, we can instead view this interaction as an example of dynamic bilingualism. Rubén and Hernán were intent on explaining their strategies, and Ms. Acuña was intent on making sure she represented their oral strategy with symbolic numeric notation in order to better provide access to the rest of the class. In sharing his strategy so easily in this discussion, Hernán both acknowledges that his strategy was different than Rubén's, which was stated in Spanish, and goes on to share his own strategy and choosing to use English. The very fluidity of the conversation appeared to aid, rather than hinder, the eliciting of mathematical ideas. Rubén's strategy allowed the class the opportunity to explore the use of negative numbers and to discuss rich mathematical ideas about subtraction. The teacher was supporting students as they engaged in complex content through her questioning and scribing of student ideas.

This interaction can also be analyzed from a mathematics teaching perspective as an example of a teacher who understands children's multidigit addition and subtraction strategies (Carpenter et al., 2014) and is purposefully sharing strategies that will also shift the mathematical understanding of the entire class. Eliciting strategies that made sense to students meant that students in Ms. Acuña's class often shared invented strategies for subtraction that relied on the properties of operations and not on a traditional algorithm. It was important that Ms. Acuña scribed the strategies shared. This can be viewed as an example of the “text” of mathematics. Providing visuals to the children's oral strategies is another way to support students as they engage with content and provides opportunities for students to develop linguistic practices in mathematics. When teachers begin to make instructional decisions based on how their students understand mathematics, teachers can begin to view their multilingual students' mathematical thinking in an

empowering way, by positioning the students as mathematical thinkers and creating a shift in who “does” mathematics (Turner et al., 2013). Thus, making space for students’ bilingualism and ways of knowing was also the norm for what it meant to do mathematics in this classroom.

In a translanguaging space, supporting students’ bilingual identities and socioemotional development means bringing together all members of a classroom community, along with their languages (García et al., 2017). In mathematics classrooms, this support intertwines a focus on children’s thinking with linguistic and cultural awareness. As an example of what this looks like, we turn to this problem that Ms. Acuña posed to her class in second grade: *La semana pasada recolectamos 38 dólares en la jarra para mandar dinero a Flint. Si después de una semana, tenemos 60 dólares almacenados, ¿cuánto dinero recolectamos durante la semana?* (Last week we collected \$38 in our jar for Flint. If after this week we have \$60, how much would we have collected during this week?) This problem, a Join Change Unknown problem (Carpenter et al., 2014), was part of a unit in which Ms. Acuña and her students explored how children were affected by the Flint water crisis. The students suggested they should raise money for Flint children, extending the unit and providing a meaningful context for problems (Dominguez, 2011). After unpacking the problem and having students work individually, Ms. Acuña gathered the class to discuss strategies. She first asked Hernán to share his strategy.

Ms. Acuña: *Hernán cuando empezó hoy tuvo una idea de que iba a ser la respuesta. ¿Qué pensabas que iba a ser la respuesta?* (When Hernan started today, he had an idea that he knew was going to be the answer. What did you think the answer was going to be?)

Hernán: 32

Ms. Acuña: *32. Y yo escuché esa idea de 32 de muchas personas. Que no me parece una idea loca. Me parece una idea que muestra mucho pensamiento. Tenemos la idea de que 30 y 30 hacen 60. También tenemos la idea de que 8 con 2 hacen el 10. Entonces esa idea enseña mucho pensamiento. ¿Pero qué pasa si tratas de sumar 38 con otros 30?* (32. I heard the idea of 32 from many people. It does not seem like a crazy idea either. It seems like an idea that shows a lot of thinking. We have the idea that 30 and 30 make 60. We also have the idea that 8 and 2 make 10.

So this idea shows a lot of reflection. But what happens if you try to add 38 and 30?)

Kellys: 68

Hernán: *Le quito un 10...* (I take away a 10.)

Ms. Acuña: *¡Ah! Eso es lo que te diste cuenta. So Kellys dice que si le sumas 30 a 38 vas a tener 68. Te vas a pasar de más. Pero él dice le puedo quitar un 10. ¿Con qué empezaste aquí? Me dijiste, vamos a empezar con 38 y él pensó que el 32 era de más. Se había pasado, hizo demasiado. Entonces le quitaste un 10 al 32 y pensaste mejor 22. ¿Por qué le quitaste el 10?* (Ah! That is what you realized? So Kellys says that if you add 30 and 38, you’ll get 68. You’re going to go too far. But he says I can take a 10. What did you start with here? You told me, let’s start with 38 and I think that 32 will be too much. He had passed, he did too much. So, you took a 10 from 32 and thought instead 22.)

Hernán: *Porque le puedo pasar esto aquí y esto va a hacer un 10...* (Because I can pass this here and it will make a 10.)

Ms. Acuña: *¡Ah! Hernán se dio cuenta de que esa decena del 30 ya la tenía escondida en las unidades. Dijo ¡ah! Estos 2, si yo los uno con los 8, hacen el 10. Y puedo encontrar 30 más 30 ó 40 más 20. Muy interesante. ¿Qué otras estrategias usaron hoy?* (Ah! Hernán realized that the tens in 30 was hidden in the ones. He said, ah! These 2, if I put them with the 8, they make 10. I can find 30 and 30 or 40 and 20. Very interesting. What other strategies did you use today?)

Enrique goes on to share his incremental strategy of starting at 38 and counting up to 60. Ms. Acuña mentions this was similar to Gina and Mariel’s strategy and has the class count along as they re-enact Enrique’s strategy on the whiteboard to arrive at the answer of 22.

In this example, Ms. Acuña purposefully asks Hernán to share his mathematical idea, even though it is not a correct solution. As Hernán shared his strategy, Ms. Acuña validated the idea that it is understandable why Hernán and others might have thought the solution was 32. As a class, Ms. Acuña’s students built not only mathematical language together, but they also built an understanding of what is commonly called *regrouping* in the elementary mathematics classroom. As Hernán shared his solution, he chose to share his ideas in Spanish. In this particular episode, allowing him to use his linguistic repertoire provided a space not only for Hernán, but for the entire class. Language was a tool through which

mathematical understanding occurred. Ms. Acuña did not force a particular format upon the student's explanation. The teacher took the role of listening and amplified students' strategies as they made sense of each other strategies and the mathematics (Kleyn & Yau, 2016).

Moschkovich (2015) posits that mathematical literacy is not just about the cognitive aspects of mathematics, it is also situated in experiences, such as within discussion and being able to express one's ideas in a known language. Thus, it is the participation in mathematical practices, in this case discussion of strategies, that leads to mathematical proficiency for multilingual students. In these interactions, Ms. Acuña was both supporting students as they engaged with and comprehended complex content and texts and also supporting students' bilingual identities and socioemotional development. She made a purposeful decision, due to her knowledge of the students' literacy in mathematics, to flexibly use Spanish as the means through which to encourage a powerful strategy in the classroom space.

Final Thoughts

The translanguaging *corriente* was a resource in Ms. Acuña's TWDL classroom, and due to the classroom's shared mathematical work throughout not just one, but two years, both bilingual and mathematical identities were developed and encouraged. Recognizing the capacities and thinking that students already bring into our classrooms (Turner & Drake, 2016), and centering them with a view of language as a fluid, communicative tool (García & Wei, 2014) promotes students' mathematical agency. This is accomplished by amplifying the connections and mathematical comprehension generated (Kleyn & Yau, 2016) and allowing multilingual students to demonstrate the extent to which they are already mathematical doers. We encourage educators to begin to view the language practices of their multilingual students and to consider a translanguaging stance. When they do so, students not only have the opportunity to explain and understand in more than one language, but also simultaneously create mathematical agency around multilingual identities.

For those worried about the dissolution of "language separation," we reiterate that emphasizing mastery of English (or Spanish) is using language to obfuscate the racialized experiences of multilingual students, for they

will still be critiqued even when they are speaking English "proficiently" (García et al., 2017). Let us instead view translanguaging as an empowering and critical act for multilingual students. Viewing mathematics learning in multilingual classrooms through the lens of language practices and continuing to research and explore how the translanguaging *corriente* builds strong mathematical literacy for multilingual students by building on the strengths of their full linguistic repertoires.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Why is it important for a teacher of multilingual students to know the historical and political consequences of language policy programs on how multilingual students' language proficiency is determined?
2. How could you bring your students' full linguistic repertoire into the mathematics classroom?
3. If you are not bilingual, how could you begin to be aware of the translanguaging *corriente* in your classroom and use it *juntos* with your students for mathematics learning?
4. A good resource for further understanding translanguaging is the CUNY-NYS Initiative on Emergent Bilinguals. Download their free Translanguaging Guides at <https://www.cuny-nysieb.org/>. What translanguaging practices will you build into your mathematics teaching?



TODOS Live!

TODOS Live! began with funds obtained through a NCTM Mathematics Education Trust grant. Through the years TODOS Live! has had many excellent sessions and presenters. These sessions are generally an hour in length and occur in the late afternoon when classroom teachers can participate. A list of previous recordings and upcoming sessions can be found at <https://www.todos-math.org/todos-live>. Since mid-March 2020, 13 TODOS Live! sessions have occurred and many are planned the remaining of 2020. Please note that due to limitations to online storage, access to some of our sessions has been lost. Currently, sessions are being stored on a TODOS Live! Vimeo Channel (<https://vimeo.com/user56336191>).

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Translanguaging to Persevere: Supporting and Recognizing the Meaning-Making Process for Latinx Bilingual Students of Mathematics

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Abstract

This paper describes the translanguaging and perseverance practices of Latinx bilingual students and the careful preparation of their English-speaking, monolingual teacher to establish a supportive classroom environment. Drawing on the constructs of translanguaging mathematical practice and perseverance in problem-solving, we share our observations of a group of four Latinx bilingual students as they leverage their bilingualism to productively struggle to make sense of an exponential function. We discuss this vignette to reveal the pedagogical decisions that helped support these students to dialogically leverage their communicative resources to help persevere with in-the-moment mathematical obstacles. Such decisions included selecting and enacting a challenging mathematical task conducive for perseverance, encouraging a linguistically sensitive learning environment, and providing access to mathematical tools as learning resources.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What does translanguaging and perseverance look like for secondary bilingual students?
2. In what ways could a monolingual English-speaking teacher encourage translanguaging and perseverance in their bilingual students, for the purpose of learning mathematics with understanding?
3. What challenges might a monolingual English-speaking teacher face when enacting a task that invites collaboration and bilingualism? How might this teacher be supported?

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Translanguaging to Persevere: Supporting and Recognizing the Meaning-Making Process for Latinx Bilingual Students of Mathematics

Joseph DiNapoli and Hector Morales, Jr.

There is a growing call for teachers to implement equitable, rigorous, and coherent mathematics instruction for their bilingual students (TODOS, 2019). However, dehumanizing school practices continue to marginalize bilingual students' linguistic, social, and cultural capital, especially those in the Latinx community (García, 2017; Gutiérrez, 2017; Langer-Osuna et al., 2016). Monolingual English-speaking teachers especially need support with authentically supporting their bilingual students' mathematical meaning-making (Wickstrom & Gregson, 2017). As authors, we position ourselves with a rehumanizing perspective (Gutiérrez, 2017) through which bilingual learners, when positioned in supportive learning environments, can act as agents of their own translanguaging use and are capable of collaboratively persevering toward authentic conceptual learning (Morales, Jr. & DiNapoli, 2018). In this paper, we describe, discuss, and reflect on a group of Latinx bilingual students' translanguaging and perseverance practice as they collaborated to make meaning in a supportive classroom environment. We detail the bilingual students' learning progression with mathematical functions as well as the classroom environment established by the monolingual English-speaking teacher that helped facilitate it.

Translanguaging to Persevere with Mathematics

Powerful classroom practice in a bilingual setting must encourage both translanguaging and perseverance with Latinx bilingual students. Recognizing and employing these lenses is paramount for mathematics teachers. Avalos, Medina, and Secada (2018) argue that the best way to support bilingual learners in mathematics is to explicitly teach and model mathematical semiotics (language, symbols, visuals) while students are engaged in challenging meaning-making interactions. We draw on the concept of translanguaging to reconceptualize bilingualism as a liberating and empowering communicative practice, and a resource capable of

transforming learning that goes beyond the transition to a dominant school language (Maldonado, Krause, & Adams, 2018).

Translanguaging is an interrelated communicative practice that makes up bilinguals' linguistic repertoire (Cenoz, 2017). To make meaning, speakers can use their languaging, bodies, multimodal resources, tools, and artifacts in dynamically coordinated, interconnected, and entangled ways (García, 2017). For learning mathematics, effective teaching must support translanguaging mathematical practice (TMP, see Figure 1) by encouraging fluid movement between mathematical and everyday speaking across languages (Morales, Jr. & DiNapoli, 2019). This includes using everyday linguistic features and mathematics register resources in dialogically entangled ways with the intention to make meaning (García -Mateus & Palmer, 2017). Classroom teachers supportive of TMP provide opportunities for bilingual students to use their linguistic, multimodal, and mathematical repertoire to make meaning as they persevere with challenging ideas.

Perseverance in the classroom is a cycle of initiating and sustaining in-the-moment productive struggle in the face of one or more obstacles, setbacks, or discouragements (DiNapoli, 2018). Because productive struggle can help develop mathematical meaning (Hiebert & Grouws, 2007; Middleton, Hatfield, Tallman, & Davis, 2015), we draw on the Three-Phase Perseverance Framework (3PP, see Figure 2) (DiNapoli, 2018) to help illuminate the ways in which students first wrestle with an uncertain mathematical situation, and, if necessary, how they amend their first effort to continue to make mathematical progress. Paramount for supporting student perseverance is selecting and enacting a mathematical task that supports student uncertainty (Bass & Ball, 2015; DiNapoli, 2018). Specific to Latinx bilingual learners, mathematical tasks necessitating perseverance should be complex enough such that students engage at all levels of language proficiency to help make their own connections (Driscoll, Heck, & Malzhan, 2012), yet invite engagement

with multiple entry points and resources (Aguirre et al., 2012). For monolingual teachers of bilingual students, the

enactment of these tasks must carefully preserve students’ opportunities to translanguage to persevere.

Figure 1

Translanguaging Mathematical Practice

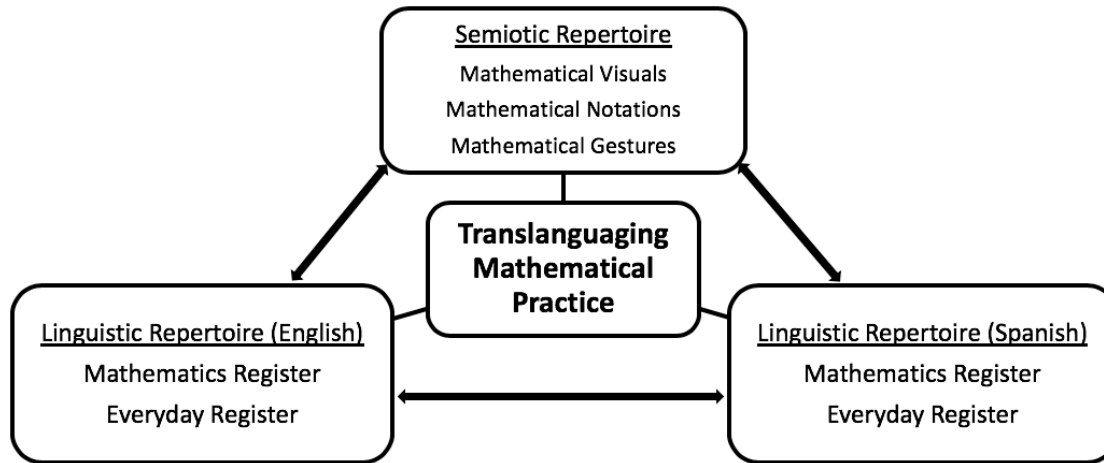


Figure 2

Three-Phase Perseverance Framework

Entrance Phase	
Clarity	Objectives were understood
Initial Obstacle	Expressed or implied that a solution pathway was not immediately apparent
Initial Attempt Phase	
Initiated Effort	Expressed intent to engage with task
Sustained Effort	Used problem-solving heuristics to explore task
Outcome of Effort	Made mathematical progress toward a solution
Additional Attempt Phase (after perceived setback(s))	
Re-initiated Effort	Expressed intent to re-engage with task
Re-sustained Effort	Used problem-solving heuristics to explore task
Outcome of Effort	Made additional mathematical progress toward a solution

Context, Participants, and Task

This study took place in an urban 12th-grade classroom (Morales, Jr., 2004). For this paper, we focus on one group of four Latinx bilingual students and their monolingual, English-speaking teacher. The students, Carina, Jessica, Elena, and Ines (all pseudonyms), were chosen because they had worked in the same small group for much of the school year and often leveraged their

bilingualism when collaborating. Carina, Elena, and Jessica were born in the United States, but their parents were from Mexico. These three students had been enrolled in bilingual programs during their elementary grades and had transitioned into mainstream classrooms by the time they were in middle school. Each of them grew up speaking both Spanish and English at home, and they all stated that they felt comfortable speaking Spanish but not comfortable reading or writing in Spanish. Ines

was born in Mexico to Mexican parents, immigrated to the United States at age 12, returned to Mexico for 9th grade, and then returned to the United States to complete her high school education. Ines' formal educational experiences included speaking, reading, and writing in Spanish. These students represent typical students in the school with a history of average achievement in their mathematics courses. For all four of these students, Spanish has always played a major role in their meaning making process for mathematics.

The teacher, Ms. O (pseudonym), was a monolingual English-speaker with 20 years of mathematics teaching experience. Ms. O was trained by her school district to use the *Interactive Mathematics Program* (Fendel et al., 2015) curriculum and thus was a proponent of student-centered mathematical activities and problem-solving. Ms. O's pedagogy incorporated many aspects of Smith and Stein's (2011) *5 Practices for Orchestrating Productive Mathematical Discussions* and she often explicitly encouraged her students to collaborate with challenging mathematics in any language and use mathematical tools to explore multiple resources and representations while doing so. Ms. O was not formally trained to support translanguaging in her teaching.

The mathematical task with which Carina, Jessica, Elena, and Ines engaged was a multi-day activity necessitating perseverance. The Function Analysis Task was introduced by Ms. O and invited students to revisit a function (couched as a story) they had explored in a past unit. With this function, students were prompted to (1) describe the mathematical context and define the independent and dependent variables, (2) describe the utility of the function, and (3) describe the family of function mathematically (e.g., linear, quadratic, exponential, etc.). The group chose a function presented as a magical story in which a character's height is doubled by eating a special cake. From this story comes the basic principles for working with exponents and an introduction to exponential growth and decay.

The design and facilitation of this task supports Latinx bilingual students' perseverance by (a) encouraging collaboration with other Latinx bilingual peers, (b) involving the exploration of a familiar mathematical idea that was not yet well-formed, and (c) affording the autonomy to choose a past function and to describe its characteristics in their own way. Also, Ms. O

explicitly stated that the students should work in any language with which they were comfortable.

Classroom Vignettes and Findings

Here we describe observations from Carina, Jessica, Elena, and Ines' engagement with the *Function Analysis Task*. This group exemplified translanguaging and perseverance practices while conceptually exploring exponential growth.

Translanguaging to Persevere on the *Function Analysis Task*

Consider the following vignettes from Carina, Jessica, Elena, and Ines' three-day collaboration around the *Function Analysis Task*. With Ms. O's help, this group demonstrated evidence of understanding the goal of the task during the last five minutes of class on Day 1. They read the task together and agreed on a past function to revisit, but they did not exactly know how to complete it. Beginning in Table 1, we share observations and analysis from this group's collaboration during the last 20 minutes of class on Day 2, as they made their Initial Attempt at solving without interaction with Ms. O.

The students made their Initial Attempt with the Function Analysis Task by coordinating their meaning making actions and deploying their linguistic repertoires. After reading, the students began to interpret the problem context and the nature of doubling. In dialogically entangled ways, they expressed linguistically across Spanish and English what doubling meant to them (e.g., double, two, dos, grows twice, sumando el mismo número, multiplicas por dos, times it). They also questioned how to represent their ideas symbolically, which demonstrates their productive struggle regarding understanding the nature of doubling. This translanguaging exchange is an example of students moving between everyday and mathematics registers across both languages, as well as drawing on mathematical gestures, notations, and visuals. Drawing on such resources is also evidence of the careful classroom culture Ms. O had cultivated, one that was supportive in students spending diligent effort using linguistic repertoires and multiple representations to explore the nature of the function. They did not,

Table 1*The Group's Translanguaging Practice in the Initial Attempt Phase of Perseverance***Transcript (English Translation)**

JESSICA: ¿Qué era la primera, se hace así? (*What was the first one, do you do it like this?*) If [she] eats one ounce, that means that she grows twice, dos ¿qué? (*two, what?*) Double, no double, two... See, so when two is four, and then three is six, and four is eight, y así, y así vamos hacer la graph (*like this, and this is how we are going to make the graph*). Going like that [gesturing], para arriba (*up*). You get it?

ELENA: Um hmm. Pero (*But*), how to times it?

JESSICA: Porque mira (*look*), two, times two. Well no... Double it by, nomas (*just*) double the number of ounces, so if she takes...

ELENA: Two times two, y luego (*and then*) four times two, y luego (*and then*) six times two, is that what you are saying?

JESSICA: Más o menos como sumando el mismo número. (*More or less like adding the same number.*)

CARINA: Pero es lo mismo de sumando si lo multiplicas por dos. (*But it is the same as adding if you multiply by two.*)

INES: Lo que parece es como hicimos un in/out table y ya lo sacamos. (*It looks like we just did an in/out table and that's it*). (See Figure 2, left)

CARINA: Yeah. In times two equal out... ¿Ya no tenemos que hacer su altura? (*We don't have to use her height?*)

Table 2*The Group's Translanguaging Practice in the Additional Attempt Phase of Perseverance***Transcript (English Translation)**

INES: Empezamos de cuatro pies. Si toma si come un pedacito son ocho, si come un pedacito son dieciseis, el tercer pedazo dieciseis y dieciseis. Treintaidos ¿no?

(*We start at four feet. If she drinks, if she eats one piece it becomes eight, if she eats one piece it becomes sixteen, the third piece, sixteen and sixteen, thirty-two, no?*)

JESSICA: Pero, ¿cómo sacastes eso? (*But how did you get that?*)

INES: Porque si empezamos con cuatro pies, como yo les digo, si come un pedacito y sale, aumenta de altura de doble [gesturing up]. (*Because, if we start at four feet, like I'm telling you, if she eats one piece and it comes out to, her height grows double (gesturing up)*).

JESSICA: Ohh, her height doubles!

ELENA: You know it's the same thing mira (*look*). Dos (*two*), you multiply one times two is two, two times four is eight, y si pones (*and if you put*) two times two is four, four times four is sixteen.

CARINA: In squared times 2 equals your out.

however, immediately realize they were multiplying the number of ounces of cake by two, instead of the character's height. Their equation correctly spanned the table of values, yet these representations did not model an exponential function.

Not completely agreeing with the other students' mathematical representation, Ines' metacognition during their Initial Attempt helped collectively move the discussion in a direction that considered the character's initial height. The entire group collaborated around this new idea and persevered together as a learning community. Near the end of class on Day 2, Ines recalled her prior experience with this problem from her

sophomore year and raised the issue of starting with an initial height. This helped the others rethink about the mathematics, cross out their first In/Out table (Figure 3, left), and they worked for about 10 minutes during class on Day 3 to think about starting a new one (see Table 2).

Ines tried to help the group understand that the character's initial height is necessary to compute subsequent heights. The task was written in English, yet Ines leveraged her native language to re-voice the problem. She modeled the concept of doubling using her mathematics register in Spanish, and she also mathematically gestured to demonstrate how the character's height doubles for each ounce of cake she eats.

This incited Jessica to have her moment of clarity, in which she realized the character’s height needed to double, not the number of ounces of cake.

This translanguaging exchange exemplified further perseverance by exploring what it means to double and illustrated how this group became aware about the flaws in their first attempt to make sense of the function. What makes this so significant is the students’ commitment to collaboratively and fluidly draw on their mathematical and everyday registers in English and Spanish as they continue to try to grapple with the concept of doubling. The combination of these interactions affords Jessica her liberating moment of understanding, which leads these students to amend their plan and adapt their thinking to continue to persevere.

Following this discussion, the students created a new In/Out table (Figure 3, center) that correctly modeled the character’s exponential growth. This new approach demonstrated the end of their Initial Attempt and the start of their Additional Attempt by the group amending their plan and making a second attempt to make sense of the function. This new approach also demonstrated the interaction within TMP as they drew dialogically on their full semiotic repertoire by considering their mathematical visuals, notations, and gestures. Unfortunately, their equation was not correct and did not span all of their entries. Again, this is an important opportunity for the group to recognize more mistakes and continue to persevere.

Near the end of their time working on Day 3, the students used a graphing calculator to make a table of values for $y=2x^2$ (Figure 3, right) and discovered that it did not match their current table. Consequently, they were ready to continue to explore ways to change their equation and make another Additional Attempt, but time was up and Ms. O needed to move on to a discussion of solution processes. Although this group did not yet find the proper equation to span their In/Out table in this episode, they were agents of their own translanguaging repertoire and leveraged this practice to help persevere and think more deeply about exponential functions.

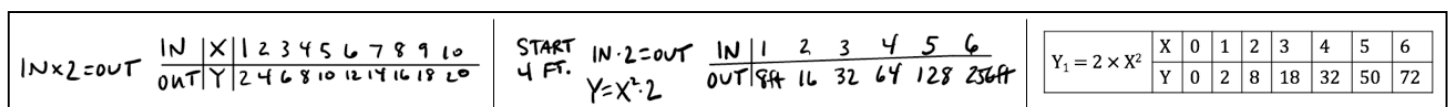
Discussion

Reflecting on these classroom episodes, we use Ramirez and Celedón-Pattichis’ (2012) five guiding principles for teaching mathematics to Latinx bilinguals to help reveal the specific ways Ms. O fostered a classroom environment supportive of mathematics learning for her bilingual students. Consider the Five Principles: *challenging mathematical tasks, linguistically sensitive social environment, support for learning English while learning mathematics, mathematical tools and modeling as a resource, and cultural and linguistic differences as intellectual resources*. These principles articulate effective ways to engage Latinx bilinguals to persevere with rigorous mathematics while engaging in TMP. In the vignette we shared, Ms. O’s careful efforts were mostly in preparation with intent to build a classroom culture that aligned with Ramirez and Celedón-Pattichis’ (2012) principles and supported Carina, Jessica, Elena, and Ines’ translanguaging to persevere. We believe it was the building of this classroom culture that helped lay the foundation for the mathematics learning of this group of Latinx bilingual students.

Carina, Jessica, Elena, and Ines’ meaning-making trajectory was one filled with rich translanguaging practice and authentic productive struggle. The thoughtful selection of a challenging mathematical task designed with several supports for bilingual learners demonstrates Ms. O’s commitment to nurturing her bilingual students toward mathematical understanding, as well as her commitment to gain knowledge and training, since nurturing alone may not always be enough. Assigning the Function Analysis Task was an important decision because it allowed students to access mathematical ideas through their prior work and created an environment conducive for productive struggle. When introducing the task, Ms. O insisted her students use their native language to engage with the mathematics, which suggests that she genuinely intended to create a productive, inclusive, and linguistically sensitive learning environment. It was

Figure 3

Progression of the Group’s In/Out Tables (recreated by authors)



within this environment that Carina, Jessica, Elena, and Ines spontaneously leveraged each other's linguistic repertoire and viewed their linguistic resources as assets to help persevere past obstacles and make progress toward a solution. Ines' strong academic discourse in Spanish was particularly celebrated and played a crucial role in the meaning making process for the group.

Ms. O also encouraged collaborative learning to allow her students to communicate with each other in their groups. Such emphasis on student-to-student communication in any language suggests Ms. O believes that cultural and linguistic differences are intellectual resources and valuable to the meaning-making process when learning mathematics. Additionally, Ms. O encouraged her students to first read their textbook in English and then discuss the mathematical meaning in both English and Spanish. This supported their language and literacy development by helping make the mathematical content more comprehensible. Further, Ms. O intended to create a classroom environment where using mathematical tools was the norm. Students' understandings were mediated by the use of the graphing calculator, In/Out tables, and other multimodal visual and symbolic representations. Access to these mathematical tools proved to be instrumental in the ways in which the group persevered past their mathematical obstacles.

Carina, Jessica, Elena, and Ines' engagement within Ms. O's classroom illustrates how an effective learning environment can be established by a monolingual teacher of bilingual students. In our view, Ms. O thoughtfully established her classroom culture with strategies aligned to Ramirez and Celedón-Pattichis' (2012) five guiding principles for teaching mathematics to Latinx bilinguals to ensure that her bilingual students had the time and space to productively struggle with a well-designed task. It was the careful planning of the classroom environment that helped nurture the group's evolving mathematical understanding of exponential functions. Although Ms. O advocated for native language use, we recognize it can be difficult for monolingual teachers to view bilingualism as an asset and not yet truly understand the advantages of translanguaging practice for learning with understanding (Maldonado, Krause-Rueda, & Adams, 2018; Mazzanti & Allestaht-Snyder, 2018). We encourage future scholarship to help connect research to practice in ways that help monolingual teachers recognize examples of

powerful classroom practices to support the meaning-making process for Latinx bilinguals, as well as to develop a belief in the educational value of bilingualism.

Conclusion

This paper offered a description and discussion of a group of Latinx bilingual students' translanguaging and perseverance practice as they collaborated to make meaning of an exponential relationship in a supportive classroom environment. We hope the reader appreciates the intentions of the monolingual, English-speaking teacher, Ms. O, and learns from her effective practices that informed task selection, design, and enactment. The students' engagement within Ms. O's classroom environment demonstrates inspiring collaborative activity and is a true example of translanguaging to persevere within the context of learning mathematics with understanding. We also hope the reader appreciates that more work is needed to support teachers like Ms. O to recognize what the process of meaningful learning looks like, especially for bilingual students. Bilingual learners need explicit support to encourage translanguaging and perseverant practice, and this study shows that teachers also need ample support to consistently facilitate such student engagement.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What methods would you suggest for encouraging bilingual learners' translanguaging to persevere?
2. What advice would you give Ms. O (or teachers like her) when she enacts the Function Analysis Task next time? Is there a way to root your advice to mathematics education research?
3. Is it enough for teachers to just allow their bilingual students to work in their native language? What do you think were some of the challenges Ms. O faced supporting her bilingual learners? Identify and talk through the challenges posed by having bilingual students work in their native language.

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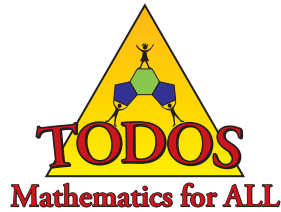
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Transmodalising for Equitable Mathematics Instruction for Multilingual Classroom

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Abstract

This paper introduces a repertoire of practice called *transmodalising* to support discursive practices in the multilingual mathematics classroom. Using the transmodal framework, we describe a classroom vignette of a mathematics classroom using translanguaging and transmodalising that promoted discursive practices and equitable access to rigorous mathematics for emergent bilingual learners. Our discussion highlights the ways in which transmodalising practices align with research-based recommendations for mathematics instruction for English learners (ELs) by treating language as a resource, supporting ELs' participation in mathematical discussions while learning English, and drawing on all meaning-making resources, including home languages, multimodal tools, and out-of-school experiences.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What do you know about translanguaging or transmodalising pedagogy that draws on students' home language and other meaning-making modes for classroom instruction?
2. What kinds of instructional supports have you used to enhance your English learners' access to the mathematics concepts, as well as their participation in mathematics discussion?

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Transmodalising for Equitable Mathematics Instruction for Multilingual Classroom

Sujin Kim and Jennifer M. Suh

The increasing diversity of classrooms offers opportunities for classroom teachers to draw on the diverse capacities of their students for classroom instruction and interaction. For example, an English-speaking teacher might teach a class of 19 students, among which 18 students are English learners (ELs) or who we prefer calling emergent bilingual learners (EBLs) from an asset-based orientation¹ whose different national, ethno-racial, cultural, and linguistic backgrounds can build a wealth of cultural and linguistic knowledge and experiences. However, such classroom diversity, often called *superdiversity* (Vertovec, 2007), has also generated numerous challenges for teachers without specialized preparation in serving EBLs. Challenges persist partly due to the misconception among content teachers that language development and content learning are two separate areas, and it is the English as a second language (ESL) specialist's job to support ELs' language and content learning. Many teachers, while endorsing cognitive skills such as reasoning, inferencing, and critiquing, have not attended to the critical role of language in conducting these cognitive tasks (Zwiers, 2014). Mathematics is one such content area where many conceive that language does not play as big of a role as in language arts or social studies (Gunderson, 2000; Wright & Li, 2008). The language of mathematics, however, is highly complex and necessary to access and comprehend mathematical concepts (Moschkovich, 2012; Veel, 1999). In fact, the Common Core Standards for Mathematical Practices (CCSSM, 2010) heavily rely on discursive skills to make sense of problems, construct viable arguments, and critique the reasoning of others (Moschkovich, 2012).

Mathematical content knowledge is accessed, constructed, and acquired through participating in mathematical discourse practices composed of both verbal and non-verbal activities (Moschkovich, 2015). This requires a broader participation of the learner in an

environment that is rich with the specialized language or discourse of mathematics that Turkan et al. (2014) referred to as Disciplinary Linguistic Knowledge (DLK). An inviting classroom culture promotes participation in such academic discourse practices and increases students' access to rich curricular opportunities, which in turn contributes to students' academic achievement (Zwiers, 2014).

We propose that the pedagogy of translanguaging and transmodalising is an essential feature for equitable mathematics instruction for multilingual classrooms and has the potential to transform power of participation in the mathematics classroom. Drawing on Gutiérrez's (2009) equity framework of access, achievement, power, and identity, we see that teachers have a pivotal role in how *power* transformation can occur by measuring the "voice in the classroom" (p. 6), who gets to talk in the classroom. The shifted power distribution recognizes what students already know, provides students *access*, and engages every student as a creator of mathematical knowledge, while constructing what "competent mathematical behavior" looks and sounds like in the mathematics classroom (Gresalfi, Martin, Hand, & Greeno, 2008, p. 68). In other words, by allowing all students to have voice, teachers can ensure equitable ownership of ideas and opportunities to learn about mathematics, and thus increase *achievement* by encouraging mathematical justification. By using different modalities (i.e., concrete/digital manipulatives, pictures, and gesture) and drawing on multiple resources of knowledge including linguistic resources, we posit that transmodalising and translanguaging supports the equity-based mathematics teaching practices as described by Aguirre et al. (2017) to enhance learners' *identity* by leveraging multiple mathematical competencies and has the potential to invite participation of ELs who typically are marginalized in the mathematics classroom.

¹ We use both terms, English learners (ELs) and Emergent Bilingual Learners (EBLs), for different contexts but mainly to refer to students from families of non-English speaking

backgrounds. We kept the less asset-oriented term "ELs" for the context in which students are officially designated with the term through school assessment system.

Attending to the call for more equitable mathematics education for EBLs, in this paper we share a case study of an elementary classroom teacher who incorporated the pedagogy of translanguaging and transmodalising to address her EBLs' needs to learn language and content at the same time. The guiding research question was: How can an elementary mathematics teacher enhance equitable learning experiences of EBLs through the translanguaging and transmodalising mathematics pedagogy? In the following sections, we first provide a brief review of translanguaging and transmodalising pedagogy and description of the graduate TESOL (Teachers of English to Speakers of Other Languages) course context in which the teacher participated. Then, we examine how the teacher responded to the pedagogy, reflected in her lesson planning and implementation.

Defining Core Practices around Translanguaging and Transmodalising Pedagogy

Translanguaging pedagogy prioritizes instructional practices that build on students' entire meaning-making repertoires (García, 2009). It views students' communicative repertoires and practices as fluid and flexible without dichotomizing one's language of home/community and language of school. For example, a translanguaging classroom allows students to flexibly use both English and their home language(s) in order to access the content, brainstorm and share ideas, and express their learning (Celic & Seltzer, 2011). Such linguistically inclusive pedagogy needs teachers to forgo the idea of teacher control over students' talk and to adopt the collaborative instructional model that acknowledges student agency in drawing from a broader set of funds of knowledge, and ways of thinking, viewing, and expressing their learning and identity (Moll et al., 1992).

Transmodalising pedagogy is an extended translanguaging pedagogy that includes all meaning-making modes such as linguistic, audiovisual, gestural, and digital modes (Hawkins, 2018; Kress, 2011). All modes, separately or combined, contribute to classroom learning. Extending from the *multimodal pedagogy* as providing multiple modes for students' meaning-making and express, *transmodalising pedagogy* (Newfield, 2014) engages both the teacher and students in a cognitive process of choosing and translating messages from one

mode to another, or combined modes. As such, transmodal translation occurs as active and ongoing meaning-making processes, rather than as a one-time event using one or the other mode (Kress, 2000). Such coordination of meaning across different modes is a powerful stimulus to learning. For example, a teacher can verbally introduce key mathematics vocabulary words using a visual dictionary accompanied by gestures for students whose English proficiency may not support understanding of key concepts if delivered in English only (Shein, 2012). Total Physical Response, sing-aloud, gallery walk, and smart board, for example, are multimodal tools to transform a message in one mode into another or a combined modal status with increased potential of getting the message across.

In our study, teachers were first provided with opportunities to understand and practice the frame of translanguaging pedagogy as a means of allowing students to draw on their full set of linguistic resources, including their home language. Then the transmodalising pedagogy was highlighted to expand teachers' understanding of discursive practices to include how language and other modes can work together to increase comprehensible input (Krashen, 1985) for EBLs. In this paper, we present a detailed vignette of *transmodalising* moments as an example of how a teacher and her students can shift from one mode to another and combine diverse modes in order to teach, learn, and communicate key content concepts (Kress, 2000; Newfield, 2014).

Study Context and Data

The case teacher, Lydia, took a 16-week Spring 2019 graduate course, *Methods of Teaching Culturally and Linguistically Diverse Learners*. Students in this course were a cohort group of 15 in-service teachers in an increasingly diverse school district. For our case study, Lydia (pseudonym) was selected for the following reasons: (1) her class was a superdiverse classroom, where more than 50% of students were EBLs, and (2) Lydia was responsive to the transmodalising and translanguaging pedagogy evidenced in her lessons, providing a good model for teachers to adopt for their mathematics lesson with diverse students.

Lydia was a first-grade classroom teacher with 11 years of teaching experiences in a school, where 65% of students were classified as ELs. In her classroom, just one

of her 19 students was a native English speaker; the rest were all EBLs from diverse immigrant backgrounds, such as Mexico, El Salvador, Guatemala, Honduras, India, Vietnam, and Morocco. Two students were former ELs, who were at the monitoring stage before exiting the English as a Second Language (ESL) service, and one started school fully proficient in English and spoke Vietnamese at home. Fifteen students were ELs with 10 student at WIDA (World-Class Instructional Design and Assessment test) English proficiency level 1 (entering stage), one at level 2 (developing stage), three at level 3 (developing stage), and one at level 4 (expanding stage).

For our case study, we analyzed Lydia's online discussion posts on Blackboard, teaching videos, simulated teaching demonstration, reflection papers, and final unit plan. Written data were analyzed using the constructive grounded theory approach of open and axial coding, constant comparison, and memo writing (Charmaz, 2010), yielding main categories such as linguistically and culturally responsive teaching strategies, teacher perception of translanguaging pedagogy, transmodalising moments, and remaining challenges. Secondly, Lydia's teaching video was analyzed using the multimodal discourse analysis approach (Kress, 2009) to capture how the mathematical discourse has taken place and evolved in her lesson, especially attending to the transmodal moments of how the teacher employed and transitioned across multiple modes to facilitate EBLs' mathematical discourse practices.

Translanguaging and Transmodalising Mathematics Classroom in Action

In this section, we present how Lydia's understanding of translanguaging and transmodalising pedagogy has evolved and manifested in actual lessons, particularly through a detailed sketch of one mathematics lesson in her classroom.

Emerging Translanguaging Practice

My concept of translanguaging has been improved throughout the semester... It is extremely useful and students should be allowed to use their native languages. I do think that I'd like to learn even more

about strategies for implementing it with young children... I use Spanish labels in my classroom. My students often work with a partner to talk out what they are going to write about and plan for writing. This is extremely useful as they can talk through in whatever language they are most comfortable (most of my students speak Spanish) and they can help each other come up with the word/phrase they want to use... I read books that are bilingual. I also sometimes give directions using my rudimentary Spanish skills if I have students who are newly arrived to the country.

This excerpt, from Lydia's unit plan analysis paper, illustrates how she viewed her perspective and practice of translanguaging change during the course. Lydia had adopted a number of strategies for supporting students in fluidly moving between languages. For mathematics instruction, for example, Lydia employed a range of translanguaging strategies to facilitate students' mathematical discourse practices. She expressed that she might better facilitate translanguaging if she were a bilingual or had better proficiency in Spanish. However, her practice demonstrated how even a monolingual teacher with limited multilingual experiences can promote translanguaging in the classroom using such strategies as sharing bilingual (English-Spanish) content and language objectives (LOs) (see Figure 1), reading bilingual books, providing key vocabulary words in both English and Spanish as well as translated directions for class materials, and allowing students to brainstorm and discuss in their home language. She also shared how the use of translanguaging strategies enhanced students' performance across content areas: "I allow my students to speak in their preferred language. This allows students to communicate their ideas effectively, and to understand the directions which leads to higher performance on assessments." Connecting to the students' extended linguistic repertoires, and thus, their ways of thinking and communicating beyond what monolingual English classrooms normally allow, is to recognize students' abilities to make sense of the concepts and discourses through their expert language and identity resources. This, in turn, increases students' motivation and engagement with the content learning and discourse practices even when the teacher does not share students' languages and knows only one or two languages at best (Celic & Seltzer, 2011).

Figure 1

English-Spanish Bilingual Learning Objective (Final Unit Plan of Lydia)

Content Objectives – **As a result of this lesson, students will be able to:**

- 1. Identify a penny and its value as 1 cent.**
- 2. Count a collection of pennies and identify their value.**

Objetivos de contenido- **Como resultado de esta lección, los estudiantes podrán:**

- 1. Identificar un centavo y su valor como 1 centavo.**
- 2. Cuente una colección de centavos e identifique su valor.**

Language Objectives – **As a result of this lesson, students will be able to:**

- 1. Identify a penny and name the value as 1 cent orally and in writing using anchor charts and their money book.**
- 2. Count pennies using real pennies and pictures and orally name and write the value.**

Objetivos del lenguaje- **Como resultado de esta lección, los estudiantes podrán:**

- 1. Identifique un centavo y nombre el valor como 1 centavo oralmente y por escrito utilizando tablas de anclaje y su libro de dinero.**
 - 2. Cuente los centavos usando centavos reales e imágenes y nombre y escriba oralmente el valor.**
-

Strengthening Transmodalising Pedagogy

Throughout the course, Lydia’s use of multimodal instructional tools became more purposeful and tailored for EBLs with increased transmodalising moments across written, spoken, audiovisual, and gestural modes of communication. Lydia reflected on the practice and effect of her transmodalising pedagogy:

I also use a lot of visuals and realia, and explicitly teach vocabulary. This allows my students to connect with the words being spoken. I also use a lot of total physical response which allows students to connect a gesture to a concept. Similarly to translanguaging, I often see a stronger performance from my students when I do these things.

With 11 years of teaching experience, Lydia’s multimodal instruction and interaction with the students may have already been a routine in her classroom with young learners. What we noticed and highlight in this paper was her awareness of why and how transmodalising (and translanguaging) can help EBLs better understand content concepts that otherwise would be difficult to grasp and express. Below is a vignette of one such transmodalising mathematics lesson Lydia used with her first-graders.

Lesson on equation. Lydia started by sharing content and language objectives of how to solve the equations, *written*

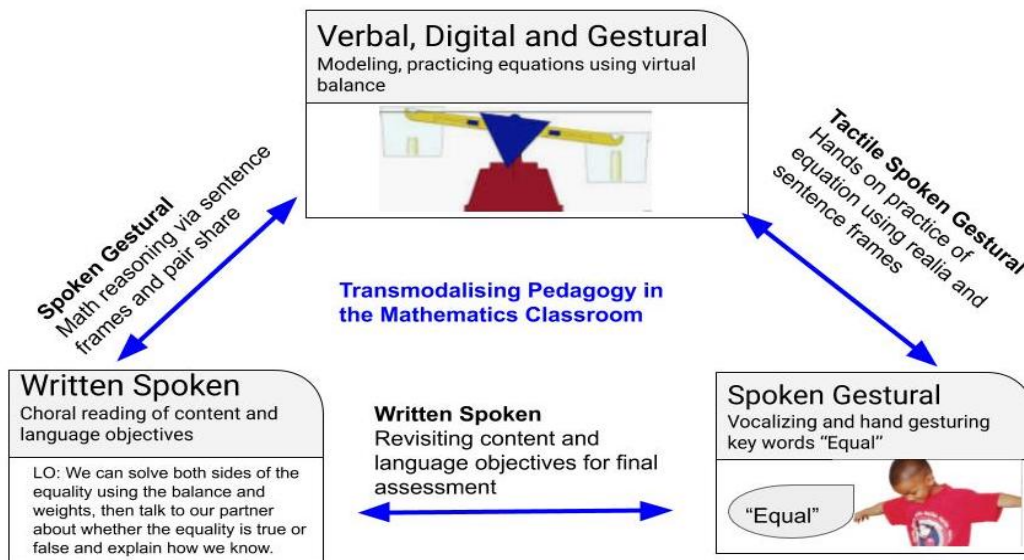
in student-friendly language, voiced using *choral reading* with students. She then *asked* students, “Who can remind me what ‘equal’ means?” Repeating a student’s answer, she then *gestured with her hands* to make a balance to describe how both hands can hold “equal” amounts to reinforce the concept and the meaning of the word “equal,” the key mathematical disciplinary vocabulary and linguistic knowledge (DLK) for this lesson. With her prompt, students all *motioned* to create a body balance following the teacher’s gesture, a brief formative assessment of students’ reception of the concept through gesture. While explaining the definition of ‘equal’ and ‘equality’ through *verbal and gestural modes*, she also added a *digital mode* of showing and solving the equation problems on the smart board. Subsequently, repeated modeling of solving equation problems was accompanied by Lydia’s *hand gesture* and using the *verbal* discourse patterns of “what do we do to make this balanced?” “How do we know?” to enhance students’ mathematical reasoning processes and discourse usage. After the teacher modeling, Lydia introduced a routine called True and False Equation for which she had her students *turn and talk* to justify their reasoning by using *sentence frames* stating, “I think it’s true because...” or “I think it’s false because...” Students’ reasoning came in the form of the full sentence structure, guided by the sentence starter and the modeled hand gesture of balancing on

either side of their hands. Alternating between the *pair-share* and the *whole group discussion*, Lydia provided multiple opportunities for students to produce, express, and share the mathematical discourse using mathematics-specific academic language such as, “equal”, “balance,” and “equality”, as well as expressions to support reasoning, such as “it is true[false] because...” Later in the same week, Lydia hosted a small group session for those who needed additional scaffolding for equations, using manipulatives (a balance scale with weights) for the *hands-on* experience, and making a *verbal reasoning* with the help of modeled sentence starters and teacher prompts asking, “How do we know? We need to decide if this is true or false, and why. *Turn and talk* to your partner.” Finally, Lydia revisited the learning objectives by *orally reading and checking* in with students’ comprehension as the final assessment. Figure 2 illustrates the synergetic collaboration across multiple modes through transmodal mathematical discourse practices in Lydia’s classroom.

In this written vignette and the multimodal teaching video, one can see and hear how Lydia strives to work on shifting the *power* by inviting her students, most of whom were EBLs (those who are typically marginalized), to

have voice through reading, speaking, watching, listening, and gesturing, and to collaborate using academic mathematical discourses to comprehend concepts of equation and express their reasoning linguistically and multimodally. Lydia’s transmodalising instruction provided multiple entry points into mathematics concepts that were being taught. Along with the increased *access* to the mathematics concept of equation, transmodalising also enabled students to express their learning in more than one way, which in turn promoted opportunities for students to actually use mathematical DLK and mathematics-specific language for their reasoning and problem solving as evidenced in Lydia’s lesson. For example, in her teaching video, Lydia did not dominate the classroom talk, but facilitated teacher-student dyads, as well as student-to-student conversation using combined modes of academic sentence starters, gestures, and realia. In this transmodalising lesson, all EBLs actively participated in mathematical discourse practices for greater *achievement* in understanding the mathematics concept and acquiring mathematics DLK, fostering their *identity* as competent mathematics learners at the same time.

Figure 2
Transmodalising Moments in Lydia’s Mathematics Classroom



Interactive Mode: Whole Group → Pair-Share → Whole Group

Implications

As teacher educators in mathematics education and TESOL, we provided an example transmodalising mathematics lesson in which students' funds of knowledge, ways of communicating and interacting outside of school, and other semiotic modes were incorporated to increase students' mathematical discourse practice. Despite potential challenges from the highly diverse context and the district policy of English-only instruction, Lydia went beyond her comfort zone to establish a more equitable and effective mathematics classroom where students were invited to a mathematical discourse community and could flexibly draw on all communicative repertoires (Rymes, 2014). The transmodalising moments described in this paper present one such classroom example, where almost all students were EBLs and supported by the classroom teacher with transmodal facilitation of students' mathematical reasoning, problem solving, and discourse practices.

For mathematics lessons with EBLs, we suggest that teachers promote students' engagement in mathematical discourse practices by: (1) drawing from students' full language repertoires including their home language practice, for example, by allowing students to talk, brainstorm, and write in whatever language(s) they feel comfortable, and (2) teaching and reinforcing mathematical concepts through different types of linguistic and multimodal practices including teacher-student dyads, small-group student discussions, teacher modeling, and activities using manual and digital realia. Such purposeful designing of transmodalising lessons can maximize students' access to and output of their mathematics learning. Likewise, the transmodalising approach can help teachers address the often unsupported mandate for ELs to simultaneously acquire a new language and new challenging content within a short window of time (Menken, 2006).

Transmodalising practices align with research-based recommendations for equitable mathematics instruction for EBLs (Gándara & Contreras, 2009; Moschkovich, 2013), supporting the idea of (1) treating language as a resource, not a deficit; (2) addressing much more than vocabulary and support EBLs' participation in mathematical discussions as they learn English; and (3) drawing on multiple resources available in classrooms (i.e., objects, drawings, graphs, and gestures), as well as

home languages and experiences outside of school. Enacting the core practice of translanguageing and transmodalising as one of the instructional repertoire will ensure that the mathematics classroom embodies the four dimensions of equity proposed by Gutiérrez (2009) by increasing *access* to content through rich tasks as well as greater participation in a discursive classroom, which will provide students more ownership in their learning, leading to greater student *achievement*, *empower* students' agency in learning, and provide more opportunities for *identity* work as EBLs.

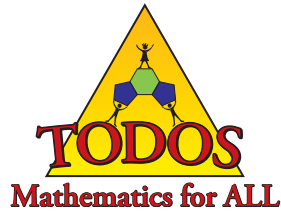
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Discussion and Reflection Enhancement (DARE) Post-Reading Questions

1. How does translanguaging and transmodalising help teachers transform the power distribution in the mathematics classroom in ways that enhance EBLs' participation in mathematical discourse practices?
2. How can you draw on different modalities (i.e., concrete/digital manipulatives, pictures, and gestures) and multiple resources of knowledge, including linguistic resources, to foster mathematical competence among all your students?
3. What other challenges and/or benefits do you envision adopting related to translanguaging and transmodalising pedagogy?



“Whenever My Mom Speaks Spanish at Home, It Helps Me Understand More in Math”: Reflections on the *Testimonios* of Bilingual Latinx Students

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Abstract

In this paper, we share three insights from our conversations with 46 upper elementary Latinx students at predominantly white schools: 1) Students’ were silenced, but they persisted; 2) Students’ relationship with mathematics was enveloped in language; and 3) Students’ mathematics and community were tied together through language. By highlighting the *testimonios* of our participants, we hope teachers reflect on how they empower and raise the voices of their bilingual Latinx students to counter deficit storylines. We also provide opportunities for growth in creating more equitable spaces for bilingual Latinx students.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What is a mathematical identity? How does or does not a students’ mathematical identity manifest itself in school? Are all children asked to share their identities in school in the same way?
2. How do you promote students to use their home language when doing mathematic activities?
3. How do you show students you value their mathematical thinking that may not be in English?
4. How do you construct a classroom environment where use of multiple languages is promoted and valued?

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“Whenever My Mom Speaks Spanish at Home, It Helps Me Understand More in Math”: Reflections on the *Testimonios* of Bilingual Latinx Students

Carlos Nicholas Gomez, Stacy R. Jones, and Hilary Tanck

Latinx¹ students have historically experienced the effects of racism within social institutions like schools (Feagin & Cobas, 2014). This includes marginalizing students based on the color of their skin, their cultural differences, or use of a non-dominant language (Yosso, 2006). As a field, we recognize the existence of structural racism and the role of schools in the oppression of students of color (e.g. Martin, 2000; Yosso, 2006). Latino critical theory (LatCrit) helps us focus on how individuals’ identities as doers-of-mathematics are impacted by race, language, and power in the everyday experiences of Latinx students. For example, some teachers continue to have a deficit perspective of bilingual students (García & Gonzalez, 1995). These deficit perspectives are perpetuated by some teachers believing students require English proficiency before they can do, for example, mathematical tasks with high cognitive demand (de Araujo, 2017)—thereby constructing a master narrative that mathematics is an English action and mathematical thinking should be communicated only in English. Students then see themselves as good at mathematics only if they have strong English proficiency. We argue deficit perspectives can be reflected on and challenged by teachers listening to the *testimonios* (stories) of marginalized students. *Testimonios* (Solórzano & Yosso, 2001, 2002) are part of the mathematical narratives of Latinx students and provide insight into their mathematical identity or what it means to be a doer-of-mathematics (Zavala, 2014). Martin (2006) describes mathematical identity as follows:

Dispositions and deeply held beliefs that individuals develop, within their overall self-concept, about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives. A mathematics identity encompasses a person’s self-understanding of himself or herself in the context of doing mathematics. (pp. 206-207)

Focusing on Latinx students’ *testimonios* help in understanding their developing mathematical identities and their perspectives on teachers’ practices.

In this paper, we reflect on our experiences interviewing 46 upper elementary Latinx students. We recognize not all Latinx students are bilingual, but most participants described speaking Spanish as a valuable part of their life or were identified as bilingual (receiving services called English for Speakers of Other Languages [ESOL]). Our discussions focused on their experiences learning and doing mathematics in predominantly white schools² (PWSs). Latinx people are racialized in American society because Latinx people are seen as needing to assimilate to the dominate (white) culture (Feagin & Cobas, 2014). Thereby, Latinx students’ racial and linguistic experiences in school need to be better understood as their experiences differ from other students. Consequently, we focus on phenomenological underpinnings (Sokolowski, 2000) and highlight these assumptions to help teachers reflect on their own experiences and ways of thinking about bilingual Latinx students. By highlighting the *testimonios* of our participants, we hope teachers reflect on how they can empower and raise the voices of their bilingual Latinx

¹ We use the term Latinx (La-teen-x) to be in solidarity with queer, trans, non-binary, gender fluid, and gender non-confirming Latinos/as. The ‘x’ is a variable to represent the different gender performances of individuals and to go beyond the strict male/female binary.

² The term ‘predominantly white schools’ has greater implications than just the percentage of white students in

comparison to minoritized students (see Bourke, 2016). The term names the dominant culture and emphasizes the power structures in play that minoritized students learn to navigate. We characterized a PWS as having more than 50% of the student population registered as white and Latinx students as being 20% or less of the student population. We choose 20% based on Kanter’s (1977/1993) work on tokenism. We increased Kanter’s percentage from 15% to 20% based on local district data.

students in developing strong mathematical identities that counter deficit storylines.

Critical Race Theory and Latino Critical Theory

We use critical race theory (CRT) and Latino critical theory (LatCrit), aligning with other critical race theorists, who “attempt to explain the implicit and explicit consequences of systemic, policy-related racism; and [who] work to disrupt and transform policies, laws, theories, and practices through the exposure of racism” (Milner, 2007, p. 391). We recognize racism and linguistic discrimination are normal and prevalent within bilingual Latinx students’ being-in-the-world (see Delgado Bernal, 2002; Solórzano & Delgado Bernal, 2001; Solórzano & Yosso, 2001, 2002). Moreover, Latinx students navigate an education system not originally built with them in mind (Yosso, 2006). CRT emerged from legal scholars in the 1970s after they saw the progress made during the Civil Rights Movement gradually dissipate (Duncan, 2018; Ladson-Billings, 1998). Therefore, they worked to investigate and theorize the relationship between race, racism, and power (Delgado & Stefancic, 2001). Since then, CRT has branched into other critical theories (e.g.,

FemCrit, AsianCrit), centering the histories and experiences of other marginalized groups (Yosso, 2006).

Our project aimed to understand Latinx students’ experiences learning mathematics in PWSs with the intent of understanding students’ *testimonios* about what it means to learn and do mathematics. Hence, LatCrit played a major role in the conceptualization of our study because LatCrit “is concerned with a progressive sense of a coalitional Latina/Latino pan-ethnicity and addresses issues often ignored by critical race theorists such as language, immigration, ethnicity, culture, identity, phenotype, and sexuality” (Solórzano & Delgado Bernal, 2001, p. 311). LatCrit helped to center the role of race, racism, power, and language in the mathematics education of upper elementary Latinx students. Solórzano (Solórzano & Yosso, 2002; Solórzano & Delgado Bernal, 2001) described five tenets foundational to CRT and LatCrit in education research and methodology (see Table 1).

Each of the tenets aid in understanding students’ *testimonios* and develop a richer understanding of the experiences of elementary Latinx students. Previous research has found that students’ *testimonios* provide insight into the mathematical identities of students. For example, Zavala (2014) found that secondary Latinx

Table 1
 CRT and LatCrit Tenets (Solórzano & Yosso, 2002; Solórzano & Delgado Bernal, 2001; Yosso, 2014)

Tenets of CRT and LatCrit in Education	Description
The centrality of race and racism and their intersectionality with other forms of subordination	Ways of knowing directly in contrast with Eurocentric epistemologies and focus on the intersection of race with gender, language, class, and other social aspects.
A challenge to dominant ideologies	Cultural and linguistic experiences provide different ways of knowing and being challenging traditional notions of knowing and knowledge construction
A commitment to social justice	Focus on the ways resistance is used when pursuing social justice.
An emphasis on experiential knowledge	Experiences of students are valuable and a strength. Embrace the counter-stories and other experiences which provide the unique insight of the student.
Importance of transdisciplinary approaches	Provides opportunity to use tools and methods from other disciplines like women’s studies and law to understand and improve the experiences of students of color.

students' perspectives of being successful in mathematics related to racial stereotypes (e.g., violent, illegal) and students who did not speak English "properly" found mathematics classrooms to be threatening spaces. LópezLeiva and Khisty (2014) identified microaggressions that an elementary Latinx student experienced from other students and teachers because of her language minority status. The microaggressions prevented her from experiencing productive group work and doing rigorous mathematics. These works, and others (see also Guerra & Lim, 2014), suggested it was necessary to talk with students about how they are experiencing school and mathematics. Moreover, LatCrit positions students' *testimonios* as valuable to our understanding of mathematics teaching and learning. Our conversations, therefore, focused on collecting *testimonios* and investigating how they were challenging dominant ideologies of race, language, power, and mathematics learning. Our guiding question was: What are the developing counter-stories of learning and doing mathematics of Latinx students? In this paper, we highlight our reflections and insights from our initial analysis of our conversations with these students.

Description of Study

The participants have entrusted us with the retelling of their *testimonios*; therefore, it is our responsibility to be reflective on how our being-in-the-world impacts our understanding and dissemination of the students' *testimonios*. Consequently, we needed to be critically conscious of our gender, race, class, sexuality, ethnicity, and nationality and how they impact all aspects of our investigation (Pillow, 2003). Our research team consists of a bilingual white-presenting Chilean cisgender male

who immigrated to the US at a young age, an English dominant Mexican-American white-presenting cisgender female, a monolingual white cisgender female, and a monolingual Black cisgender female. This project was a pilot study designed to inform the team of the experiences of Latinx students in local schools and meant to begin a partnership with the participating district. Therefore, the researchers had no previous relationship with the students. We worked with school administrators and were dependent on their relationship with the community for students to trust us.

Fazil, Kemp, and Leona elementary schools are all part of the Kingwood County School District (all names are pseudonyms) located in the southeastern United States. The three elementary schools had the largest percentage of Latinx students (17%, 20%, and 11%) in the Kingwood District with most of the students in these schools being white (61%, 59%, and 72%). We conducted 46 semi-structured interviews with 3rd, 4th, and 5th grade students whose school records identified them as Latinx. Table 2 below provides the number of students by grade, school, and those receiving ESOL services. The consent form was distributed in both Spanish and English and all students were given the option of being interviewed in either language. All students, however, chose English. Interviews ranged from 22 to 50 minutes.

A LatCrit methodology emphasizes the experiences of Latinx students and their response to our education system (Solórzano & Yosso, 2002). So, the first portion of the interview focused on participants sharing their *testimonios* about learning mathematics at a PWS. Questions included, "How do you think being Latina/o plays a role in how you learn mathematics?" and "How does knowing Spanish help you in learning mathematics

Table 2

Number of Participants by Grade Level and School (participants receiving ESOL services)

School	3 rd Graders	4 th Graders	5 th Graders	Total
Fazil	6 (3)	6 (4)	11 (9)	23 (16)
Kemp	6 (0)	4 (3)	7 (6)	17 (9)
Leona	1 (1)	1 (1)	4 (4)	6 (6)
Total	13 (4)	11 (8)	22 (19)	46 (31)

at a PWS. Questions included, “How do you think being Latina/o plays a role in how you learn mathematics?” and “How does knowing Spanish help you in learning mathematics?” Moreover, a series of questions were developed based on Martin’s (2006) conceptualization of mathematical identity: “Would you describe yourself as a good math student? Why or why not?” and “How do you think learning mathematics will change your life?” For the second portion of the interview, a semi-structured task-based interview approach was used. For this paper, however, we concentrate only on students’ responses to the first part of the interview.

In (re)telling the participants’ *testimonios*, we followed Solórzano and Yosso’s (2002) description of critical race methodologies. Accordingly, we constructed their *testimonios* from (a) the data collected during the research process; (b) the literature informing the study; (c) the research team’s professional experience; and (d) the research team’s personal experience (Solórzano & Yosso, 2001). To analyze the data collected, Solórzano and Yosso (2002) recommend the use of grounded theory (Glaser & Strauss, 1967) along with a critical lens. We began by transcribing all interviews, reading them multiple times, and taking notes about possible emerging ideas. We met weekly to discuss our reflections regarding our own personal and professional experiences caused by our reading of the data. For this article, we focus on three points of reflection we shared during our conversations. We connect these points to data from our study and the literature to help readers reflect on their own experiences and what they could do in the classroom to create more equitable learning spaces for bilingual Latinx students.

What We Learned from the Students

We emphasize three facets of students’ phenomenological ways of being-in-the-world (Sokolowski, 2000): 1) Students were silenced, but they persisted; 2) Students’ relationship with mathematics was enveloped in language; and 3) Students’ mathematics and community were tied together through language.

Students were Silenced, but They Persisted

Many of the students we interviewed determined when it was appropriate and when it was inappropriate to use Spanish at school. This prompted us to reflect on the

choices students make in regards to the linguistic resources they bring to the mathematics classroom and how they may choose not to access linguistic resources because of the risk it may pose to their self-concept as doers-of-mathematics. This is further complicated for bilingual students who use multiple languages to make sense of their world, particularly, when teachers do not create an environment where multiple modalities of expression are valued. For example, Antonio described how the use of Spanish was construed as “keeping secrets” by his teacher.

Interviewer: Ok, how do you think your teachers feel about you talking in Spanish, using Spanish or talking in Spanish with Juanita?

Antonio: Um, kind of like mad because all the time me and Juanita would speak Spanish and they said ‘no secrets in class’ and it’s basically not a secret because we’re just um telling each other like jokes when it’s raining outside so we have to stay inside. And then um, like we would say it in English and then they’d be like oh wait that’s what they were saying so they got kind of mad that we were, doing secrets but we weren’t.

Interviewer: Does that make you not want to speak Spanish in school?

Antonio: No, not really. I only do it at recess.

Antonio’s use of Spanish was silenced by his mathematics teacher. Consequently, his communication with Juanita in the mathematics classroom will be based on a limited linguistic repertoire (i.e., English only). Although he may still use his entire language repertoire when thinking about mathematics and in specific spaces (“at recess”), his verbal communication is restricted in mathematics class because of the risk of being seen as a misbehaving student. This is also an example of how multilingual students are socialized into believing their linguistic resources are not valued in regards to their participation.

Our participants highlighted how teachers kept their language as a present-absence (Sokolowski, 2000), or recognizing the existence of the child’s linguistic practices but not acknowledging their language through mathematical activity. We recommend teachers reflect on how they make visible a student’s use of language and the role it plays in the student learning and doing of mathematics. Literature about translanguaging—seeing bilingual students as not having two autonomous languages but instead one linguistic repertoire (García & Wei, 2014)—may be particularly helpful in thinking

about classroom environment and how to promote holistic linguistic practices (see García, Johnson, & Seltzer, 2017; García & Wei, 2014). Translanguaging is not limited to multilingual individuals. Even those who see themselves as monolingual are translanguaging because linguistic repertoires are also culturally constructed (see Translanguaging Reading Group, this issue). García et al. (2017, p. 63) recommend the following actions to practicing teachers: hang bilingual posters, and signs; put up students' work in English and in the students' home language; create word walls incorporating students' home languages. We also advocate for teachers to incorporate ways for students to use the entirety of their linguistic resources in doing mathematics. In the same way we promote students to use multiple representations to explain their mathematical thinking, we should also encourage them to use their multiple linguistic resources as ways of knowing and doing mathematics. This may include drawing connections to similar-sounding words in different languages (e.g. *adición/addition*; *suma/sum*), promoting agency in the language students use to discuss their mathematical thinking, and developing gestures and visuals collectively to communicate mathematical ideas (see Texas Math Sign Language Dictionary: <http://www.tsdvideo.org>).

Students' Relationship with Mathematics Is Enveloped in Language

From our reflections on our conversations with students, we were able to discern different ways students described how knowing Spanish impacted learning mathematics. When asked, "Does knowing Spanish help you when learning math?", most students said yes, but some were not able to state how. Others described how knowing Spanish is a resource to learning mathematics. Estifan, for example, described how doing mathematics is not necessarily different in either language, but learning mathematics can be easier in one language compared to the other depending on the problem.

Interviewer: How do you think knowing Spanish helps you learn math?

Estifan: Because sometimes you can—instead of just every time saying it in English you can also say it in Spanish.

Interviewer: Oh ok and is it helpful whenever you can say it in Spanish too?

Estifan: Yeah.

Interviewer: How does it help you?

Estifan: By it is just—they're the same numbers. I don't know. That's all.

Interviewer: Is it sometimes easier to understand if it's in Spanish?

Estifan: Not really.

Interviewer: No. Ok

Estifan: Sometimes it's hard, sometimes it's easy.

Interviewer: Is it—is sometimes it's easier in English and sometimes it's easier in Spanish?

Estifan: Mmhm [yes]

Interviewer: So it just depends

Estifan: Yeah. It depends what the problem is.

Estifan determined mathematics can be done with his complete linguistic repertoire, and therefore, mathematics can be done in either language depending on his need. We describe Estifan's and other students' perspective like this as seeing mathematics and their languaging as a fluid and dynamic practice.

There was no evidence the students had considered or been given the opportunity to use Spanish as a way of communicating their mathematical thinking at school. The students did not describe their use of Spanish to quickly access ideas as being publically legitimized. Even though a couple of students described thinking about their mathematics activity in Spanish before communicating their thinking in English. For example, Evita recognized knowing Spanish as a resource for doing mathematics; however, when it came to sharing and communicating her ways of knowing, she had to communicate in English.

Interviewer: How does knowing Spanish help you learn math?

Evita: Well for me it probably more easy counting in Spanish the numbers. Then counting the numbers in English.

...

Interviewer: And when you share your answer, you speak Spanish or English?

Evita: Well mostly I count in my head. Then speak English.

Interviewer: Oh okay. So you count in Spanish in your head but then when you are sharing with the class -

Evita: I speak English.

Evita shared how she prefers to count in Spanish, but the norm in the mathematics classroom is to communicate mathematical thinking in English. She does not challenge this norm. Her linguistic repertoire is then constrained

when sharing her mathematical thinking even though in her head she used her full linguistic repertoire to solve the problem. Hence, Evita was subtly resisting the idea mathematics needed to be done in English. Her example also highlights the importance of her linguistic repertoire as a resource for doing mathematics. Does Evita's teacher recognize and legitimize this aspect of Evita's mathematical thinking? Teachers need to promote students' access to their linguistic resources publically. By not opening space for students to use their complete linguistic repertoire, teachers are limiting students' access to an important resource for learning and doing mathematics. Evita's *testimonio* demonstrates how mathematics was not just an English action, but accessing her full linguistic repertoire could happen only internally.

Students' Mathematics and Community Connected Through Language

Some of the students when asked how Spanish helps them in learning mathematics discussed how knowing Spanish provided them access to their parents as a resource for helping them think about the mathematics they were learning. At first, we did not think too much of this, but reflecting on how taken-for-granted this is for English speaking students, we realized this is an important relationship as it highlighted how students' community, particularly family, can legitimize their complete linguistic repertoire as useful for learning and doing mathematics. This again, helps in the construction of the narrative that mathematics is not just an English action. Alfredo provided us one of these narratives.

Interviewer: how does knowing Spanish help you in learning math?

Alfredo: Because, well in school not a lot, but at home yeah. Because whenever my Mom speaks Spanish at home, it helps me understand more in math, like, what to do and stuff.

For Alfredo, not only is his mother a resource for understanding mathematics, but doing and learning mathematics is discussed in Spanish. Moreover, this interaction helps him understand the mathematics better. Consequently, the experience legitimizes mathematics as a Spanish action, thereby countering the stories of doing and learning mathematics he hears at school. However, he also emphasized how knowing Spanish—a part of his language repertoire—is a lot of the time not useful in a

school context. This is where the teacher needs to work as a cultural broker and help the child in bridging in- and out-of-school mathematics experiences through the use of the child's full linguistic repertoire. Previously, we discussed how teachers can construct spaces that empower bilingual students. Alfredo demonstrates how those environments may also legitimize doing and learning mathematics with a students' full language repertoire and subsequently the community's ways of doing mathematics.

This reminded us of the aspects we, at times, take for granted when teaching because we assume a particular norm. In regards to family or a student's community, it is more than seeing them as a resource, it is also recognizing how the students' language may be bridging the out-of-school mathematics and in-school mathematics. Teachers need to recognize this bridging action and the impact it may have on their activities in the classroom. The questioning strategies used by teachers to learn about students' mathematical thinking can provide evidence of students' perspectives on the relationship between in-school and out-of-school mathematics.

Conclusion

In this paper, we reflected on what we learned from our conversations with elementary Latinx students. Limiting a student's linguistic repertoire not only constrains his or her forms of communicating, but also masks from the teacher the relationship the student has with mathematics and their community. Moreover, students not accessing their complete linguistic repertoire keeps teachers from having a full picture of the students' mathematical thinking. Thereby, to counter deficit perspectives of bilingual Latinx students, teachers must listen and legitimize students' *testimonios* and empower students to use their complete linguistic repertoire in learning and doing mathematics. Students need to see they can be successful in mathematics and still use their complete linguistic repertoire.

Our study pushed us to reflect on the opportunities we provide students to bring their entire identity and linguistic repertoire to the mathematics classroom. In addition, their *testimonios* had us thinking deeper about what support systems could look like for bilingual students. We recommend teachers use reflective tools like mathematics journals, discussion boards (e.g., Flipgrids),

or brief interviews with students to learn about their *testimonios*. Listening and reflecting on students' stories can help in the construction of stronger relationships and in designing meaningful mathematics activities. Learning about the community can also help in redefining how to leverage bilingual Latinx students' linguistic resources for the benefit of all students.

Although we focused on bilingual Latinx students, counter-stories are developed by other marginalized populations (e.g. Black, Indigenous, neurodiverse, LGBTQ+, low socioeconomic status). We recommend reflecting on how your mathematical practices allow the entire child to be part of the classroom's mathematics. What opportunities do you provide the child to bring in their culture and alternative ways of reasoning? How and when is language separation detrimental to mathematics learning? To rehumanize bilingual students we need to focus on the 'ands.' A child is bilingual and mathematically brilliant and Latinx. Each of these aspects of their identity work together to construct the child's mathematics and will deeply influence their mathematics learning (Aguirre, Mayfield-Ingram, & Martin, 2015). It is our responsibility to see each child's mathematics how they want it to be seen and heard.

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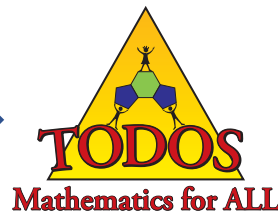
Chicana/Chicano educational pipeline. Taylor & Francis Group.

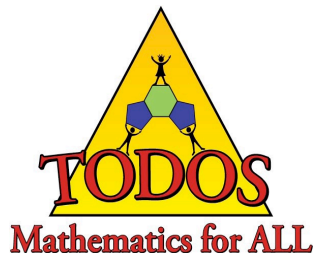
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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. How can children's linguistic practices be promoted in the mathematics classroom?
2. In what ways can children be empowered through their use of language in the mathematics classroom?
3. How can your mathematical practices promote the entire child to engage in the mathematics classroom?
4. What consequences are there to students learning mathematics if they continue hiding other identity aspects?
5. What norms can be established to promote students to use their entire linguistic repertoire?

"DARE to Reach ALL Students!"





TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

<http://www.todos-math.org/teem>

Teaching for Excellence and Equity in Mathematics (TEEM) is a refereed journal published by **TODOS: Mathematics for ALL** and available via membership in TODOS. The intended audience of *TEEM* includes mathematics teachers, leaders, administrators, and mathematics teacher educators. The articles in *TEEM* must align with the mission of TODOS:

“To advocate for equity and high quality mathematics education for all students—in particular, Latina/o students.”

The journal aims to address topics involving excellence and equity (simultaneously) in the teaching and learning of mathematics, in a way that connects research to classroom practice and can directly inform practice. To this end, manuscripts are welcomed that relate to:

- ALL students learning mathematics as problem solvers beyond computational abilities;
- A vision of exemplary practices in the K-12 classroom and professional development;
- Attention to the roles that language and culture have in teaching and learning mathematics.

The journal defines equity broadly, including (but not limited to) issues of language, gender, ethnicity, and culture. *TEEM* welcomes manuscripts on issues of language, culture, access, equity, and quality from diverse viewpoints.

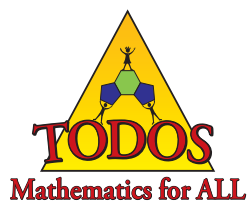
Call for PAPERS

We encourage the submission of manuscripts, including applied or action research, literature surveys, thematic bibliographies, commentary on critical issues in the field, professional development strategies, and classroom activities and resources. While contributions in English are recommended, *TEEM* will also consider contributions in languages such as Spanish. The *TEEM* Editors welcome query emails about the suitability of proposed topics: email at [teem@ todos-math.org](mailto:teem@todos-math.org).

TEEM is very interested in receiving manuscripts from classroom teachers and / or teacher educators. The following are suggested ideas for manuscripts in this category:

- A description, discussion or reflection on implementation of a particular teaching strategy
- A specific classroom-tested TODOS -oriented “excellence and equity” activity accompanied by a blackline worksheet for classroom use
- A focus on some aspect of the TODOS mission and related goals:
 - to advocate for an equitable and high quality mathematics education for all students;
 - to implement lessons and programs that incorporate the role that language and culture play in learning mathematics;
 - to inform the public, including parents, and influence educational policies in ways that enable students to become mathematically proficient: and
 - to inform teacher education programs.

For more details on the guidelines for papers, see <http://www.todos-math.org/teem>.



TODOS Position Statements

The Mo(ve)ment to Prioritize Antiracist Mathematics: Planning for This and Every School Year

2020 Position Statement

<https://tinyurl.com/y3ojztro>



Four Key Areas to Weave Together in Mathematics

Commentaries on two of the key areas for consideration:

Centering Our Humanity: Addressing Social and Emotional Needs in Schools and Mathematics Classrooms

<https://tinyurl.com/y3z4fvre>

Student and Family-Centered Mathematics Assessment

<https://tinyurl.com/y38gb9mb>

Mathematics Education Through the Lens of Social Justice: Acknowledgment, Actions, and Accountability (2016), A joint position statement from the National Council of Supervisors of Mathematics and TODOS: Mathematics for ALL

<https://www.todos-math.org/socialjustice>



2020-21 ELECTED LEADERSHIP

Linda Fulmore President	Mathematics and Educational Equity Consultant Cave Creek, AZ
Diana Ceja Past President	Riverside County Office of Education Riverside, CA
Sylvia Celedón-Pattichis Vice President	University of New Mexico Albuquerque, NM
Gloria Brown Brooks Director	Mathematics and Educational Equity Consultant Hollister, CA
Zandra de Araujo Director	University of Missouri Columbia, MO
Nichole Lindgren Director	Buncombe County Schools Asheville, NC
Luz Maldonado Rodríguez Director	Texas State University at San Marcos San Marcos, TX
María del Rosario Zavala Director	San Francisco State University San Francisco, CA

APPOINTED LEADERSHIP

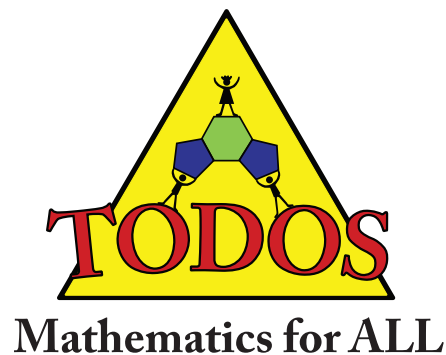
Nora Ramirez Executive Director	Mathematics Education Consultant Tempe, AZ
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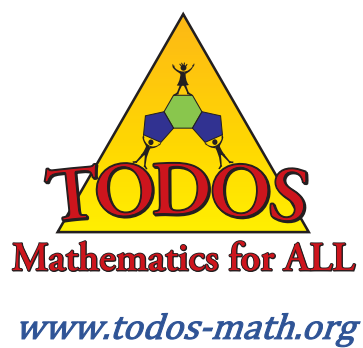
SAVE THE DATE

#TODOS2021

2021 TODOS Conference
Activating Agency For Student Access,
Engagement and Advancement in
Mathematics

June 28-30, 2021
Pre-Conference on June 28, 2021
Scottsdale, Arizona





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