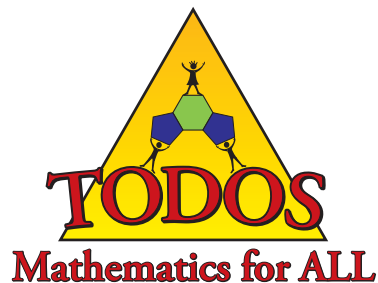


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TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS



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TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

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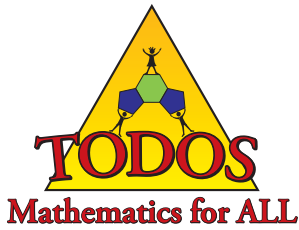
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TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

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From the Editors

The first issue of *TEEM* appeared in Fall 2009. The editors of that first issue wrote, “we hope *TEEM* serves as an inspiring pedagogical and scholarly resource for the broader mathematics education and education communities.” Ten years later, as current editors, we continue to share this hope and are looking forward to many more issues of *TEEM* in support of the mission of TODOS: Mathematics for ALL, which is *to advocate for equity and high quality mathematics education for all students—in particular, Latina/o students*.

TEEM uses a rigorous double-blind review process to ensure that a paper is judged on its merits without the external reviewers knowing the identity of the author(s) and vice-versa. For information (and an archived webinar) on reviewing or writing for *TEEM*, please see the *TEEM* webpage <http://www.todos-math.org/teem>. As editors, we are extremely grateful for the dedication and expertise of all our reviewers and authors. We are also very appreciative of the excellent editorial support provided by Associate Editor Lawrence M. Lesser and Layout Editor Susie W. Håkansson.

This tenth issue of *TEEM* has three externally-reviewed articles covering a wide variety of topics: addressing the needs of emergent bilinguals and students with learning disabilities; using the CHAT theoretical framework to analyze the notion of competence in a first-grade bilingual classroom; and engaging preservice teachers in conversations about equity.

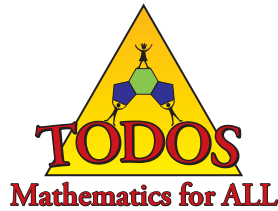
The first article, “Why Recreate the Wheel when Collaboration is Key? Implementing Strategies for Success across Student Groups,” written by Amanda E. Lowry, Melinda (Mindy) Eichhorn, and Kristen Burke, argues that all students can benefit from a collaborative approach between teachers and specialists. The article considers some practical strategies that the authors have used in their own classrooms, and that benefit all mathematics learners—in particular, emergent bilinguals and students with learning disabilities.

The second article is by Cathery Yeh and Ansley Wong and is entitled “The Co-Construction of Competence: An Activity System Perspective for Leveraging and Strengthening Students’ Language and Mathematics Competencies.” This article presents and illustrates a theoretical framework, Cultural Historic Activity Theory, that can be used by researchers and practitioners. The authors use a series of vignettes from a first-grade bilingual classroom to situate the framework in practice.

In the final article, “Using ClassChatter to Mediate Controversial Discussions in Small Teacher Preparation Programs: A Case Study,” Rebecca Dibbs, Laura Beene, and Kelly Lewis describe a History of Mathematics course that used an online journal and discussion board, ClassChatter, to facilitate students’ discussions about equity in the mathematics classroom. The authors report that the online platform resulted in increased and more equitable participation both in online and face-to-face discussions.

TEEM gratefully acknowledges the support of all the leaders in our sponsoring organization, TODOS: Mathematics for ALL. We hope *TEEM* continues to serve the TODOS membership and that this issue serves as a resource for the community and a source of inspiration for future contributions to the journal.

Marta Civil, Ksenija Simic-Muller, M. Alejandra Sorto, and Craig Willey



Why Recreate the Wheel when Collaboration is Key? Implementing Strategies for Success across Student Groups

Amanda E. Lowry
Rutgers University

Melinda (Mindy) Eichhorn
Gordon College

Kristen Burke
Gordon College

Abstract

This paper explores strategies used in diverse K-12 mathematics classrooms to meet the needs of emergent bilinguals and students with learning disabilities. We do not assume that all students have the same needs but recognize that there is some overlap in research. All students can benefit from a collaborative approach between teachers and specialists, and we explore three strategies that we use in our classrooms. We hope that this paper reduces teachers' stress by demonstrating how some strategies can be implemented to teach mathematics to all students regardless of different learning needs.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What strategies are successful when trying to adjust instruction for different needs of students in your mathematics classroom?
2. How do you currently collaborate with other teachers and specialists in your school? How has this collaboration affected student engagement or progress?

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Why Recreate the Wheel when Collaboration is Key? Implementing Strategies for Success across Student Groups

Amanda E. Lowry, Melinda (Mindy) Eichhorn, and Kristen Burke

Introduction

Mathematics can be a gatekeeper to educational and economic success (Impecoven-Lind & Foegen, 2010; Gonzalez, 2012; Koestler, 2012). For our classrooms to be places of equity and access with regard to mathematics, teachers need support to meet the diverse needs of students in their classroom.

Mathematics classrooms include students with varying learning needs. Two common identifications in schools are “English learners” (what we will refer to as emergent bilinguals) and students with learning disabilities (LD). As researchers and practitioners, we explore how to provide all students access to mathematics and achieve “mathematics for all” by focusing on best practices for emergent bilinguals and students with LD to support learning in the general education classroom. We explain reasons for focusing on these two particular groups later in the manuscript. We choose to use the term emergent bilinguals because we believe that these students will become proficient in both languages and successful bilingual students and adults, while English learners posits them as deficient in their English skills and devalues their other language skills.

We acknowledge that different states and school districts use varying criteria for identifying emergent bilinguals and students with LD. For this paper, emergent bilingual is any student who is identified as a non-native speaker of English and is receiving additional language acquisition support by their school (as determined by state and local policies). We use the term students with LD to refer to students who have an Individualized Education Program (IEP) as determined by their local and state regulations and are eligible for modifications to the general education curriculum due to persistent difficulties and lack of response to targeted instruction. Each of these groups is identified using different criteria, and thus the focus of our empirical analysis is to identify strategies that help both

groups independently. While these strategies are beneficial for students with these two identifications, they are also helpful for students with just one or even none. This analysis of classroom strategies is aimed at helping educators feel less overwhelmed by the number of methods needed to assist the diverse students in their mathematics classrooms.

Teachers need knowledge, but also time and resources to foster collaboration with ESL and special education specialists (Avalos, Zisselsberger, Langer-Osuna, & Secada, 2015). Recent research has highlighted the need for collaboration between ESL and special education specialists (Kangas, 2018) and the need for culturally and linguistically responsive practice for all teachers (Linan-Thompson, Lara-Martinez, & Cavazos, 2018), but has focused on general best practices rather than specific strategies for the mathematics classroom. More often, the literature separates strategies for these two groups of students. In this paper, we argue that educators and researchers can learn from each other without reinventing the wheel every time students have different needs.

We recognize that emergent bilinguals and students with LD have different needs, which stem from different root causes, and there is no “typical” student with either classification. It is also important to remember that receiving ESL services should be temporary, as students do eventually learn the language. Being bilingual is not a disability and is an asset to classroom instruction, and we do not intend to ascribe language acquisition as a learning difficulty. However, while these identifications are not the same, we have found certain strategies to be broadly beneficial to both groups.

Assets-Based Approach

All students have a diverse profile of strengths and learning differences. They bring important experiences, attitudes, and behaviors to the learning of mathematics

(Nieto, 1999; Torres-Velasquez & Lobo, 2004). Emergent bilinguals bring linguistic and cultural perspectives to classrooms, regardless of the grade level, and those assets should be recognized and encouraged by the classroom teacher. As with emergent bilinguals, students with LD have a wide range of abilities, cognitive profiles, and overall cognitive scores. For further background on Specific Learning Disability classification, see Zirkel (2012), Bateman and Cline (2016), and Overton (2016). However, there is no average student – each student brings to the classroom an individual profile of various strengths and areas to be strengthened (Rose, 2015). Each student has cognitive processes that are an area of strength for them, and they might use that area of strength to compensate for another area that is not as strong in their “jagged” learning profile (Rose, 2015). Using strategies such as the three listed in this article, teachers can discover what their emergent bilinguals can do while their English skills continue developing.

Strategies

As both practitioners and researchers, we prepared this paper as a guide for mathematics teachers to alleviate some challenges faced with students’ diverse learning needs. We show teachers that although student needs vary, strategies can be implemented to address different learning needs and improve mathematics learning in the classroom. The purpose of this paper is not only to demonstrate the need for more collaboration among the general education, ESL, and special education communities but also to provide teachers with specific strategies that can be used for all students in the mathematics classroom.

As authors, we met through professional conferences and graduate courses and came to this topic through conversations about our individual experiences with research and classroom teaching. We each observed them as beneficial, from the perspectives of our different roles (as a special education and mathematics education researcher, as a doctoral student and high school math teacher, and as an elementary ESL specialist and special educator). Strategies were selected for analysis if our students appeared more engaged with the mathematics as

observed by the teacher, showed increased mastery with the concepts through formative assessments and rubrics, and/or expressed preference for the strategy in terms of overall learning. We reviewed literature to identify which evidence-based practices are beneficial for both emergent bilinguals and special education students, regardless of what the strategy is called in the literature. We found that many strategies can be helpful for mathematics teachers to meet the needs of all students, including emergent bilinguals and students with LD. This review focuses on three overarching strategies we found effective through our personal practice, literature review, and reflection: Concrete-Representational-Abstract (CRA) approach, vocabulary development, and supporting mathematical discourse.

Concrete-Representational-Abstract

Based on our experience, teachers have relied heavily on oral and written language to convey ideas to students. This becomes a problem for students who have difficulty understanding the language of instruction or who have difficulties processing language (receptive and/or expressive). Receptive language describes linguistic input (i.e., reading and listening skills) and expressive language describes linguistic output (i.e., writing and speaking skills). To address the language demands and increase access to instruction for both emergent bilinguals and students with LD, it is imperative that teachers encourage students to use pictures or graphic organizers, rather than words solely, in order to sort, analyze and share information with their teacher and peers. One possible approach to provide access to instruction in all classrooms is the use and connection of multiple representations, which can include physical phenomena or manipulatives, natural language (written and spoken), tables, diagrams, and symbols (Huinker & Bill, 2017; Dougherty, Bryant, Bryant, & Shin, 2017). Teachers can promote conceptual understanding by using objects or manipulatives in the Concrete-Representational-Abstract (CRA) approach or sequence of instruction (Maccini & Gagnon, 2000, 2002; Bryant, Kim, Hartman, & Bryan, 2006). The CRA approach helps students understand concepts at a concrete level and internalize their understanding through multi-sensory learning. A verbal explanation accompanies concrete manipulatives, which later transitions to a

representational drawing. Finally, students work with numerals and algorithms (the abstract stage) so that students understand the connection between each stage (Maccini & Gagnon, 2000).

In the CRA approach, physical manipulatives are used at the concrete level (e.g., counters, base-ten blocks, algebra tiles, and geoboards), drawings, pictures, and virtual manipulatives are tools used at the semi-concrete or representational stage, and at the abstract level, students use mathematical notation (numbers, symbols, and variables) (Witzel, 2005; Witzel, Mercer, & Miller, 2003; Strickland & Maccini, 2010). Students need continued access to concrete materials while they are transitioning to the representational and abstract stages to support their learning and bridge the language to the mathematical idea (Allsopp, Lovin, & van Ingen, 2018).

While using and connecting mathematical representations is an effective teaching strategy that enables all students to deepen their conceptual and procedural understanding, some students may need support to access abstract concepts, state their understanding of relationships between quantities, and to move flexibly among representations (Huinker & Bill, 2017). Using the CRA sequence of instruction, teachers can help students with LD and emergent bilinguals make explicit connections between the various materials, while supporting their language development and allowing them to show their understanding and thinking (Witzel & Little, 2016).

In an urban third-grade math class with both emergent bilinguals and students with LD, the third author used multiple means of representation to introduce and reinforce the concept of multiplication. The topic was first introduced with a short video (Math & Learning Videos 4 Kids, 2015) to explain the process as well as give a visual. Then the teacher used concrete manipulatives for groups, such as cups or drawn circles, and number of objects in each group, such as blocks. See Figure 1 for an example of how you can use blocks and drawn circles to represent groups. Once students are comfortable with the concrete stage of this strategy, they can begin drawing the groups for multiplication on paper (the representational stage). Lastly, students begin to learn the multiplication facts and

can solve multiplication problems without manipulatives or representational drawings (the abstract stage).

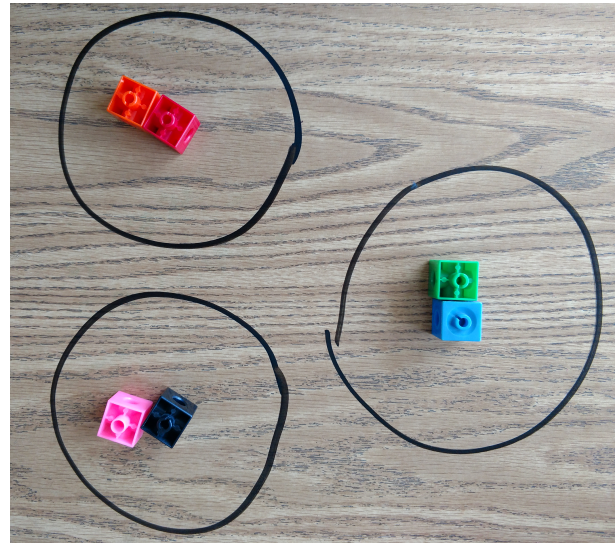


Figure 1. Representing Multiplication with Manipulatives (Blocks) and Drawn Circles.

In a foundations of mathematics class for newcomer high school students with interrupted formal education, addition and subtraction of fractions is introduced using Cuisenaire rods to help students visualize concepts of common denominator. These students are considered newcomers as they have recently enrolled in a U.S. school for the first time. Additionally, the amount of time they have been out of school ranges from one year to having never been enrolled in school. Students begin by working with the Cuisenaire rods (the concrete stage) to identify different size fractions through comparing sizes with other ones. They sketch their results on paper to visualize the relationships (the representational stage). Once students have a firm understanding of the size of the Cuisenaire rods, they can begin adding different-sized fractions of their choosing using the rods and figuring out how to get the common denominator. Typically, students say that the denominator is just the multiplication of the denominators of the two pieces they are adding together. They then use this “discovered rule” to apply it to other problems, but without the rods. Eventually, students are comfortable enough with the skill that they can work on addition of fraction problems without representational sketches or manipulatives. At this point, students work through algorithms (the abstract stage) for adding fractions.

Overall, the CRA approach is modified from a graduated instructional sequence to concurrent presentation of concrete manipulatives, sketches, and abstract notation due to limited time and increased content demands (Strickland, 2017), and can be used effectively at all levels of mathematics instruction. The CRA approach links new mathematical concepts to students' previous knowledge, provides multiple pathways for showing their understanding and thinking, and uses multi-sensory cues to highlight conceptual and procedural processes. As students make explicit connections between new concepts and what they already know, meaning is enhanced (Allsopp, Lovin, & van Ingen, 2018). While effective for all students, the CRA approach provides for emergent bilinguals and students with LD access to content that they may not otherwise receive. Specifically, the use of visual representations is an effective reasoning tool for all learners but is also a valuable communication avenue for emergent bilinguals and students with LD for showing what they know (Driscoll, Nikula, & DePiper, 2016).

Vocabulary Development

While the increased use of concrete materials and visual representations is helpful for promoting learning, students still frequently “encounter mathematics instruction that is wordy, abstract, and defined by symbols” (Coggins, Kravin, Coates, & Carroll, 2007, p. 68). Standardized test problems often use complex linguistic structures and teachers do their best to prepare students for them. This becomes even more challenging for emergent bilinguals because they are continuing to develop proficiency in the English language and mathematical discourse (Ramirez & Celedón-Pattichis, 2012). Both emergent bilinguals and students with LD can be confused by figurative language and words that have multiple meanings. This is especially difficult for these two groups of students as they can have challenges processing language, possible code-switching (alternating between languages) and taking more time to decipher meaning. Emergent bilinguals also take more time as they tend to mentally translate instruction into their first language (Domínguez, 2011; Moschkovich, 2005). Mathematical language can be dense and abstract, from mathematical vocabulary to the syntax of word problems (Krasa & Shunkwiler, 2009; Little, 2009).

Teachers can assist emergent bilinguals and students with LD in acquiring language and mathematics simultaneously through the use of strategies such as graphic organizers and diagrams. Graphic organizers are beneficial in helping students organize all aspects of a problem in one place, as well as aid in processing information and connecting new knowledge to prior knowledge (The IRIS Center, 2010). Graphic organizers develop understanding and reasoning by arranging words and information needed to approach mathematics problems successfully. Diagrams are a type of graphic organizer that can help students visualize how a mathematical problem is structured. Diagrams reorganize the mathematics problem to separate the mathematics from the language and context of the problem. They can help students internalize what is asked of them mathematically to link vocabulary and context, in order to build conceptual understanding.

The Frayer model is one such graphic organizer that helps students understand mathematical vocabulary (Frayer, Fredrick, & Klausmeier, 1969). Frayer models can be used across grade levels for introducing new concepts and vocabulary simultaneously. A graduate student of one of the authors used a Frayer model (see Figure 2) to reinforce the definition of a rational number, and explicitly teach that pi is not a rational number in her middle school classroom. Frayer models can also be used in second grade as students develop proficiency in adding and subtracting with regrouping because it allows students to understand the process in different ways. In a Frayer model, the vocabulary word is placed in the middle, and in the quadrants, the students write a definition, characteristics, examples, and non-examples. Emergent bilinguals and students with LD can use this model to interact with language and see the visual representation in different ways. Using both examples and non-examples is more beneficial than merely using examples alone to promote student understanding and retention of concepts (Booth, 2011; Siegler, 2002). When students identify examples and non-examples, teachers can assess that the students have obtained conceptual understanding. Additionally, the Frayer Model can be used in small groups or posted on anchor charts around the classroom.

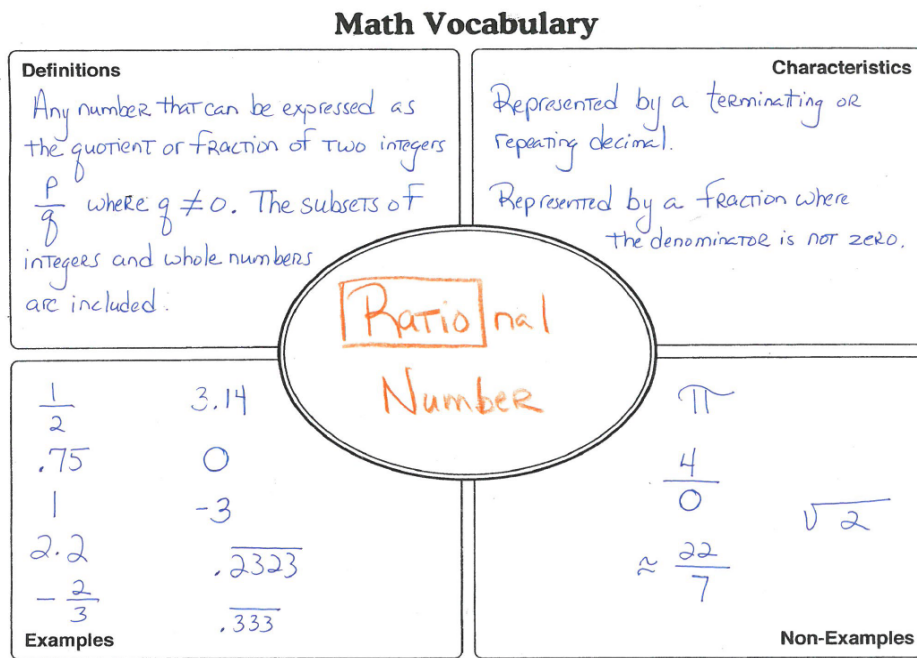


Figure 2. Frayer Model for 7th-Grade Learning of Rational Numbers.

There is a close connection between vocabulary and conceptual understanding. Students can be provided multiple means (speaking, writing, drawing, using concrete manipulatives and graphic organizers) of applying their vocabulary knowledge to aid in their understanding and expression of mathematical ideas and reasoning (Allsopp, Lovin, & van Ingen, 2018). Explicit instruction in vocabulary helps some struggling students integrate the word with its meaning as they analyze the mathematical terminology (Witzel & Little, 2016; Allsopp, Lovin, & van Ingen 2018).

Supporting Math Discourse

It is important to note that while vocabulary development is important, it cannot be decontextualized from social context or the mathematical ideas represented by certain words and phrases. Students learn mathematics in a “linguistically sensitive social environment” fraught with complex linguistic demands and classroom discussions (Ramirez & Celedón-Pattichis, 2012, p. 21). By focusing on the social context surrounding mathematical discourse, teachers can develop mathematical understanding beyond purely cognitive forms of learning (Willey, 2010). The presence of mathematics discourse within classrooms develops mathematical reasoning and can be supported through multimodal approaches to content delivery (as

discussed earlier) (Willey, 2010; Musanti & Celedón-Pattichis, 2013). Vocabulary can be taught alongside, or after, contextual understanding, rather than without alignment to the content.

Standards for Mathematical Practice (Common Core State Standards Initiative, 2010) place considerable emphasis on knowing how to justify and talk about mathematics. To be successful with either of these tasks, students must be comfortable with mathematical discourse. While emergent bilinguals and students with LD continue to develop their linguistic strengths in the mathematics classroom, they are expected to conceptualize and perform mathematics tasks with high levels of linguistic complexity on some standardized tests (Chval & Khisty, 2009).

Students with LD may also have difficulty with expressive language, which can affect their ability to engage in small and whole-group discussions (Bryant, Kim, Hartman, & Bryant, 2006). In the study of Butterworth (2005), students with LD in math reported that they did not forget what the teacher said, but that they just did not understand it. Language difficulties can be compounded if students are distracted or have difficulty with organization and can lead to difficulties in math skill acquisition (Krasa & Shunkwiler, 2009; Fletcher, Lyon,

Fuchs, & Barnes, 2007). By using sentence frames and supporting mathematical discourse, teachers can minimize the confusion students may be feeling about the language of the mathematics classroom.

Through experiencing continual modeling of mathematical discourse by both the classroom teacher(s) and peers, students internalize those skills and forms of talking about mathematics. Over time, students can adequately use mathematical discourse to justify and talk about mathematics if provided with ample time to practice and through continual demonstration by all teachers and students in the classroom (Humphreys & Parker, 2015).

Teachers can demonstrate not only how to have appropriate mathematical discourse with a partner and within a group, but also how to use the resources that are available to students such as vocabulary word walls and sentence frames (Chval & Khisty, 2009). See Figure 3 for an example of sentence frames posted in a classroom.

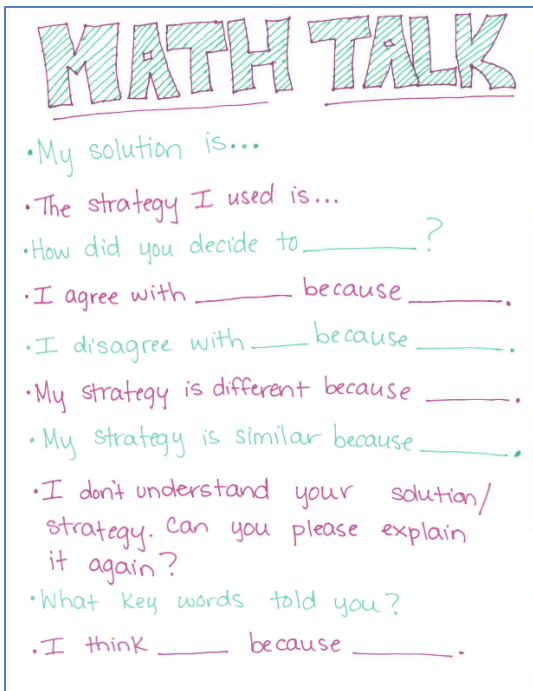


Figure 3. Sentence Frames Posted to Support Discourse in 3rd-Grade Classroom.

The most important thing to remember when using these resources is that we need to explicitly teach our expectations for their use. Additionally, just posting the sentence frames will not ensure that students will utilize

them. For example, when teaching a geometry unit on describing and listing the attributes of 3-dimensional shapes, words such as vertices, faces, angles, and edges are to be posted on the word wall with pictures to allow students to find the word they seek. One of the authors posts sentence frames for students to see, introduces them to the class, and practices each one by one. A sample sentence frame used with intent and structure by the same author follows, as well as how to explicitly teach and model it:

Teacher: Our first sentence frame is listed on this anchor chart. It has certain words that are left blank for you to fill in as you are using it in your mathematical discourse with your partner. It says, “A _____ has _____.” I will show you how to use your sentence frame correctly. It’s very important that you look at the word wall and use the precise mathematical vocabulary in your sentences. Your first blank is asking for a shape. The second blank is asking for a number. And the last blank is asking for an attribute. (A shape has # of attributes.) I am going to describe a cube and use the attribute, vertices. “A cube has eight vertices.” Now it is time for you to try. Partner A, I want you to describe a cube using the attribute: edge. Tell your partner how many edges a cube has using our sentence frame [20 seconds]. Now Partner B, I want you to describe a cube using the attribute: angles. Tell your partner how many angles a cube has using our sentence frame [20 seconds].

While these strategies do not promote the rich mathematical discourse that we aim for, they do help start conversations in classrooms where students are reluctant to talk due to their confidence levels and/or abilities.

Graphic organizers, word walls, and posted sentence starters can reduce the cognitive demand and confusion for students with LD and emergent bilinguals by providing a visual schema that makes associations between words and representations of mathematical ideas (Allsopp, Lovin, & van Ingen, 2018). These tools themselves do not teach vocabulary or aid in student

understanding. The teacher and students must use the tools to engage in connecting the vocabulary to the associated mathematical idea and allow students multiple ways to engage with the content during mathematical discussions. By modeling and providing think time or wait time for students with LD and emergent bilinguals to frame their response and to make use of the vocabulary tools around the room, teachers can increase student engagement in mathematical discussions (Allsopp, Lovin, & van Ingen, 2018).

Conclusion

Teaching mathematics for equity is challenging, but it is not impossible. Students of mathematics have varying learning needs, and there are many strategies teachers can use to meet individual students' needs every day. The strategies suggested by specialists can be numerous depending on the diversity of learning needs in each classroom. While emergent bilinguals and students with LD do not have the same learning needs, some strategies are useful for all students. Implementation of the CRA approach helps students of all backgrounds understand mathematics through multiple representations. Vocabulary development can be supported through the use of diagrams and graphic organizers that connect terminology to content. By modeling mathematical discourse through sentence frames and mathematically appropriate language use, teachers can help students talk about and justify mathematics regularly in the classroom.

While teachers sometimes feel overwhelmed by the diverse learning needs in their classrooms, the strategies described in this paper can help a wide variety of students, including emergent bilinguals and students with LDs. By collaborating across disciplines in their schools, mathematics teachers can simplify their teaching using strategies like these that benefit all students. These are not the only strategies that benefit multiple learning needs in the classroom, but these strategies have proven successful in our classrooms, and we encourage teachers of mathematics to collaborate with colleagues to streamline how they support different types of learners.

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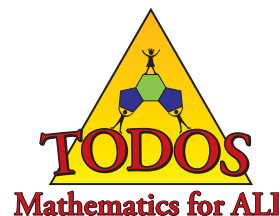
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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Think about the students in your classroom. Which of the strategies discussed do you think you can implement successfully for the benefit of all of your students?
2. Can you think of any other strategies that you use for one student or group of students that can be tweaked to improve mathematics learning for more students? What would this look like in your classroom?
3. What is one action step that you can take this academic year to increase collaboration with other teachers and specialists in your school? What resources do you need to begin your plan of action?
4. Discuss with your colleagues how you would apply the CRA approach in your setting.
5. Which mathematical terms do your students regularly confuse or use incorrectly? Pick one or two of them and think of how you can use diagrams or graphic organizers to eliminate the confusion.
6. What is one strategy you can try in your classroom to improve mathematical discourse for all students?

“DARE to Reach ALL Students!”



POETRY CORNER

UTEP's Lawrence Lesser is often inspired to write songs or poems related to mathematics education topics. Such creative work can communicate important points with more efficiency and emotion for broader audiences, and *TEEM* readers are invited to explore this as well. Other journals that regularly include both articles and poetry related to mathematics education include *Journal of Humanistic Mathematics* (scholarship.claremont.edu/jhm/) and the *Association of Mexican American Educators Journal* (amaejournal.utsa.edu/index.php/amae). Lesser's poem below connects a current sociopolitical setting with a figure (showing varied representations for long division) from Judit Moschkovich's 2013 *TEEM* paper.

DI / VISION

by Lawrence Mark Lesser

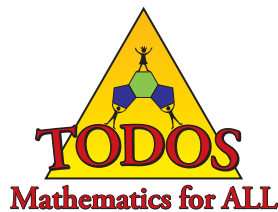
In a New Mexico primary school
In view of the new bollards,
María takes on the division problem

*To buy the \$123 medicine he needs,
How long at his \$7-an-hour job
Must Juan work?*

María does it
How her Mexican abuela taught her,
Each subtrahend kept in her head.

Not seeing
How different layout yields the answer,
Her teacher scolds: "Do it the *right* way!"

What dividends if we could share
Alternative means to reach fact,
Not alternative facts to reach mean.



The Co-Construction of Competence: An Activity System Perspective for Leveraging and Strengthening Students' Language and Mathematics Competencies

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Abstract

With this article, we share a framework positioning competence not simply as attributes of individuals but as co-constructed during interactions in the classroom. The theoretical framework, Cultural Historical Activity Theory, is shared for the purpose of examining competence construction as an attribute of participation in the activity system of the classroom. A series of vignettes in a first-grade bilingual classroom highlight ways in which components of the activity system of the classroom come together to leverage and strengthen students' mathematics and language competencies.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. "Ethnic identity is twin skin to linguistic identity – I am my language. Until I can take pride in my language, I cannot take pride in myself." (Anzaldúa, 1987, p. 253)
How does this quote speak to you as a learner? As an educator? As a researcher?
2. In what ways, specifically, does language impact the mathematics learning of students, particularly emergent bilingual students?
3. How do you interpret the notion of mathematics competence as socially co-constructed?
4. What can mathematics educators do to better leverage and strengthen students' mathematics and language competencies?

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The Co-Construction of Competence: An Activity System Perspective for Leveraging and Strengthening Students' Language and Mathematics Competencies

Cathery Yeh and Ansley Wong

Introduction

This article builds on a conception of the pedagogical and political (Freire, 1970/2007; hooks, 1994; Ladson-Billings, 1994). Classrooms are “inherently cultural spaces where different forms of knowing and being are being validated” (Nasir, Hand, & Taylor, 2008, p. 206). The activity system of a classroom determines what form of student participation is considered legitimate, what ideas are valued, and who can be considered “competent” (Gresalfi, Martin, Hand, & Greeno, 2009; Hand, 2012). We focus on classroom mathematics discussions with Latin@ and bilingual (Spanish/English) students.¹ The learning experiences of bilingual students in the United States often mirror current politics of xenophobia and assimilation (Aquino-Sterling, Rodríguez-Valls, & Zahner, 2017; Yeh, 2018). Emergent bilinguals too often sit silently in mathematics classrooms using algorithms that are not their own, in a language different than their native tongue, and solving mathematics problems irrelevant to their interests and experiences (Aquino-Sterling et al., 2017; Yeh, 2017; 2018).

Language and choice of language, more than reflecting proficiency, are integral to the identity work students engage in as they learn. Bilingual speakers use language not only as the basis of proficiency but also as an expression of identity (Cummins, 2000; Khisty & Willey, 2008). Validation and maintenance of students' linguistic identities are intricately linked to academic performance. Bilinguals who can read, write, and communicate in their native language are more likely to enroll in advanced mathematics courses and continue to higher education (Khisty & Willey, 2008). As such, honoring and extending students' mathematics and linguistic competencies should be central in any classroom serving bilingual students.

This paper focuses on leveraging and developing students' competencies, both linguistic and mathematical. In the following sections, we share a conceptual tool that builds on sociocultural theoretical frameworks to guide our analyses of competence not simply as attributes of individuals but as co-constructed interactively and discursively within specific activity contexts (Vygotsky, 1978; Gresalfi et al., 2009). Rather than viewing ability and competence as innate, a sociocultural perspective suggests examining how learning occurs as a process of change, a complex dialectic between an individual and the social/cultural context in which they mutually construct and transform each other (Vygotsky, 1978, 1987). To highlight competence as co-created in situated activity, we use classroom examples to illustrate how competence is made meaningful through classroom interactions. In the sections that follow, we explain a conceptual tool, Cultural Historical Activity Theory (CHAT), to examine competence construction within the activity system of the classroom. Then, we provide a series of vignettes and illustrations in a first-grade bilingual classroom to highlight the ways in which the components of the activity system come together to leverage and strengthen students' mathematics and language competencies.

Cultural Historical Activity Theory

Cultural Historical Activity Theory (CHAT) guides our understanding of the process of competence construction in mathematics classrooms (Cole & Engeström, 1997; Engeström, 1987). From a CHAT perspective, students' competencies are not constant or perceived as located solely within individuals. Rather, they are constructed in the relation between individuals and the opportunities available by the activity system. CHAT centers on three core ideas: 1) people act collectively, learn by doing, and communicate in and through their actions; 2) people make and use tools to learn and to communicate, and 3)

¹We employ the term bilingual students rather than English language learners to emphasize the rich funds of knowledge of

students with a heritage language other than English and that “through school and through acquiring English, children become bilingual” (García, Kleifgen, & Falchi, 2008, p. 6).

community is central to learning and identity development (Vygotsky, 1978). Figure 1 depicts the social organization of the activity system (Engström, 1987, 1999). The subject is the person or group of people whose perspective is the focus of analysis (e.g., a student or groups of students). The object is the goal of the activity system as a whole (e.g., developing language and mathematics competencies). Both subject and object are influenced by mediating tools or artifacts, the nature of the community to which the system belongs, the rules of behavior appropriate and the division of labor within the system. Division of labor relates to Marxist analysis of social relations and refers both to hierarchical power structures within the system and the way in which labor is divided within the system context (Roth, 2012).

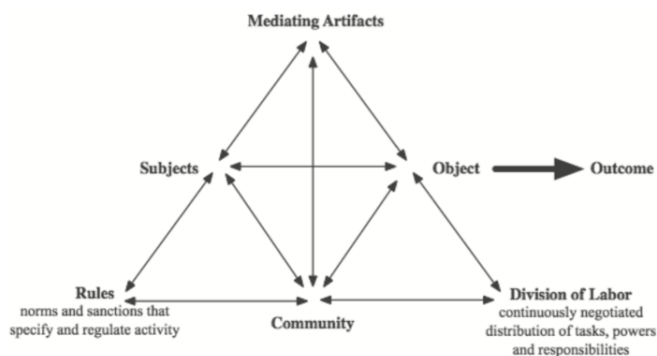


Figure 1. Cultural historical activity theory (CHAT) activity system.

Viewing learning and competence as socially constructed allows the focus of analysis to shift from the individual to the classroom in which the individual is an active participant. As such, whether a student is learning or is deemed to be competent is no longer seen as an individual attribute; rather, competence construction stems from social resources (e.g., the conceptual or physical tools and signs) made available within the social contexts. In the process of taking up and employing these tools, an individual transforms both her/his own understanding and the shared understanding of the community.

In the following section, we provide an extended case example to situate CHAT in practice. The classroom practices discussed here build from a three-year

longitudinal and observational study in which we explored mathematics teaching and the opportunities novice bilingual teachers have to learn to teach (Yeh, 2017, 2018). All case study teachers were purposefully selected during teacher preparation because of their expressed commitment to bilingual education and equity-oriented mathematics teaching. The interrelation of their mathematics teaching and opportunities to learn to teach are discussed in prior works (Yeh, 2017, 2018). This article focuses specifically on Laura, a third-year teacher who has been identified in her district for her success in developing students' linguistic and mathematics competencies as well as their excitement for learning. Our analysis for this article examined the interaction amongst students and components of the activity system as the unit of analysis. In particular, we considered the opportunities that were afforded for students to engage in mathematics, paying particular attention to the ways in which components of the classroom system came together to leverage and strengthen students' mathematics and language competencies. A model of the social organization of Laura's classroom can be found in Appendix A. The following section showcases distinct ways in which systems of linguistic and mathematics competencies are constructed and leveraged in Laura's classroom.

Rules of Behavior

Laura's classroom, like all classrooms, is a social system organized through regularities of shared practice (Hand, 2012; Nasir, Hand, & Taylor, 2008). Laura's scheduled block of math instruction begins with a math talk activity in which students explore mathematics concepts or number patterns and relationships. Then, students engage in a problem-solving activity working independently or in pairs, followed by Laura strategically calling on students to share their solution method with the whole class. Here, her first-grade students are introduced to their first Join (Change Unknown) – a joining word problem where the action is a joining of two or more quantities and one of the quantities is missing (Carpenter, Fennema, Franke, Levi, & Empson, 2015) (see Figure 2).

| | | |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|--------|
| Nostros tenemos ___ fotos en el altar. Los estudiantes ponen algunas fotos más. Ahora hay ___ fotos in el altar. ¿Cuántas fotos ponen los estudiantes? [We have ___ photos on the altar. The students put more photos on the altar. Now, there are ___ photos on the altar. How many photos did the students put on?] | | |
| 3/10 | 13/20 | 83/100 |

Figure 2. Join (Change Unknown) word problem.

There is growing expectation for equitable instruction in which each student, regardless of language and dis/ability classification, is afforded opportunity for rich mathematics engagement. However, rich mathematics engagement is not often evident in classrooms serving linguistically and culturally rich communities in which students are often expected to passively absorb information (Freire, 1970) or is limited to particular ways of engagement that are determined by the teacher. By contrast, students in Laura's class are expected to exercise agency (Gresalfi et al., 2009), through decision-making, exploration, strategizing, choosing methods and considering and developing meanings and relations of concepts using tools (e.g., manipulatives, invented algorithms, drawings, native and hybrid language practices) in different participation structures (e.g., individual or pairs or collectively).

Before Laura's students are asked to share their ideas with the class, they are typically independently or collectively problem solving. During this time, Laura works to challenge dominant conceptions of mathematics

competence as limited to accuracy, speed, and rule following. Instead of confirming these conceptions, Laura praises students for their mathematical decision-making in using certain tools or problem-solving strategies, making connections across mathematical representations (e.g., visual, kinetic, symbolic), sharing their ideas with peers, or fixing mathematical work. Her spoken praise is routinely provided to expand students' ideas about what doing mathematics involves and what counts as mathematics competence.

At the same time, Laura takes note of student strategies to determine who should share and in which order. Laura knows that calling on a student to publicly share their ideas positions students with authority and marks their contribution as valuable. In the vignette below, Laura intentionally calls up Jesus to be the third student to explain his strategy. Jesus is a quiet student, who seldom speaks out during class discussions as evidenced by our visits. Given his hesitation, Laura asked for his permission during independent work time to call on him and what follows is the public sharing of his strategy.

Excerpt 1: Jesus begins to share his strategy.

| | | |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| T: | Jesus, adelante. Escuchando a Jesus, Jesus ¿Que hiciste? ((Jesus's talk is inaudible but points at his twenty dots as Laura draws the same twenty dots on poster paper)) | Jesus, come forward. Let's all listen to Jesus. Jesus, what did you do? ((Jesus's talk is inaudible but points at his twenty dots as Laura draws the same twenty dots on poster paper)) |
| T: | ¿Jesus primero dibujo veinte que? Veinte. | Jesus first drew twenty what? Twenty. |
| Jesus: | Veinte puntos. | Twenty dots. |
| T: | Veinte puntos, cuenten conmigo. El primero puso veinte puntos. Tres, dos, uno. | Twenty dots, let's all count together. He first put twenty dots. Three, two, one. |
| Class: | Uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve, diez, once, doce, trece, catorce, quince, dieciséis, diecisiete, dieciocho, diecinueve, veinte. ((Students chorally count the dots as Laura draws them on the chart paper.)) | One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty. ((Students chorally count the dots as Laura draws them on the chart paper.)) |

In the vignette above, Laura uses various strategies to intentionally position students with authority and agency. Laura intentionally calls on Jesus to share. Jesus brings in rich ways of problem solving that may go unnoticed in many settings. During the public sharing, Jesus was not audible and pointed to his work instead of giving a verbal response. Laura encouraged Jesus to use multimodal approaches – written, visual models, and gestures – to communicate. Jesus’s work served as a mediating tool to communicate his competencies and to deepen the learning of peers. Jesus held his paper to show the class how he had solved the problem. Jesus first drew twenty dots and crossed out thirteen. As Jesus pointed to his work on paper, Laura drew his strategy on a white poster (see Figure 3).



Figure 3. Teacher representation of Jesus’ strategy.

Laura has established rules and norms to support student agency. It is a normative expectation for students to provide explanations and commentaries on the strategy shared using words, drawings, and gestures. Student choices of language (English, Spanish, or a mixture) and communication style broaden their opportunity to demonstrate competencies and contribute to the conversation. Even without verbal speech, Laura positioned Jesus as an expert as he displayed his work visually, and the class unpacked his strategy together.

Division of Labor Distribution

Laura views each student and the teacher as part of a classroom activity system in which all contribute to the classroom learning. Let’s continue with the vignette of the class discussion of Jesus’s strategy to demonstrate how Laura addresses a component of the discussion when Jesus is hesitant to discuss his own strategy.

Laura positions herself as the facilitator and learner within the learning community. As shown in the scenarios below, Laura intentionally takes on a posture of uncertainty (“Sophie, do you have an idea?”) to open up space for students to take on the expert role (Yeh, 2017).

Excerpt 2: Distributing the explanation of the solution strategy.

| | | |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| T: | El dibujo veinte puntos, ¿Por que dibujastes veinte Jesus? | He drew twenty dots. Jesus why did you draw twenty? |
| Jesus: | No me acuerdo | I don’t remember. |
| T: | Él no se acuerda porque dibujo veinte. ¿Sophie tu tienes una idea? | He doesn’t remember why he drew twenty. Sophie, do you have an idea? |
| S2: | <i>((Student walks up to the chart poster and stands next to Jesus.))</i> El dibujo veinte porque ahí, veinte fotos <i>(points to the space for “twenty” in the word problem written on the chart paper)</i> | <i>((Student walks up to the chart poster and stands next to Jesus.))</i> He drew twenty because of that, twenty photos <i>(points to the space for “twenty” in the word problem written on the chart paper)</i> . |
| S3: | Habían trece al principio <i>(uses finger to circle the thirteen crossed out dots)</i> y luego habían veinte y estos son los que ponen <i>(uses finger to circle the seven circled dots)</i> | There were thirteen <i>(uses finger to circle the thirteen crossed out dots)</i> in the beginning and then there were twenty and those are the ones they put there <i>(uses finger to circle the seven circled dots)</i> . |

During interviews, Laura frequently discussed the role of power, status, and positionality in students' sense of self and their learning. In most classrooms, students fluent in the dominant language and practices dominate the classroom discussion. By recognizing that certain groups have been positioned with higher status than others, Laura knows she must make strategic decisions to distribute (and disrupt) power relations and broaden competency in her classroom.

Note the focus of the strategy share was not on the answer but on collective sense-making of a student idea. Laura intentionally calls on other students to explain the reasoning behind Jesus's strategy. The class had an expected norm for a student to initiate explanation but others to contribute and build on from the explanation. This was an instructional move to hold students accountable for making sense not only of their own thinking but also of the thinking of others. By distributing the task of explaining the solution strategy across participants, Laura not only positioned Jesus as competent but allowed multiple students in class to also be positioned as competent.

Student Strategies as Mediating Artifacts

Student strategies can serve as a central mediating artifact to support language and mathematics development. The following vignette (See Excerpt 3) shows how Laura uses the comparison of two students' strategies as a way to analyze the context of the word problem.

The first student Jesus called on (notice Jesus called on students to explain his thinking) used his finger to circle the 13 crossed-out dots, and then pointed to the words "thirteen photos" in the word problem to explain the relationship between Jesus's strategy and the problem context. This opened up the floor for a total of six students to explain his strategy. The explaining and clarifying of Jesus's strategy led students to arrive at the discovery that Sammy's counting-up strategy (See Figure 4) was "opposite," or "the reverse" (an inverse relationship) of Jesus's subtraction strategy. Seeing counting up as a viable strategy has been shown to allow students more versatility when solving subtraction problem types (Carpenter et al., 2015).

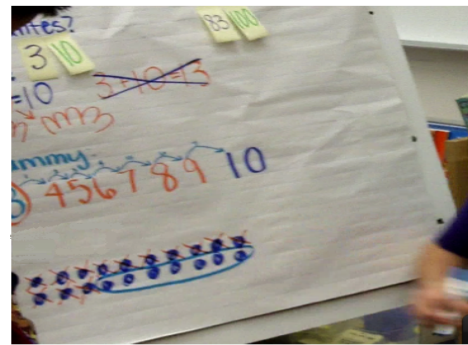


Figure 4. Display of subtraction (Jesus) and counting up (Sammy) strategies.

In Laura's class, student strategy shares were not always fully articulated ideas; partial explanations and even incorrect solutions were common. Laura often positioned a student's incorrect solution as the central discussion

Excerpt 3: Student strategies as a mediating artifact to language and mathematics development.

| | | |
|------------|-------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| T: | Hmm, okay. ((puts finger on check to show a pensive look.)) Alguien. ((Jesus points to a student to speak.)) | Hmm, okay. ((puts finger on check to show a pensive look.)) Anyone. ((Jesus points to a student to speak.)) |
| S4: | Porque Jesus did it backwards a Sammy... so it's twenty take away thirteen is seven... | But Jesus did it in a way opposite that of Sammy... so it's twenty take away thirteen is seven... |
| T: | Veinte quita trece es igual a siete. ((Writes 20-13=7 under the dot representation)) Tú dijiste él lo hizo al revés a Sammy. Qué quieres decir? | Twenty minus thirteen is equal to seven. ((Writes 20-13=7 under the dot representation)) You said that he did the reverse to Sammy. What did you mean? |
| S5: | Sammy counted up. Sustracción es oposite. | Sammy counted up. Subtraction is oposite. |
| S6: | Sammy lo hizo una suma. Sustracción es el revés. Es opposites. | Sammy added. Subtraction is reverse. They are opposites. |

piece, highlighting the mathematical understandings and competencies the student exhibited. She explicitly positioned mistaken solutions as valuable to challenge the notion that only correct solution strategies can be counted as competent. Below are the identified actions Laura takes

to broaden competence and to leverage students' linguistic and mathematical experiences. The following table (see Table 1) highlights each activity system component in practice.

Table 1
Components of the Activity System in Practice

| ACTIVITY SYSTEM COMPONENTS | COMPONENTS IN PRACTICE |
|-------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Mediating Tools/ Artifacts (tools available to mediate student participation) | <ul style="list-style-type: none"> - Design mathematics curriculum that focus on mathematical understanding and reasoning and leverage students' experiential, linguistic, and mathematical knowledge - Affirm students' home and everyday language, code switching, and interactional patterns familiar to students - Use and promote multimodal communication (verbal, written, physical materials, models, gestures,) to represent ideas - Display student work/thinking |
| Rules (expectations and norms) | <ul style="list-style-type: none"> - Recognize student voice has implications for power and student agency - Explicitly communicate high academic expectations for all students - Expect students to solve problems on their own in ways that make sense to them - Construct social structures that enable students from non-dominant backgrounds to serve as principal players in the classroom discussion - Develop norms for students to take risks, construct meaning, and to collectively seek reinterpretation of knowledge |
| Community (students and teacher) | <ul style="list-style-type: none"> - Acknowledge that learning is a social endeavor - Distribute math agency and authority across participants (teachers, students, families) |
| Division of Labor (the hierarchical power structures and ways in which labor is divided within the classroom) | <ul style="list-style-type: none"> - Recognize that certain groups have been positioned with higher status than others - Share power in the classroom by providing student agency in problem-solving, communication structure, and classroom decision-making - Position students as mathematical and linguistic resources for each other |

Reflection

The central aim of this article is to problematize the assumption that students' mathematics competence or lack of competence is an attribute of the individual. Despite the social turn in educational research, mathematics education research and classroom interventions still focus heavily on labeling, diagnosing, and fixing individual students, particularly students of linguistic and cultural diversity, rather than examining closely and fixing the activity system in which students participate (Celedón-Pattichis & Ramirez, 2012; Moschkovich, 2013; Turner, Dominguez, Empson, &

Maldonado, 2013; Yeh, 2018). Competency models with the focus of analysis on the individual student ignore the systemic disenfranchisement of linguistically and culturally diverse communities or the socio-political processes that privilege some forms of mathematical activity over others and thus position some students, often those white and English dominant, as mathematically competent and others as incompetent (Cummins, 2012; Flores & Bale, 2016; Moschkovich, 2013).

Viewing competence as constructed has important implications for research and practice, as it allows assessment and remediation to shift from the individual to

the classroom in which the individual participates. By highlighting the participation structure available in Laura's classroom, we hope to provide an analytical lens for future research on the learning opportunities of bilingual students and a conceptual tool for teachers to better understand the complexities of classroom systems.

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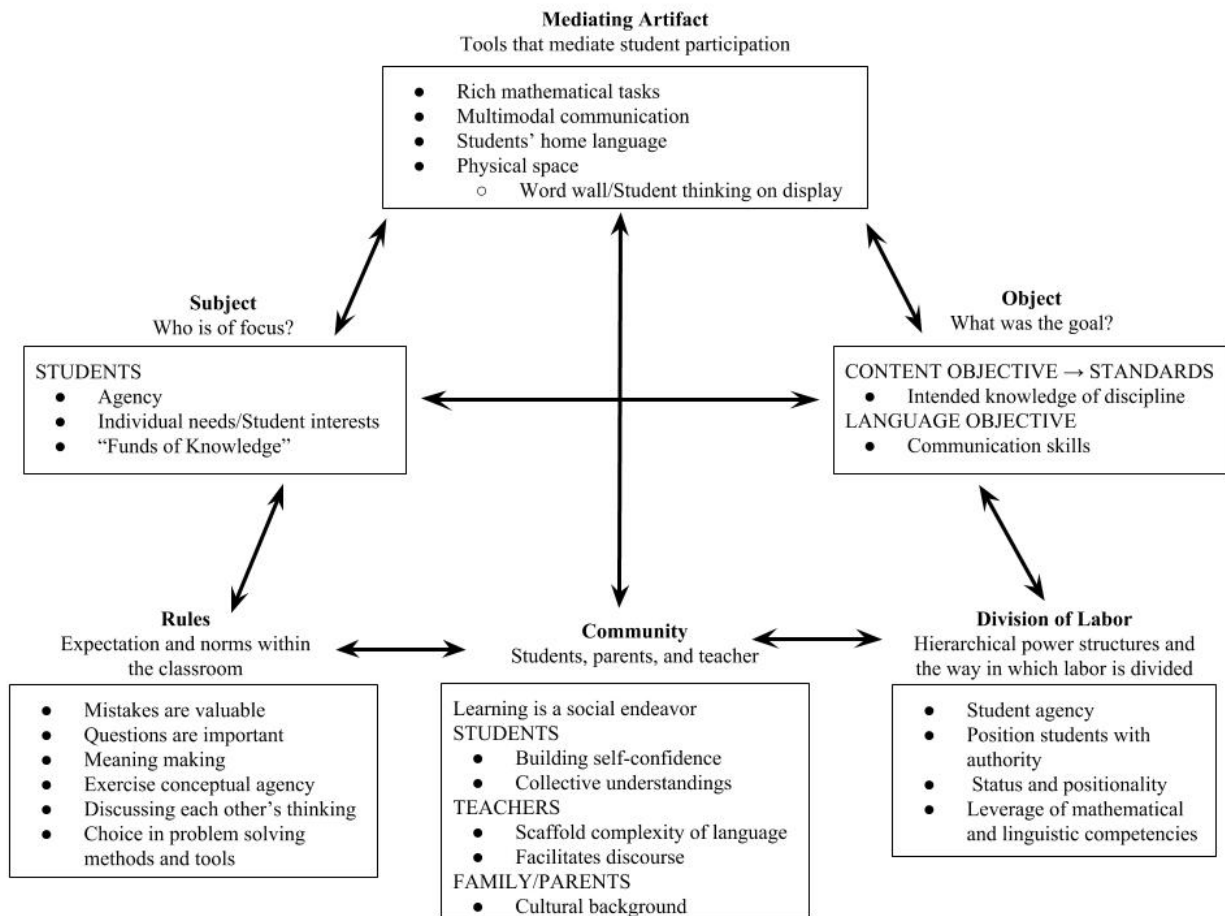
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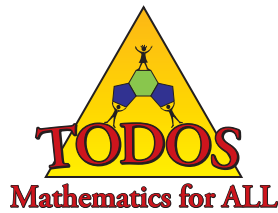
Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What actions did Laura take to broaden competence and to leverage students' linguistic and mathematical experiences?
2. In what ways do teachers unknowingly position some students as more competent than others? What are some concrete ways teachers can broaden how mathematical competence is defined and allow more students opportunities to showcase their mathematical brilliance?
3. How would you use Culturally Historical Activity Theory (CHAT) to reflect upon and analyze your practice?
4. In what additional ways can you reflect on your teaching practice to better leverage the linguistic and cultural competencies of your students?

Appendix

Laura's Classroom Activity System





Using ClassChatter to Mediate Controversial Discussions in Small Teacher Preparation Programs: A Case Study

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Abstract

While small teacher preparation programs allow pre-service teachers to bond with their cohort, the established social norms within cohorts can sometimes make it difficult for students to discuss equity in a face-to-face setting. In this History of Mathematics course, we used an online journal and discussion board, ClassChatter, to facilitate students' discussions about equity in the mathematics classroom. In the online setting, pre-service teachers participated more equally, and all students believed that the format was vital to facilitating more open face-to-face and online communication in a geographic area known for racial tensions.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. How, in your experience, is having a classroom discussion easier if you have taken several previous courses with your classmates? Harder?
2. How do you participate differently in online vs. a face-to-face discussion?
3. What factors might hinder you from participating in a classroom discussion? Are these factors still a hindrance in an online discussion? (And would it depend on whether that online discussion is anonymous?)
4. How can equity and culturally responsive pedagogy be seamlessly included in mathematics content courses?
5. What kinds of activities support equitable participation in discussions about equity and culturally responsive pedagogies?

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Using ClassChatter to Mediate Controversial Discussions in Small Teacher Preparation Programs: A Case Study

Rebecca Dibbs, Laura Beene, and Kelly Lewis

One strength of small teacher preparation programs are the strong cohort bonds that form amongst pre-service teachers (PSTs) as they develop an established classroom community. However, these same cohort bonds create one of the greatest challenges for teacher educators: helping PSTs engage in uncomfortable conversations about equity when the cohort has established communication patterns in which students of color, non-traditional students, transfer students, first-generation students, and students whose native language is not English are less likely to participate (Civil, 2007; Plett et al., 2014). To expand access and create a more equitable classroom where all pre-service students have equal access to the conversation, such discussions may be moved online (Glover, Parkin, Hepplestone, Irwin & Rodger, 2015).

Equitable conversations can be particularly challenging when PSTs have little experience with cultures other than their own. In our program, like most teacher education programs, PST cohorts are less diverse than the classrooms they will teach in terms of race, culture, and socio-economic status (Ramirez & McCollough, 2012), and have few personal multicultural experiences (Simic-Muller, 2015). Embedding PSTs in the cultural activities of their students may help to broaden their perspectives (Ramirez & McCollough, 2012).

There is evidence suggesting that stand-alone multicultural education and diversity courses are not effective in some contexts, particularly for pre-service STEM teachers (Barton, 2000; Krummel, 2013; Ladson-Billings, 2000; Smith, 2009). This is due, in part, to the perspective that mathematics is a culture-free enterprise (Aguirre, 2016). An alternative view suggests that multicultural standards should be integrated throughout each content area (Association of Mathematics Teacher Educators, 2017), helping PSTs better understand the issues of diversity within the context of their content; and teaching them how to create a more culturally responsive pedagogy, a more culturally diverse curriculum, and a

more culturally aware style of relating to students (Krummel, 2013; Ladson-Billings, 2000). However, when multicultural topics are addressed in STEM teacher education courses, they are often engaged only at surface levels because race and culture remain taboo topics in the United States (Krummel, 2013; Smith, 2009). Yet, these activities are important; classroom modeling of culturally relevant activities prepares PSTs to provide engaging learning experiences for their own students (d'Entremont, 2014). The History of Mathematics course was chosen as the pilot course for infusing multicultural standards into the content course because of the interplay between PSTs' cultural experiences and the history of mathematics; this course invites students to talk about content-specific cultural responsiveness and how to implement it in their future classrooms (Averill et al., 2009; Spader, 2015).

History of Mathematics, an upper-level content course in our undergraduate teacher education program, has an advantage over a stand-alone multiculturalism class because PSTs are likely to have an established classroom community, particularly in smaller teacher preparation programs. This sense of classroom community is a major predictive factor of student success (Bahr, Toth, Thirof, & Masse, 2013), and it is especially important for non-traditional and transfer students. Students who experience success in a mathematics classroom report feeling like a part of a classroom community, and they internalize those successful classroom norms as representing the ideal mathematics classroom (Ulriksen, Madsen, & Holmegaard, 2015). However, in mathematics classes, classroom communities are often based upon content authority (Rios & Dibbs, 2016), and this can cause difficulties when personal experience or feelings need to be discussed (Langer-Osuna & Engle, 2010).

Although there has been little research on using technology to facilitate teachers' discussions of cultural awareness, two studies suggest the potential for success in moving such discussions to an online format. Kerr (2010) found that in-service middle school teachers

benefitted from professional development courses in culturally responsive teaching, but it was difficult to establish communication norms between the participants because they had an established set of relationships. Leggett's (2016) research with in-service teachers suggested technology can open new lines of communication and provide a safe space to grapple with uncomfortable topics, particularly civic development and intercultural awareness.

The technology we chose to use for this history of mathematics course was ClassChatter. ClassChatter was a free online platform that did not require student information that allowed us to create a pseudo-anonymous discussion board for our students to interact with each other on equity topics. By pseudo-anonymous, we mean that the students knew they were speaking with their classmates and only their classmates, but the privacy settings on the discussion board gave students random names on the discussion board so that they did not know who made which response. We explored how the use of ClassChatter in a History of Mathematics course for pre-service middle school teachers with a major ethnomathematics component added to the usual textbook (Berlinghoff & Gouvea, 2004) changed cohort social dynamics, facilitated communication, and supported the participation of non-traditional and transfer PSTs. In this study we explore the following question: To what extent does the use of ClassChatter facilitate contributions from PSTs who were uncomfortable participating in a face-to-face discussion?

With the exception of the first ethnomathematics activity, the ethnomathematics portion of class dealt with mathematics done from African and South American cultures, ending each activity with a discussion for how they could be implemented as in-class activities or at Family Math Nights. We used the Brenner (1998) framework on culturally relevant teaching to frame the activities for the students and to have students write reflection papers following the conclusion of the activities. These reflection papers were different from the ClassChatter discussion boards. For the purpose of this course, we took our definition of cultural responsive teaching to include knowledge about culturally diverse groups used strategically to help students and teachers

form partnerships to increase engagement and improve achievement (Gay, 2002).

Mathematics History and ClassChatter

Twelve PSTs conducted extensive mathematics history research and writing projects outside of class as well as in an online asynchronous component of the course via ClassChatter. All PSTs were required to participate in these projects, but one did not consent to participate in the research. During class, PSTs participated in activities designed to incorporate mathematics history and grade-appropriate history activities they could use in their future classrooms. About 25% of all class meetings were explicitly dedicated to ethnomathematics or equity. These planned equity lessons were spread evenly throughout the semester.

The PSTs were required to complete one or two journal assignments each week using the online classroom management tool ClassChatter, a password-protected website that allowed students and teachers to interact with each other outside the social pressures of the traditional classroom. We chose to make 25% of the course consist of two elements showing PSTs how to apply history in a culturally responsive manner in their classrooms: through an online discussion board where PSTs could reflect on their future practice, and in-class culturally responsive lessons we created for future Family Math Night activities.

There were two online ClassChatter prompts to answer each week. These prompts were either journals (viewable only by the author and instructor) or discussion boards (viewable by the whole class) with randomly assigned user names. Every other week, a discussion board assignment replaced the second journal entry. A typical journal response appears in Figure 1. The discussion boards, however, were public to the PSTs and instructors in the course, and discussion posts appeared anonymous to students, although author names were visible to the instructor. ClassChatter assigned students a random moniker for each discussion board assignment so that there was little chance of online identities being matched to participants. The primary source of data for this study were the ClassChatter discussion board and journal posts;

The image shows a screenshot of the ClassChatter.com interface. At the top left is the ClassChatter.com logo with the tagline 'Web Tools For Blended Learning'. On the left side, there is a dark sidebar with two menu items: 'Assignment Blogs' and 'Grade Overview'. The main content area is divided into two sections. The top section is titled 'Journal #5 Due: Wed 2/3' and contains a yellow prompt box with the following text: 'Where do you want to teach? (Describe the type of school, grade levels, type of students) How does your personal culture compare with the type of culture you want to teach in? Will it give you an advantage or disadvantage? What kind of challenges would you expect to face if you worked in a completely different/opposite culture than that which you grew up in as a K-12 student or that which you are?'. The bottom section is titled 'Student Response' and contains a white text box with a student's response: 'My school of choice would be a campus that allows individuality and innovation in teaching. I would encourage and introduce my students to learn with music songs, and interactive learning. The classroom environment would be one of knowledge and freedom to express the students concerns and interest. Respect and acceptance will be our number one rule. My inner-city classroom will be an eclectic representation of their neighborhood filled with goals, challenges and results. Teaching in the inner-city is important to me because of my personal insight of the of the schooling system and my own experiences that I can somehow help those who are less privilege. I understand the importance of parent's participation and I understand the possible limitations of parents who may have 2 jobs or limited education. My most effective advantage is my personal disadvantage I experienced from a failed educational system. Some challenges that I may come across working in a completely different or predominantly white environment would be, the perception of students that may not'.

Figure 1. Typical ClassChatter prompt and excerpt of student post.



informal interviews were also conducted with each participant after the semester ended to provide additional data about how they perceived the results of the online discussions.

Our intention for assigning the ClassChatter discussions was to incorporate a second strand of equity ideas into the course in addition to the ethnomathematics activities. We used Gutiérrez's (2009; 2012) four-dimensional definition of equity: access, achievement, identity, and power; we focused less on the achievement than the other categories though all were addressed in the discussion boards and journals. Examples of the discussion board prompts may be found in the Appendix.

The ClassChatter prompts were typically classroom vignettes that related to the challenges of students from a non-dominant group in mathematics or more global

questions for PSTs to ponder about how equity might relate to math history in and beyond this course. These vignettes were written by the authors; our major goal was to help PSTs confront some of their implicitly held beliefs about equity. For instance, in Figure 2 below, the prompt asked the PSTs to discuss what, if any, relationship there was between culturally responsive teaching and the history of mathematics. Each PST needed to start a thread with their response of what they would do, and respond to two other PSTs (Figure 2).


The PSTs who participated in this study were enrolled in a History of Mathematics course for middle school teachers in the spring semester of 2016. Table 1 provides a summary of participants' self-written descriptions. We did not provide labels for the participants to circle, rather each PST chose his or her own descriptive phrases that they felt were most important to share about themselves.

Journal 20  

I think that there is a relationship between the History of Mathematics and cultural awareness. When you teach the history of math, you are also providing the students with cultural awareness. Mathematics came from all kinds of cultures and when you teach the foundations of mathematics to students they will see that people from their culture succeeded in mathematics. This may inspire them to become more than a low or middle scoring student when it comes to math. I think that if students have role models in mathematics and see others who are like them who have succeeded and made considerable contributions to the subject they will try harder because they realize they can do it too; because of this, I believe that the number of jobs in the U.S. would increase when it comes to STEMM jobs. I think that if more jobs stay in the U.S. then America can start to come out of the slump that we are in as far as economics, but I don't think that America will ever be the economic leader and innovator again.

[Add Comment](#) [Hide Comments](#)

I couldn't agree more. Every. Single. Word!! There are certainly relationships between all of these, and they are far more important than people (including myself before this class) can even see or realize!!

Posted By: Hope 

I agree with you, if students have a role model that is currently alive and contributing to mathematics then they will succeed because they'll realize they can do it too. It's sad but true, I also don't believe America will

Figure 2. Typical discussion board¹.

Table 1
Participants

| Participant | Description | Participant | Description |
|-------------|-----------------------------------------------------------------|-------------|---------------------------------------------------|
| Andrea | African-American, female, heterosexual | Ann | White, female, middle class, interracial marriage |
| Brandi | White, female, middle class | Brilliana | Mexican, female, heterosexual |
| Fana | African, female | Gayle | Black, female, gay, middle class |
| Holly | White, female, heterosexual, middle class, interracial marriage | Jillian | White, female, middle class |
| Karis | African-American, female, lower class | Kevious | African-American, male, heterosexual, disabled |
| Paige | White, female, lower class | Yeny | Mexican-American, female, middle class |

Student Interaction with and Perceptions of ClassChatter

Initially, students were nervous about discussing equity and justice within the context of a mathematics history class, but found participation to be much safer on the pseudo-anonymous ClassChatter. Andrea, a traditionally-aged Black female student, mentioned in her final

interview, “I was worried about talking about equity in class. I didn’t wanna be representing all black folk. I said more [on ClassChatter] than I would have offline because my name wasn’t on my posts.” Paige, a non-traditionally-aged white student, agreed, saying “I grew up in [a local town known for hate group activity]. I know I know ignorant things, but I don’t know what they all are. Since my name wasn’t on my posts, I could just ask without anyone thinking less of me.”

¹ Though students could not see who posted, the faculty view showed students’ real names so we removed them in these figures.

Participation patterns were distinctly different in class and on ClassChatter—different leaders emerged online. Students who did not often speak in class or who were part of an underrepresented group stated that they liked the opportunity to participate without having to be a spokesperson, and most students appreciated that the asynchronous nature of the discussion board allowed them to reflect on what their classmates had said before replying. As Gayle explained:

I'm older than these kids. So sometimes it takes me awhile to think of a way to say something in a way they [my classmates] would understand. The discussion boards gave me a chance to interact with my classmates on a different level, because I had the time to think before I interacted with them.

We began the analysis by looking at participation patterns to see how they differed between in-class and ClassChatter discussions. Previous research indicated that this cohort had formed a strong social network in the previous semester (Rios & Dibbs, 2016). This social network is a statistical measurement of how close the relationship is between each pair of students. The social network was generated with a survey where PSTs checked all of the activities they had participated with each person in and outside of class. The thickness of the lines is proportional to the strength of the relationship between participants. Yeny, Karis, and Ann (Figure 3, black nodes) were the leaders of the class with the highest average social network scores. Yeny was considered an authority on the mathematics, Karis was the most organized, and Ann was the best person at explaining problems to her classmates. Only Karis regularly spoke forcefully in class discussions; Yeny and Ann were more comfortable in small group settings. However, during in-person discussions, the consensus opinion of Karis, Yeny, and Ann was adopted by the rest of the class on all but two occasions.

On the discussion boards, PSTs were required each week to post a new thread and respond to two other threads. Surprisingly, threads from three PSTs with the lowest social network connection scores (Kevious, Andrea, and Paige) consistently attracted the most responses on the

discussion board. (Figure 3, white nodes). All three are students who identify as students of color, with Kevious and Andrea being African-American and Paige being Mexican, and their posts often disagreed with the opinions presented by those of their peers. However the pseudo-anonymity of the discussion board was why Kevious, Andrea, and Paige felt empowered to speak out on the discussion board. As Andrea explained:

I'm a pretty quiet person, and there are some really [pause] strong personalities in this class. I disagree with Holly sometimes, but it isn't worth arguing with her most of the time. But with ClassChatter, I could say what I wanted, without having to be the quiet person talking all the sudden, or speaking for all Black folk. Since our name changed on every assignment, it was even hard to keep track of which posts were mine when I looked back. So it was a lot less about who said what and more about ideas.

Kevious and Paige also had reasons they did not feel as comfortable sharing with the class. Kevious was the only male student in the class and Paige was a non-traditional student who was repeating the class; this was the first mathematics course she had taken with this cohort.

When asked about the discussion board in their interviews at the end of the semester, all participants indicated three things about the discussion board that helped them: the randomly assigned monikers, time to reflect on what was said before responding, and knowing that their classmates were the only ones besides their teacher who could see their discussion responses. Paige explained in her interview: "You know, sometimes I just didn't know how to say something right. So I typed something and hoped I didn't sound too hick-ish. I probably wouldn't have done that if my real name was up there. I probably never would have asked the question at all." Holly, one of the most vocal students in the class, said she was grateful that the discussion boards gave her time to think before she replied, and she elaborated as follows:

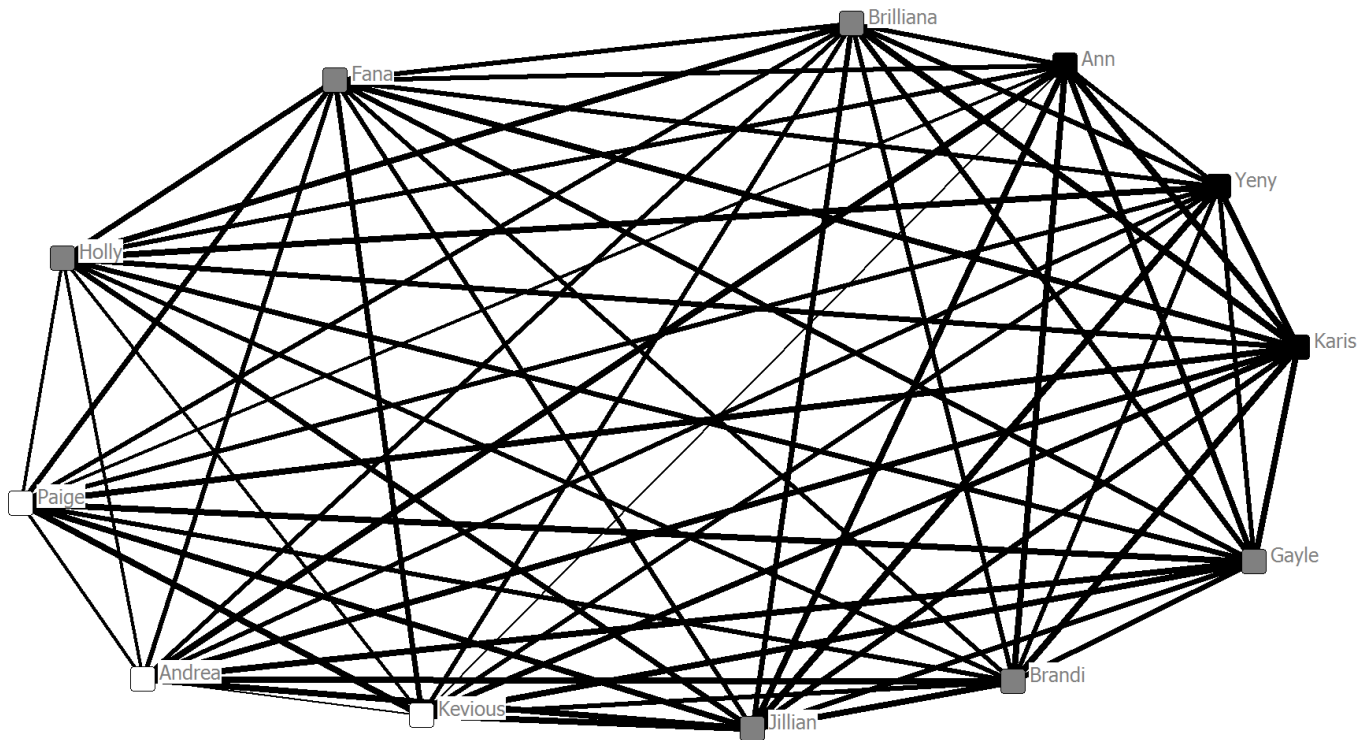


Figure 3. Participants’ social network.

You know, I just say what I think. That can get me into trouble sometimes. And some of the journals, like the one about Bart and Ken², really made me upset, because I wanted to give Ken a higher grade and felt like I couldn’t, because “Ken isn’t annoying” isn’t really a good enough reason. That’s why I brought it up in class that week. I never knew those things about gifted students acting out. That was probably the best thing from the discussion board. Now I know what to look for in my classroom for students who got missed...If we had had that discussion in class, I probably would have gotten pretty hot. It was good to talk about it in class, but after I cooled down.

There was evidence that the ClassChatter assignments also helped to facilitate more equitable in-class discussions. The journal that appeared to be pivotal for most PSTs occurred in the seventh week of class. The class meeting after the Ken and Bart journal response was due, the PSTs asked to discuss their responses to the journal entries as a class. Prior to this, white PSTs in general, and Holly in particular, dominated class discussion. During the discussion of the Ken and Bart journals, Kevious, Gayle, and Yeny were outspoken and led the discussion, while Holly was notably quiet. After Journal 7, every class began with a 10-20 minute discussion about the journal postings at the beginning of class. While the PSTs who led the discussion differed from week to week, the in-class discussions in the latter half of the semester were no longer dominated by any one group of students.

² You have two students: Ken and Bart, who are African American males from the same neighborhood. Ken always does his work on time and gets low grades on tests. You really enjoy working with Ken and told his parents during the yearly conference that he was a “joy to teach”. Ken’s course average is 79.4% at the end of the semester. Bart doesn’t normally do his homework, but gets higher grades (than Ken)

on assessments. He is difficult to work with and only comes to school 75% of the time. You have had a few meetings with the school counselor and with his mother regarding Bart’s effort and his disposition. Bart’s course average at the end of the semester is 79.4%. Who do you think has learned more mathematics? What letter grade would you assign to each student? Justify your decision.

Discussion

Overall, the evidence suggests that the use of ClassChatter and online journaling in the History of Mathematics class provided a bridge to more equitable participation in-class discussions on equity. The randomized discussion board monikers removed some of the anxiety these PSTs had about voicing their thoughts about culturally responsive teaching. In every interview, participants mentioned how much they valued not knowing who they were talking with, and all three of the students who participated least in class mentioned the value of having a different venue where their voices could be heard.

All PSTs also mentioned the asynchronous nature of the medium as an important factor in keeping discussions civil and helping them become more culturally aware. Furthermore, the discussion boards allowed participants to build trust, which made it easier to have in-person discussions later in the semester. Although some PSTs like Holly still held problematic views, we were able to have challenging conversations about equity and culture in an area where such topics are especially taboo. While she and some of the other PSTs are not where we would like them to be, they are more open to new ideas than they were at the beginning of the semester (Lewis, 2016). That said, the use of ClassChatter necessarily made the early discussions more structured than they would have been in class. While this allowed us to moderate responses and avoid the faux pas of anyone saying something potentially offensive to their peers, the lack of spontaneity could have been a drawback with PSTs who had a higher initial level of cultural awareness.

The PSTs' responses align with the suggestions of Kerr (2010) and Leggett (2016) about introducing culturally responsive teaching practices to groups of teachers that have established networks. Our findings suggest that History of Mathematics can be combined successfully with cultural responsiveness. For mathematics teacher educators, using technology to mediate discussions also means most class time is history of mathematics content, and the participants in this study all maintained they learned more through the discussion board than through classroom discussion. Although ideally both equity and mathematics history could be entwined throughout the

course, this separation may make it clear to mathematics departments that the mathematical learning objectives are being met while still yielding the opportunity for students to engage meaningfully in equity. Furthermore, by combining these two topics into a single course, credits in the degree program may be reallocated to align programs more closely with the teacher preparation program standards proposed in Association of Mathematics Teacher Educators (2017). Since these standards call for both equity interwoven into mathematics content and a mathematical history course, this pairing has a natural synergy. Finally, after we taught this course, ClassChatter stopped its services. However, we recommend Chatzy (www.chatzy.com) for similar levels of functionality for those looking for a free discussion board resource.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. How did using ClassChatter help the pre-service teachers to participate more openly during discussions?
2. How could such an online discussion board be used in classes you teach? In classes you have taken in your own education coursework?
3. How do the findings of this study relate to the prior research on equity and classroom technology?
4. Why do you think these students found ClassChatter effective?
5. Based on upon this article and your prior knowledge and experiences, what suggestions would you make for incorporating equity and culturally responsive pedagogy into content courses for pre-service mathematics teachers?

Appendix: Sample Journal and Discussion Board Prompts

Achievement: You have two students: Ken and Bart, who are African American males from the same neighborhood. Ken always does his work on time and gets low grades on tests. You really enjoy working with Ken and told his parents during the yearly conference that he was a “joy to teach”. Ken’s course average is 79.4% at the end of the semester. Bart doesn’t normally do his homework, but gets higher (than Ken) grades on assessments. He is difficult to work with and only comes to school 75% of the time. You have had a few meetings with the school counselor and with his mother regarding Bart’s effort and his disposition. Bart’s course average at the end of the semester is 79.4%. Who do you think has learned more mathematics? What letter grade would you assign to each student? Justify your decision.

Access: Describe your future classroom. How is it laid out? How will you decorate it? What steps will you take to ensure all students have access to the classroom space? What steps can you take to ensure all students have access to the conversational space?

Power: This week, we will focus on power in the classroom. First, read the article (Langer-Osuna & Engle, 2010) posted to the class website. Then write a paragraph describing in what ways the student was able to use his power to influence his classmates to his point of view. What advantages did this student have? How might the teacher have done things differently to include more students and redirect the discussion?

Identity: What is your identity as a teacher? Thinking back to the previous journal, where you described your ideal first school, how does your identity as a teacher relate to the identity of your hypothetical students? In what ways might your identity help you connect with your students? How might your identity hinder building connections with students? What are some things we discussed throughout this course that might help you overcome these hindrances and build rapport with students?



TODOS Live! ← → TODOSonLine!

TODOS Live! began as an interactive webinar series in 2006. Topics were divided between hour-long content/grade level sessions and sessions that focused on the mission and goals of TODOS. While some of the sessions were lost due to lack of storage space, many of the sessions are archived on Vimeo (<https://vimeo.com/user56336191>).

TODOS Live! is in the process of reorganizing. We currently have two types of sessions. Content sessions will continue, with the length set by the presenter. The second type are Quick Talks on Equity (QTEs) which are short sessions that are recorded, and attendees have a chance to interact with the presenter at the end of the session. The QTE session will be rebroadcasted with interaction taking place through twitter chats.

Become involved with TODOSonLine! Contact todoslive@todos-math.org to volunteer or to share comments, questions, or suggestions. Hope to hear from you and “see you” online.

TODOS Blog

Ethnomathematics: Mathematics de TODOS

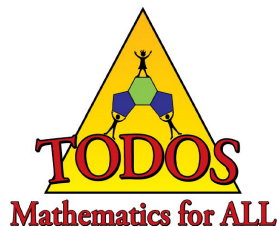
CarlosLópez Leiva, Kyndall Brown, and Silvia Llamas-Flores

Go to the TODOS Blog at <https://www.todos-math.org/the-todos-blog> to read and respond to the May 23, 2019 post on ethnomathematics. The introduction gives an overview of ethnomathematics and its relevancy in today’s classroom. The following comes from the blog:

These notes and blog were developed with the goal of sharing available resources around ethnomathematics. Our hope is that mathematics teachers and educators can access and use them as needed. This blog includes three main sections:

1. How is ethnomathematics relevant and critical?
2. What has been learned and done in ethnomathematics?
3. What can be done in the classroom?

We hope you enjoy it, and if you experience some of these or new ideas in your classroom, please share with us here, so more teachers and researchers can learn about what of ethnomathematical approaches can be implemented in the mathematics classroom.



Teaching for Excellence and Equity in Mathematics

<http://www.todos-math.org/teem>

TEEM Special Themed Issue

TEACHING MATHEMATICS WITH MULTILINGUAL LEARNERS: ACTIONS AND INNOVATIONS

Call for MANUSCRIPTS

SUBMISSION DEADLINE: SEPTEMBER 15, 2019

There is growing awareness of the need for teachers to implement equitable, rigorous, and coherent mathematics instruction (TODOS, 2019). This type of instruction requires that we *purposefully* include the multilingual students in our classrooms. Though aware of this need, many teachers are still unsure of how to *act* on this awareness as evidenced by their lack of confidence in their ability to teach diverse groups of students (Banilower et al., 2018). For this special issue on multilingual learners, *TEEM* seeks manuscripts from classroom teachers and/or teacher educators that provide evidence-based examples of how to enact and/or learn how to enact effective instructional strategies and/or learning activities with multilingual students. We are particularly interested in manuscripts that (1) promote and utilize deficit-free language, (2) include positive, evidence-based examples and/or vignettes, and (3) bridge research and practice. We are interested in articles that span the continuum of K-16, preservice, and inservice learning spaces. The following are suggested manuscript topics for this special issue:

- A description, discussion, or reflection on implementation of effective teaching practice(s) in multilingual classrooms and/or learning environments. This work could be situated in teacher education contexts.
- A specific classroom-tested TODOS-oriented “excellence and equity” mathematics activity exploring innovations and practices for teaching mathematics in multilingual settings. This activity could be accompanied by a blackline worksheet for classroom use.
- A description of advocacy work with students, parents/families, or colleagues that foster effective policy and practices for teaching mathematics with multilingual students.
- A description of professional development initiatives aimed at developing practices for teachers to support the learning and success of multilingual learners in mathematics.
- A description of teacher preparation initiatives aimed at helping prospective teachers to effectively teach multilingual students.

The *TEEM* Guest Editors for this special issue, Zandra de Araujo (University of Missouri), Craig Willey (IUPUI), Sarah Roberts (University of California-Santa Barbara), and William Zahner (San Diego State University), welcome query emails about the suitability of proposed topics: email dearaujoz@missouri.edu. Please write “Multilingual Learners Issue” on the subject line.

Teaching for Excellence and Equity in Mathematics (TEEM) is a refereed journal published by **TODOS: Mathematics for ALL** and available via membership in TODOS. The intended audience of *TEEM* includes mathematics teachers, leaders, administrators, and mathematics teacher educators.

For more details on the guidelines for papers, see <http://www.todos-math.org/teem>.

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TODOS. (2019). Mission and goals. <https://www.todos-math.org/mission-goals>



2018-19 ELECTED LEADERSHIP

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*There is an even year (2020) coming up.
That means a TODOS conference is in the making!*

Announcing

TODOS 2020 Conference

***Activating Agency for Student Access, Engagement, and
Advancement in Mathematics***

and

Pre-Conference

Change Agents Taking Action on Equity in Mathematics Education

June 25 - 27, 2020

Scottsdale Plaza Resort

7200 N. Scottsdale Road
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Why Attend? Because EQUITY MATTERS!

Schedule

Thursday, June 25, 2020

8:00 – 3:00 Pre-Conference

5:00 – 8:00 Conference opening session and reception

Friday, June 26, 2020

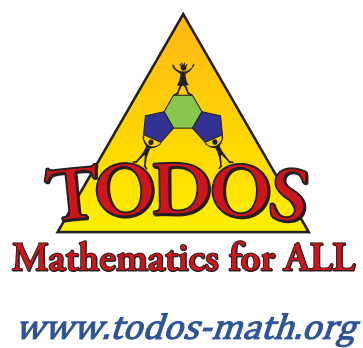
8:00 – 5:00 Keynote, a variety of sessions and networking
Dinner on your own

Saturday, June 27, 2020

8:00 – 4:30 Variety of sessions and interactions, closing session

Call for speaker proposals will be posted to the conference website
by mid-July and due September 30, 2019

<https://www.todos-math.org/todos-2020-conference>



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