Vol. 8, No. 1 Spring 2017 ISSN 2153-0173 online

TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

TODOS
Mathematics for ALL

An Affiliate Organization of the National Council of Teachers of Mathematics

TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

IN THIS ISSUE

From the Editors

The *TEEM* editors are happy to present this eighth issue of *TEEM* in time for the 2017-18 school year. Editors Marta Civil (University of Arizona) and Luciana C. de Oliveira (University of Miami) are pleased to introduce this issue as another wonderful contribution to the field of mathematics education. The journal is a vehicle to provide a scholarly and pedagogical resource for mathematics educators, practitioners, leaders, and administrators at all levels. *TEEM* uses a rigorous double-blind review process to ensure that a paper is judged on its merits without the external reviewers (or even the Editorial Panelist coordinating that paper's external reviewers) knowing the identity of the author and vice-versa. For information on reviewing or writing for *TEEM,* please see the *TEEM* webpage ***http://www.todos-math.org/teem.*** On that webpage, you will also find a link to a webinar on writing and reviewing for *TEEM*.

The current issue of *TEEM* includes two externally peer-reviewed articles and an invited article. The issue starts with a contribution by Christa Jackson (Iowa State University) and Sarah A. Roberts (University of California, Santa Barbara), "Dimensions of Equity within Preservice Teachers' Responses to Equity Quotations." This inquiry focused on the interpretation and responses to five quotations related to issues of equity in mathematics education by secondary mathematics preservice teachers (PSTs) in mathematics methods courses at three different universities. Using Gutiérrez's (2009) *dimensions of equity* (access, achievement, identity, and power) to examine PSTs' responses, the authors describe how PSTs were able to discuss issues of equity that could affect their future mathematics instruction.

The second article is by Ji-Yeong I (Iowa State University) and Zandra de Araujo (University of Missouri). Their article, "Examining One Mathematics Teacher's Decisions Regarding Mathematics and Language," shows one teacher's decisions in response to the difference between the intended meaning of a mathematical problem and her student's (an ELL) understanding. Using a vignette that illustrates the teacher's tensions when making her instructional decisions, they provide the teacher's rationale for her decisions and an analysis of the episode.

The invited article "A Framework for Modifying Math Tasks for Accessibility," by Walter Secada, Edwing Medina, and Mary Avalos, provides an illustrative summary of a four-dimensional framework (mathematical content, mathematical practices, context, and language demands) used in creating an assessment of academic language in mathematics for *Language in Math* (*LiM*), an IES-funded research and development project. In sharing this framework, the authors aim at supporting educators in their work towards improving the accessibility of mathematics tasks.

TEEM gratefully acknowledges the support of all the leaders in our sponsoring organization, TODOS: Mathematics for ALL. We hope *TEEM* continues to serve the TODOS membership, and provides an inspiring pedagogical and scholarly resource for the broader mathematics education and education communities.

Marta Civil
The University of Arizona

Luciana C. de Oliveira
University of Miami

Dimensions of Equity within Preservice Teachers' Responses to Equity Quotations

Christa Jackson
Iowa State University

Sarah A. Roberts
University of California Santa Barbara

Abstract

Secondary mathematics preservice teachers (PSTs) in mathematics methods courses at three different universities interpreted and responded to five quotations related to issues of equity in mathematics education. PSTs engaged with the quotations both individually, in writing, and as a whole class, in an inner-outer circle discussion. We used Gutiérrez's (2009) *dimensions of equity* (access, achievement, identity, and power) to examine PSTs' responses. Along with other course work, this activity created a space where PSTs were able to discuss issues of equity that could affect their future mathematics instruction.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Does equity have various dimensions, and if so, what might they be?

2. What aspects of equity, with regards to the teaching and learning of mathematics, would you anticipate preservice secondary mathematics teachers to be more (or less) likely to discuss and why?

3. What might make it more (or less) challenging for preservice mathematics teachers to discuss particular issues of equity related to the teaching and learning of mathematics?

Christa Jackson (jacksonc@iastate.edu) is an Assistant Professor at Iowa State University. Her research focuses on teachers' knowledge of equity and teaching practices mathematics teachers use that affords opportunities for students from diverse cultures, ethnicities, and socio-economic backgrounds to learn rigorous, challenging mathematics while simultaneously fostering productive mathematical identities.

Sarah A. Roberts (sroberts@education.ucsb.edu) is an Assistant Professor of mathematics education at University of California Santa Barbara. Her interests include equity in mathematics education, supporting English learners in math, and pre- and in-service teacher professional development.

Dimensions of Equity within Preservice Teachers' Responses to Equity Quotations

Christa Jackson and Sarah A. Roberts

> It's a little overwhelming, realizing just how much I hadn't thought about when it comes to the education and well-being of my future students. More and more, I feel like the majority of my focus in teaching should be addressing the different backgrounds and the different needs of my students. With classrooms becoming more and more diverse, I have a responsibility to be as informed and prepared as possible to run and maintain a diverse classroom. (Preservice Mathematics Teacher's Written Post-Discussion Reflection)

What does it mean to have preservice mathematics teachers reflect on and discuss issues of equity within mathematics teaching and learning? Why is it important to attend to equity issues in the mathematics classroom? Researchers (e.g., Malloy, 2009) argue that as teachers attend to equity in their classrooms, they build relationships, have high expectations, and support students to maintain their identities. Teachers must be prepared to make learning relevant (Malloy, 1997) and to teach diverse learners (Gutíerrez, 2009) who have varied cultures, lives, and prior experiences. Thus, it is important for preservice teachers (PSTs) to see beyond their own experiences and to understand those of their future students in order to begin creating equitable classrooms (Milner, 2006). Therefore, the purpose of this study is to examine PSTs' conceptions of equitable mathematics teaching. More specifically, the research question underlying this study is: What do secondary preservice mathematics teachers attend to when interpreting and reflecting on issues of equity in the teaching and learning of mathematics?

Framework

Attending to issues of equity is key to transforming "mathematics education in ways that privilege more socially just practices" (Gutiérrez, 2009, p. 4). Gutiérrez (2009) argues there are four dimensions of equity, which lie on two axes: the dominant axis (*access* and *achievement*) and the critical axis (*power* and *identity*). The dominant axis attends to "preparing students to participate economically in society and privileging a status quo. The dominant axis, where access is a precursor to achievement, measures how well students can play the game called mathematics" (Gutiérrez, 2009, p. 6). Access includes resources that enable students to participate in mathematics, such as quality teachers, rigorous curriculum, and adequate supplies. Achievement includes students' participation in class, their patterns of course taking, their standardized test scores, and their participation in the mathematics pipeline.

The critical axis, in contrast, ensures "that students' frames of reference and resources are acknowledged in ways that help build critical citizens so that they may change the game" (Gutiérrez, 2009, p. 6). Attending to identity through acknowledging the ways students are racialized, gendered, and classed gives students opportunities to draw on their cultural and linguistic resources (e.g., home language, frames of reference, etc.). Whereas, attention to power in mathematics classrooms occurs through supporting students in using mathematics to transform the world in which they live, such as through giving students voice in the classroom and through using mathematics to critique society (Gutiérrez, 2009).

Context

This study took place in three different mathematics methods courses at three universities in the United States (Midwest, Southeast, and New England states) with 43 PSTs (> 85% were Caucasian). There were seven to nineteen students enrolled in each methods course, split almost equally between males and females, and the course was the only secondary mathematics methods course offered to students at each university and the PSTs enrolled in that course the semester prior to student teaching.

Although the PSTs at each university take a multicultural education course as part of their teacher preparation program, those courses do not connect explicitly to education course as part of their teacher preparation program, those courses do not connect explicitly to mathematics education. Therefore, to prepare teachers adequately to teach mathematics through an equity lens, we structured our methods classes to attend to and focus on equity issues throughout the course. During our methods courses, the PSTs read and reflected on mathematics equity-related articles (see Appendix for a complete list of the equity-related articles PSTs read during the course) and also discussed equity in the context of the other key components of mathematics teaching and learning (e.g., within the context of assessment, lesson planning, and promoting classroom discourse). Equity was not a single-day lesson in the course, but instead was embedded in the day-to-day work of the class.

Equity Quotations Task

To examine secondary mathematics PSTs' conceptions of equitable mathematics teaching, we collected written responses to five quotations (see Table 1) during the sixth week of the semester. The five quotations were selected because they were representative of viewpoints our PSTs would encounter in their current and future practice. Prior to responding to the quotations, the PSTs did not read the articles and/or books from which the quotations were taken. All the PSTs responded to Quotation 2 and were randomly assigned to respond to two other quotations, for a total of three quotations each. The PSTs were asked to respond to three quotations to provide them an opportunity to delve more deeply and reflect more on their responses. We purposefully selected Quotation 2 for all the PSTs to respond to because teachers' expectations were paramount in the discussions at all three universities, and we wanted to provide a window to further explore those conversations. For each assigned quotation, the PSTs had to (1) interpret the meaning of each quotation in 1-2 paragraphs, and (2) discuss their reactions to each quotation (What do you think about the quotation?) in 1-2 paragraphs. We acknowledge that thinking about one quotation might affect how the PSTs considered others. However, we believe we selected a range of quotations that broadly represented the thoughts and ideas related to equity in mathematics education.

Table 1
Equity Quotations with Number of Preservice Teacher Responses per Quotation

Equity Quotation	PSTs' Responses
1. "Minority and linguistically diverse students have not been construed as visible players within mathematical discourse either in or out of schools." (We adapted this quotation from González, Andrade, Civil, & Moll, 2001.)	21
2. "Teachers have different expectations of their students based on their students' ethnic and socio-economic backgrounds." (This quotation is based on expectations literature, e.g. McKown & Weinstein, 2008.)	43
3. "The way teachers teach mathematics does not send any messages; mathematics is free of context." (We wrote this quotation as a negation of the following text: "Simply put, teaching math in a neutral manner is not possible. No math teaching – no teaching of any kind for that matter – is actually 'neutral'" (Gutstein & Peterson, 2006, p. 6).	23
4. "I thought math was just a subject they implanted on us because they felt like it, but now I realize that you could use math to defend your rights and realize the injustices around you… now I think math is truly necessary and, I have to admit, kinda cool. It's sort of like a pass you could use to try and make the world a better place" (Gutstein & Peterson, 2006, p. 1).	22
5. "Students can connect math with their own cultural and community histories and can appreciate the contributions that various cultures and people have made to mathematics" (Gutstein & Peterson, 2006, p. 2).	20

After completing their responses, the PSTs participated in a videotaped discussion using a discussion structure called inner-outer circle, an adaptation of a Socratic Seminar (e.g., Thompson & Radosavljevic, 2014). The students with the same quotation discussed their interpretation and reaction to it in the "inner circle" for ten minutes. The PSTs who did not respond to the quotation listened to the discussion in the "outer circle," and after the inner circle's discussion, they had five minutes to discuss their thoughts on the discussion and the quotations. The use of the inner-outer circle structure allowed the PSTs to extend their thinking about all the quotations with their peers.

Following the inner-outer circle discussion, the PSTs individually reflected on the discussion by giving written responses to these prompts: (1) What are you thinking about after the discussion of the equity quotations? (2) What are you thinking about related to your own beliefs and experiences about mathematics teaching and learning? (3) What are you thinking about related to your teaching, your future classroom, and your future students?

To analyze the PSTs' responses to the quotations, we coded each student's written responses to the quotations based on Gutíerrez's (2009) dimensions of equity (access, achievement, identity, power). The two co-authors coded 20% of the data independently to establish reliability. We discussed our disagreements and agreed to whether an item belonged with a particular code. We then used the discussion to independently code another 20% of the data. We achieved a Cohen's Kappa of 82.6%. We used the inner-outer circle discussion and the post-reflection responses to triangulate the data.

Preservice Teachers' Responses to the Equity Quotations

We categorized and aggregated the PSTs' written responses according to Gutiérrez's (2009) four dimensions of equity. Table 2 provides an overview of these written PST responses to the quotations, which occurred prior to the inner-outer circle. We discuss each dimension in the following subsections.

Table 2
Classification of Responses (from 43 PSTs) by Gutiérrez (2009) Equity Dimension

Quotation	Access	Achieve-ment	Identity	Power
Quotation 1	40%	21%	36%	3%
Quotation 2	57%	14%	28%	1%
Quotation 3	75%	6%	15%	4%
Quotation 4	24%	4%	2%	70%
Quotation 5	19%	1%	76%	4%

Access

Our PSTs discussed access most often in relation to providing opportunities for all students to learn mathematics and to having high or the same expectations for all students. In terms of providing opportunities to learn, one PST argued during the inner-outer circle discussion that if teachers were not teaching all students and giving them opportunities to learn, they were not doing their jobs. Another PST provided an example of having students in class who did not understand English. She emphasized that teachers must do "their part" to make sure the English learners know and understand mathematics vocabulary and how to explain and use it to solve word problems. Although this PST did not discuss how she would attend to language development in her mathematics classroom, she claimed it was "our responsibility" to figure out how to modify instruction in ways that support English learners in the classroom.

The second focus PSTs discussed in relation to access was providing either high or the same expectations for all students. For example, many PSTs reacted in their written responses to Quotation 2 by saying that they would have high standards and encourage all students in the mathematics classroom. To explore why high expectations were so important, during the inner-outer circle discussion one PST asked: "If a teacher has low expectations for his/her students, but is not vocal about it, how would the students know?" A peer responded,

> There's a lot of different ways it can come across. First of all, it can be by the type of assignments you are giving them. If you don't think they can do higher-cognitive level tasks, then you won't even give them

> the option of learning it. Other ways you can do that is by not providing them time to explore or think mathematically. So you just don't provide them the opportunities for them to learn. So it's not like you're telling them "I don't think you can do this," but you're not giving them the opportunities to do so. Like the second they start struggling with something, instead of letting them struggle, you say "Oh this is too hard, let me give you something easier," or "Here's the answer."

Low expectations, according to our PSTs, lessened access to opportunities to learn mathematics.

The PSTs wrote a number of recommendations about how they would provide access to their students, beyond more generally creating opportunities to learn and having high expectations: (a) relate mathematics content to students and their lives, (b) create a safe learning environment, (c) use tools to create access, and (d) learn how to adapt lessons to make students of color "visible players" in the mathematics classroom.

Achievement

Achievement is often what results from students having or not having access to rigorous mathematics (Gutiérrez, 2009). The PSTs noted this symbiotic relationship between access and achievement. For example, a PST commented during the inner-outer circle discussion, "If you have lower expectations for those students that you don't think are going to do as well in your class, then they aren't going to do well, because they know you don't expect a lot out of them." Another PST in this discussion mentioned this interconnection between access and achievement: "Assumptions…You know their ethnic background. You know their SES. So you make an assumption about it.... If you only expect them to perform at a certain level, and if they hit that level, that's all you expected out of them."

When the PSTs discussed issues focused specifically on achievement, they repeatedly mentioned students' performance in mathematics and on standardized tests. One PST articulated in her written response, "A student who is a minority or a student who is diverse and maybe is not a native speaker of the English language is not seen as an excelling student within or out of the classroom." With regards to current mathematics assessments, a PST expressed in her written response, "[S]tandardized tests seem to be [the] clearest exemplar of [the] dominate Anglo culture's ability to subjugate minority and ELL [English language learner] learners." By and large, PSTs noted that all students are capable; however, PSTs raised the concern that not all will often be seen as such in their teachers' eyes or on standardized assessments.

Identity

We found that when PSTs discussed identity, they related it to: 1) how teachers align particular characteristics or traits with groups of individuals and 2) how teachers lower expectations for particular groups of students. In attending to groups of people aligned with particular traits, one PST, whose response was representative of other PSTs' written equity quotation responses, suggested, "Students who experience diversity either linguistically or as a minority are not often thought of as people who will be prominent figures in their math classes while they are in school." The PSTs indicated these beliefs about characteristics of students often lead to lower expectations for students from non-dominant backgrounds, and often as a result of these lower expectations, teachers generally perceive these students as less capable. For example, a PST stated, "People assume that since [students] do not speak the language or do not have the same backgrounds as the general population, they do not have the ability to work with mathematics."

Most of our PSTs distanced themselves from such attitudes. As one of the PSTs explained during the inner-outer circle discussion, "You still want to get them to that eventual same place as everyone else, but you can use their individual identities, their ethnicity, their gender, etc. and use those to your advantage to help plan how you are going to get them there." Several PSTs argued the importance of teachers attending to students' identities during mathematics instruction. In particular, one PST explained why ignoring students' identities is detrimental to their mathematics learning: "We can't expect them to learn something if we're not considering their backgrounds or their culture in general…. And we're holding them back if we don't try to change our lesson plans to try to fit their needs." PSTs made the case that teachers can and must use students' cultures and identities to engage students in mathematics classrooms. However, as a PST suggested, this requires teachers, not students, to do the work: "Instead of making them adapt to us, [we must] adapt to them."

Power

Power involves social transformation through students' use of mathematics (Gutiérrez, 2009). While very few PSTs attended to power, a few did note the role of mathematics in attending to social injustices, in that mathematics "can give one the ability to identify social injustice and the means to address these issues." The following dialogue helps illustrate the PSTs' challenge with discussing power during the inner-outer circle discussion:

PST1: I guess when I first read [Quotation 4], I was really surprised, because I never really thought of math as this…sword of justice that you would use to defend your rights… And I started to understand how you could say that. And understanding why things are happening in the world and the reasons for that and whether that is right or wrong, and using the logic and the reasoning that you learn in math and applying those to those kinds of situations as to whether this is right. It kind of opened my mind up to how you would think about it.

PST2: And I think, like you said, the key elements that we teach in math classes, the critical thinking and the problem solving, they're the best ways that we can fight things, like the racism and the intolerance that a lot of our students face.

Here the PSTs were starting to make sense of how they and their students could attend to issues of power using mathematics.

Some PSTs suggested that their students could use mathematics to understand the world, which we categorized as attention to power. One student illustrated this idea, saying, "Mathematics is an invaluable tool for studying our world so the understanding of society can be deepened." In addition to understanding the world, students could use mathematics to affect the world. One PST explained, "Being able to master certain skills and understanding how to manipulate numbers and/or variables can give you the ability to have more of an impact on the world around you." These were practical pathways PSTs suggested for attending to power.

Discussion

Overall, we found that PSTs were able to engage with the quotations across multiple learning contexts in both private (individual written responses in the initial quotation responses and in the post-discussion reflections) and public ways (inner-outer circle discussion) that allowed them to think about both the dominant and critical axes. While many of the PSTs' responses focused on the dominant axis, some PSTs recognized some issues of identity and power, such as combating stereotypes, using students' culture and background when developing their lessons, and teaching mathematics for social justice.

The inner-outer circle discussions created a space for PSTs to talk through ideas with their peers and allowed them to develop more nuanced thinking about equity in their future mathematics classrooms. The discussion structure permitted the PSTs to extend their own thinking and to broaden other's thinking. The PSTs noted in their post-discussion reflections that they found the inner-outer circle helpful; they appreciated listening to each other's thoughts and opinions. From the discussions, the PSTs were able to negotiate a shared understanding of what various quotations meant and were better able to articulate their interpretations of the quotations.

We found that our equity quotation activity sparked conversations and opened a space for PSTs to think about how they could affect students' lives positively through their mathematics teaching. The PSTs' ability to articulate and engage in such powerful conversations across all three campuses leads us to believe that the combination of the structure of this activity and the prior work with readings and an explicit equity focus supported PSTs with developing an equity lens for their teaching.

We want to highlight that we did not see many, if any, deficit perspectives in the PSTs' responses both in their written and verbal work. We attribute this both to how we structured this assignment and to our focus on equity, with key exercises, throughout the course. During the course, our PSTs read and reflected on articles focused on equity, and our conversations related to equity occurred weekly and not during a single "equity week." However, even with the "right" kind of discourse, we hoped our students would go even deeper in their discussions of equity in mathematics teaching and learning. We believe we just got them to touch the surface as they were thinking about the dimensions of equity. But, we do believe that as our PSTs engaged in these activities we moved them closer to addressing equity and social justice issues in a coherent way within our mathematics methods course (Koestler, 2012). Our next step will be to see how our work with PSTs translates into their classroom practice and instruction as student teachers and inservice teachers.

Acknowledgment

We would like to thank Alejandra Salinas, our fellow STaR Fellow, who has made valuable contributions to this work.

References

González, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zone of practice in mathematics. *Journal of Education for Students Placed at Risk, 6*(1&2), 115-132.

Gutiérrez, R. (2009). Framing equity: Helping students "play the game" and "change the game." *Teaching for Excellence and Equity in Mathematics*, *1*(1), 4-8.

Gutstein, E. & Peterson, B. (2006). Introduction. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: Teaching social justice by the numbers* (pp. 1-6). Milwaukee, WI: Rethinking Schools, Ltd.

Koestler, C. (2012). Beyond apples, puppy dogs, and ice cream: Preparing teachers to teach mathematics for equity and social justice. In A. A. Wager & D. W. Stinson (Eds.), *Teaching mathematics for social justice: Conversations with educators* (pp. 99-111). Reston, VA: National Council of Teachers of Mathematics.

Malloy, C. E. (1997). Including African American students in the mathematics community. In J. Trentacosta & M. J. Kenney (Eds.), *Multicultural and gender equity in the mathematics classroom: The gift of diversity* (pp. 23-33). Reston, VA: National Council of Teachers of Mathematics.

Malloy, C. E. (2009). Instructional strategies and dispositions of teachers who help African American students gain conceptual understanding. In D. Martin (Ed.), *Mathematics teaching, learning, and liberation of the lives of Black children* (pp. 88-122). New York: Routledge.

Martin, D. B. (2009). Does race matter? *Teaching Children Mathematics 16*(3)*,* 134-139.

McKown, C., & Weinstein, R. S. (2008). Teacher expectations, classroom context, and the achievement gap. *Journal of school psychology, 46*(3), 235-261.

Milner, H. R. (2006). Preservice teachers' learning about cultural and racial diversity: Implications for urban education. *Urban Education*, *41*(4), 343-375.

Stiff, L. V., Johnson, J. L., & Akos, P. (2011). Examining what we know for sure: Tracking in middle grades mathematics. In W. F. Tate, K. D. King & C. R. Anderson (Eds.), *Research and practice pathways in mathematics education: Disrupting tradition* (pp. 63-75). Reston, VA: NCTM.

Thompson, A., & Radosavljevic, A. (2014). Fostering mathematical discourse With Socratic Seminars. In M. Civil & E. Turner (Eds.), *The Common Core State Standards in mathematics for English language learners: Grades K-8* (pp. 81-94). Alexandria, VA: TESOL International Association.

Discussion and Reflection Enhancement (DARE) Post-Reading Questions

1. How would you respond to the equity quotations?

2. If you were to add a topic and/or discussion point to the quotations, what might it be and why?

3. In what ways, if any, do the quotations influence your thinking of your mathematics instruction and/or interactions with your students?

4. What methods would you suggest for engaging preservice teachers in thinking about equity in mathematics teaching and learning and why?

5. What additional quotations might be useful for this activity? (See, for example, quotations from the first issue of *TEEM* or from readings in the 2016-17 national equity webinar series, http://www.nctm.org/webinars/EquitySocialJustice/.)

"DARE to Reach ALL Students!"

TODOS Mathematics for ALL

Appendix: Equity Readings for Mathematics Methods

Avineri, T., Belledin, C., Graves, J., Noble, R., Hernandez, M., Robinson, D., & Teague, D. (2011). Issues of equity for advanced students. In M. Strutchens & J. R. Quander (Eds.), *Focus in high school mathematics: Fostering reasoning and sense making for all students* (pp. 37-63). Reston, VA: NCTM.

Borgioli, G. M. (2008). Equity for English language learners in mathematics classrooms. *Teaching Children Mathematics, 15*(3), 185-191.

Bresser, R. (2003). Helping English-language learners develop computational fluency. *Teaching Children Mathematics, 9*(6), 294-299.

Dance, R.A., Wingfield, K.H., & Davidson, N. (2000). A high level of challenge in a collaborative setting: Enhancing the chance for success for African American students in mathematics. In M. E. Strutchens, M. L. Johnson, & W. F. Tate, *Changing the faces of mathematics: Perspectives on African Americans* (pp. 33-49), Reston, VA: NCTM.

Dieker, L., Maccini, P., Hunt, J. & Strikland, T. (2011). Minimizing weaknesses and maximizing strengths of students with disabilities. In M. Strutchens & J. R. Quander (Eds.), *Focus in high school mathematics: Fostering reasoning and sense making for all students* (pp. 37-63). Reston, VA: NCTM.

Gutiérrez, R. (2009). Embracing the inherent tensions in teaching mathematics from an equity stance. *Democracy & Education, 18*(3), 9-16.

Gutstein, E., & Peterson, B. (Eds.). (2005). *Rethinking mathematics: Teaching social justice by the numbers.* Milwaukee, WI: Rethinking Schools, Ltd.

Lovin, L. Kyger, M., & Allsopp, D. (2004). Differentiation for special needs learners. *Teaching Children Mathematics, 11*(3), 158-167.

Martin, D. B. (2009). Does race matter? *Teaching Children Mathematics,* 134-139.

McCulloch, A.W., Marshall, P.L., & DeCuir-Gunby, J.T. (2009) Cultural capital in children's number representations*.* *Teaching Children Mathematics, 16 (3),* 184-189.

Moschkovich, J.N. (2010). Language, culture, and equity in secondary school mathematics classrooms. In J. Lobato (Ed.), *Teaching and learning mathematics: Translating research for secondary school teachers* (pp. 75-80). Reston, VA: NCTM.

Mueller, M. F., & Maher, C. A. (2010). Promoting equity through reasoning. *Teaching Children Mathematics, 16*(9), 540-547.

NCTM (2007, July). Research summary: The relation between reform teaching and equity in mathematics education. Retrieved from: http://www.nctm.org/news/content.aspx?id=12906

Stiff, L.V. Johnson, J.L., & Akos, P. (2011). Examining what we know for sure: Tracking in middle grades mathematics. In W.F. Tate, K.D. King & C.R. Anderson (Eds.), *Research and practice pathways in mathematics education: Disrupting tradition* (pp. 63-75), Reston, VA: NCTM.

White, D. Y., Spitzer, J. S., & Malloy, C. E. (Eds). (2009). *Mathematics for every student responding to diversity: Grades Pre-K-5.* Reston, VA: NCTM.

TODOS
Mathematics for ALL

Examining One Mathematics Teacher's Decisions Regarding Mathematics and Language

Ji-Yeong I
Iowa State University

Zandra de Araujo
University of Missouri

Abstract

Teachers have to make many in-the-moment decisions when teaching. We investigated one teacher's decisions in response to the difference between the intended meaning of a mathematical problem and her student's understanding. The student—an English language learner—had a different interpretation of the mathematical scenario related to one particular clause in the problem that was, ironically, intended to be explanatory but ended up obscuring intended meaning and therefore impacted the student's solution. In order to reflect on the teacher's decisions, we include a vignette that illustrates the teacher's tensions when making her instructional decisions. The vignette is followed by the teacher's rationale for her decisions and an analysis of the episode. We invite readers to participate in her decision-making process and explore impacts of each decision.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Have you ever had a moment when you notice your student interprets a direction or a problem statement differently from the intended meaning when working on mathematical problems? If so, how did you recognize it and what did you do?

2. What types of decisions do teachers of English language learners (ELLs) face when enacting mathematics word problems using complicated language with students?

3. When working with an ELL on mathematics, how do you decide whether your students' misunderstandings, if any, stem from language, mathematics, culture differences, or some combination of these areas?

Ji-Yeong I (jiyeongi@iastate.edu) is an assistant professor of mathematics education at Iowa State University. She taught mathematics and science for diverse students in urban areas before she started her research career. Her research interests include English language learners, teacher education, and culturally responsive teaching.

Zandra de Araujo (dearaujoz@missouri.edu) is an assistant professor of mathematics education at the University of Missouri. Her research examines teachers' use of curriculum, particularly with English language learners, and the preparation of elementary mathematics teachers.

Examining One Mathematics Teacher's Decisions Regarding Mathematics and Language

Ji-Yeong I and Zandra de Araujo

Teachers make many pedagogical decisions daily. Schoenfeld (2011) described teachers' decision-making processes as "the selection of goals consistent with the teachers' resources and orientations" (p. 460). In other words, a teacher's decisions should be made in accordance to his or her goals for students, while also taking their individual beliefs about learning, access to curriculum materials or technology, and expectations for performance based on cultural and linguistic standards into account. Moreover, the decisions, made consciously or unconsciously, have consequences that may or may not be evident in the short term.

Imagine you are a mathematics teacher and you just discovered your student interpreted a problem statement differently from the intended meaning. Should you address this misinterpretation immediately, or should you wait until he or she realizes it on his or her own? And what if the student is an English language learner (ELL)? How can you figure out if the different interpretation is due to the student's English or to the student's mathematical fluency? Linguists have discussed the importance of distinguishing mistakes from errors when working with language learners. Brown (2007) characterized errors as fixed habits that cannot be self-corrected. Errors stem from a lack of knowledge of language conventions. For example, a student may incorrectly say "my four dog" repeatedly because he or she may not have learned the rule regarding plural nouns. In contrast, mistakes are the result of a temporary stumble. Mistakes, also referred to as slips or lapses, can be self-corrected because they do not result from a lack of understanding (Brown, 2007). For example, if someone writes "Angie is to nice" rather than "Angie is too nice" because they were typing quickly, this constitutes a mistake because the person knows the correct form. Therefore, it is essential to determine if a student's different language use or interpretation is a mistake or an error if a teacher is to enact the proper response.

In this paper, we analyze a single episode of a teacher experiencing tensions between mathematics and language when deciding how to respond to an ELL who misinterpreted a task statement. This student's response does not fall neatly into the category of mistakes or errors (Brown, 2007) because it was not due to Henry's English structure or grammar but to his interpretation of the problem statement overall. Therefore, we use the term *misinterpretation,* rather than mistake or error, to describe this situation. In addition to our analysis of the episode, we also provide the teacher's insight into how she perceived the student's misinterpretation. By providing both a researcher's and a teacher's perspective, we hope to shed light on differing accounts and the various aspects to which observers might attend (Boaler & Humphreys, 2005). The reader might similarly examine the vignette and consider the instructional decisions he or she might make in the moment regarding the same types of situations.

Context

Henry, a third-grade Chinese student, had been living in the United States for two years when he participated in our study. He attended a public elementary school located in a small city and was identified as an ELL by his school district. Though we did not have access to Henry's English proficiency level, we noted, and his teacher confirmed, that his informal English was fluent when engaging in everyday conversation. Henry was confident in mathematics and often expressed his fondness for the subject. During our interview, Henry was eager to solve the mathematics tasks and demonstrated strong computational and problem-solving skills. However, throughout the interview, Henry typically wrote only his solution; he did not show or describe how he arrived at the solution unless we asked him to explain his thought process.

The purpose of the study from which this paper is drawn was to investigate preservice teachers' use of cognitively demanding tasks with ELLs. When searching for tasks we purposefully selected those that were not solvable by applying a simple algorithm or computation. For the present study, we modified a released item (http://ccsstoolbox.agilemind.com/parcc/elementary_3775_1.html) from Partnership for the Assessment of Readiness for College and Careers (PARCC) as follows.

> Three classes at an elementary school are going on a fieldtrip to the zoo. Mrs. Ruiz's class has 23 people, Mr. Yang's class has 25, and Mrs. Evans' class has 24 people (all numbers include the teacher). They can choose to use buses, vans, and/or cars. Buses have 20 seats, vans have 16 seats, and cars have 5 seats. You are in charge of deciding how to transport all of the classes to the zoo. Explain how you would choose how many of each type of vehicle to take and why. Write a response and explain your thinking.
>
> <u>Extension</u>
>
> 1. If there cannot be any empty seats in a vehicle, how would you choose the vehicles? Explain your strategy.
> 2. If you can only take less than five vehicles, how many different ways can you choose? Explain your strategy.

The original task only required students to find three combinations of vehicles that could be used to transport the classes to the zoo. It also included images of the vehicles and a table of the relevant data. We modified the task to increase the cognitive demand by making the task more open-ended and adding prompts such as, "Write a response and explain your thinking," and, "Explain your strategy." We also removed the images and table in the original PARCC item to maximize the capacity of modification. Before we investigated the PSTs' implementation of the task with ELLs, we piloted the modified task by enacting it with several ELLs to check if it had an appropriate level of cognitive demand for this age group.

When collecting data for that pilot study, the first author, a former mathematics teacher whose native language is not English, interviewed Henry and encountered an interesting moment. We, the authors, then transcribed the interview, thoroughly reading the transcript several times. Focusing on Henry's misinterpretation and the teacher's corresponding decisions, each author wrote analytic memos and reflections of the interview. After discussing our initial analysis, we summarized the reflections from the teacher's and the researchers' view. Although the initial purpose of Henry's interview was to pilot the task, the first author remained in the role of teacher throughout the interview though she was not Henry's classroom teacher. Hence, we refer to her perspective as the teacher's perspective and juxtapose that with the second author's researcher perspective.

In the following sections, we focus on the teacher's decision-making process. We begin with an excerpt from the teacher's meeting with Henry, and then provide an interpretation of this excerpt from the teacher's viewpoint, followed by the researcher's reflection.

Teacher Decision Moment

The following vignette begins after the teacher (denoted T) has provided Henry (H) with the modified task previously stated. Henry was rushing to solve the task as soon as he read it.

T: So is there any word you don't know?
H: No.
T: Okay. Then is there anything you don't understand in this problem?
H: There's about, the teacher included, teacher… Okay. 23 plus, [whispering
to himself] Okay. 72 people and add the teachers, 75.
T: What is 75?
H: Because I just can't, all the students first, and then just add the teachers.

T: So, you understand what you have to do for this problem, right?
H: 75, then.
T: Would you explain what you have to do in this problem to me?
H: I don't…
T: Because I do not really understand what we have to do, so you can just add…? Would you explain and help me understand?
H: Um, I found 23, and then add 24,
T: Why did you add those three numbers?
H: So they're students.
T: They're students? Not teachers?
H: Yes. And then, three teachers and then, Mrs. Ruiz, Mr. Yang, Mrs. Evans.

Reflection – You have noticed Henry double counted the teachers. How would you address this misinterpretation of the task statement, if at all (decision)? What might happen next as a result of this decision (outcomes)?

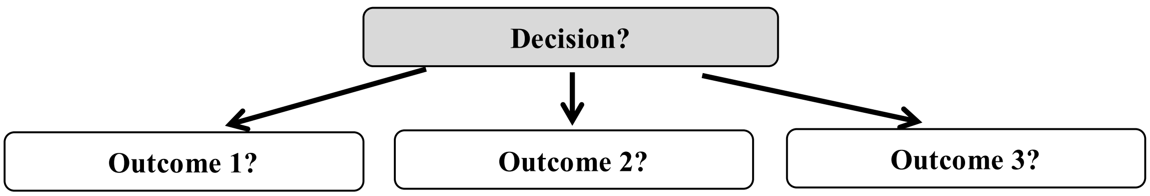


Figure 1. Possible decision and related outcomes stemming from Henry's misinterpretation.

Reflections

Teacher Perspective

At the start of my meeting with Henry, my goal was to find out if he understood all the words and the mathematical situation presented in the task. However, Henry had already started working on the problem and did not attend to my questions. I had planned to discuss what the task was about and to find an entry point together before he began to solve it because I wanted to make sure he fully understood the problem's context. Henry did not approach this task as I had planned, so I changed my plan and asked questions to address his work on the task such as, "Would you explain and help me understand?" (line 13), and, "Why do you have to add those three numbers?" (line 15). While he was responsive to these questions, I noticed he interpreted the task differently (lines 4-5). He added the number of teachers separately, resulting in his arriving at 75 people instead of 72. Hence, following the exchange above, I decided to provide some guidance in the hope that Henry would notice his double counting.

T: Could you read this sentence?
H: All numbers include the teacher is 75, so I got it, so a bus has 20 seats, so I could use… um… about… um.
T: Actually I'm not sure what this means, you know, English is not my first language, either, so I think you can help me understand "all numbers include the teacher" means you have to add three more or you don't have to?
H: Include teacher means the teachers are included.
T: Included where? Included in this number? ((points to the class totals))

28 H: Yeah.
29 T: So you have to add one more or you don't have to?
30 H: You have to add.

Initially, I had assumed Henry knew the meaning of "include," but after he insisted that 75 people were on the trip (line 20), it seemed as though he had misinterpreted it. It is possible that he had not read the sentence carefully at the start of the task because he was busy calculating numbers. He answered my question about what the sentence meant with, "Include teacher means the teachers are included" (line 26), merely repeating the sentence. I was then further convinced of a misunderstanding once I asked Henry whether he had to add the three to the sum in an effort to get him to rethink his answer (line 24), and he responded "You have to add" (line 30). Throughout this exchange Henry was confident in his understanding of and approach to the task.

At this point, I wondered if I should point out this misinterpretation to Henry. I was hesitant to tell him he had misinterpreted the task because he was confident in his understanding of English and was actively solving the task. Although stopping to clarify his misinterpretation would allow him to proceed with the intended quantities, I was afraid that it would decrease his self-efficacy (Ramdass & Zimmerman, 2008). Moreover, because his mathematical thinking was on the right track, I did not want to interrupt his problem solving process. Moreover, he may not have been ready to listen because he was very focused on solving the problem. In light of these factors, I decided to wait until he would be more responsive to listening.

Finally, I found a chance to address the meaning of the clause, "*all numbers include the teacher,"* as he worked on the second extension. In order to take fewer than five vehicles, Henry found that the 75 people could either take four buses or take three buses and a van to the zoo. I knew that using the intended amount of 72 people would yield more possibilities. I took this opportunity to take him back to the original task and reconsider the clause:

31 T: Let's go back to the beginning. Okay. This little sentence, just one sentence,
32 "all numbers include the teachers," which means the teachers are already in
33 there. One of the 24 people is the teacher. One of the 25 people is the teacher.
34 Then, 23 people, one of those are teachers.
35 H: Oh. I know. So, there's only 32 people.
36 T: 32?
37 H: Hmmm. Yeah.
38 T: What is 32?
39 H: Because I just, to show that I add them out.
40 T: So, how many people are total?
41 H: Hmm
42 T: Before, you said it's 75. Now you figured out how many people in three
43 classes?
44 H: Now. Then there are 72.

He quickly understood how this realization impacted his initial solution and changed the number of people going on the trip to 72. He then completed the final extension using 72 people and was able to, with some assistance, use a table to find all the ways to transport the people. Although I was torn over whether to intervene sooner, I did not want to stop him when he was engaged in mathematical activity because my primary goal was to support Henry's mathematical learning.

Researcher Perspective

While analyzing this situation, I first noted three important observations. First, an ELL (or any student) with sufficient mathematical capabilities could arrive at a different solution than intended because he or she misinterpreted a single clause or phrase. Second, determining whether a student's different solution to a task stems from a misinterpretation of mathematics, language, culture, or some combination of the factors is difficult. Finally, determining the most effective way to address a student's misinterpretation is challenging, particularly at the moment it occurs.

As Henry came to his initial conclusion that 75 people were going on the field trip, the teacher could have proceeded in a number of ways. For example, she could have immediately addressed Henry's misinterpretation of the clause. This intervention may have helped Henry circumvent future challenges when solving the extensions, but he may have experienced frustration because his mathematical work was overshadowed by his language misinterpretation. Alternately, she could have decided to ignore the misinterpretation completely because it was unrelated to his ability to meet the mathematical learning goal of the problem. Or, the teacher could have waited until Henry completed the task using his interpretation and then go back through the problem, asking questions such as, "What if the numbers meant the students and the teacher, would that change your answer?" Such questions may have allowed Henry to continue with the problem's intent while addressing language issues afterward.

What we see that the teacher chose a fourth approach: to wait until there was a seemingly appropriate teaching moment to address Henry's misinterpretation. The decision to focus on the context and language seemed appropriate to her in this instance because Henry was mathematically correct within his interpretation of the problem. His method was to find the total number of people and split that number into groups of 20, 16, and 5. Henry's proper approach caused her to delay addressing Henry's misinterpretation because her focus was on his mathematical thinking rather than his English vocabulary (Moschkovich, 1999, 2010). However, when working on the final extension, the teacher did intervene by telling Henry that the quantities listed contained the teachers.

In retrospect, it seems as though it would have been relatively easy for the teacher to address the misinterpretation immediately. However, it is not clear whether an earlier intervention would have resulted in Henry solving the task as intended, Henry being discouraged and losing interest as the teacher had feared, or some other outcome. Although the teacher was able to find a time to intervene, the teacher might not have addressed the misinterpretation at all if a seemingly appropriate moment had not arisen.

Discussion

Our purpose in analyzing the teacher's decision making is to encourage teachers to reflect on and consider situations when students interpret tasks differently than intended. Making purposeful decisions with regard to these instances while remaining mindful of the mathematical goals is imperative to supporting ELLs' learning. It is harder in a typical classroom setting to notice these types of instances than in an interaction with only one student. Nevertheless, teachers should keep in mind that students' misinterpretation of a single word, phrase, or clause can change their solution, so they need to pay close attention to students' reasoning process and deliberately implement strategies to uncover student's misconceptions as well as provide multiple supports to avoid the misconceptions (Sorto, Mejia Colindres, & Wilson, 2014).

Furthermore, the twofold structure of this study, attending two different perspectives of the teacher and the researcher, helped us analyze Henry's misinterpretation in depth. The teacher's perspective evidenced concern for Henry's confidence and her desire to allow him to correct his own misinterpretation, though she did ultimately intervene. The researcher voiced similar concerns, but her perspective was driven by an analysis of the pros and cons of the different approaches.

We acknowledge that every decision a teacher makes will have pros and cons; however, Ramdass and Zimmerman (2008) assert that taking a self-correction approach helps students increase their mathematical accuracy along with their self-esteem. From this perspective, waiting to intervene until Henry encountered difficulty may be appropriate. However, the best decision is probably to prevent this possible misinterpretation in advance. Henry's misinterpretation occurred during piloting tasks in which we intentionally removed all visual representations. In a classroom setting, teachers could design the task with clearly labeled visuals that show both students and teachers in classrooms as stated in the task, so students can see that the number of teachers was included in the given numbers. Another approach is using a table to show the given number information clearly. More importantly, teachers should notice the clause, "*All numbers include the teacher,"* contains semantic confusion because numbers cannot include people. Teachers could rewrite this clause to make its meaning clear, such as "There are 23 people in Mrs. Yang's class, including the teacher."

Many scholars (e.g., Coggins, Kravin, Coates, & Carroll, 2007; Moschkovich, 2002) have supported the notion of allowing students to use informal language while acquiring academic language. For example, if a student describes an angle as big rather than using the term *obtuse*, teachers can allow them to use the everyday language while reasoning and then bring in the mathematical language later. In Henry's case the word *include* was neither content nor everyday vocabulary, but a *function word* with meaning central to a task context (Echevarria, Vogt, & Short, 2010). Thus, teachers should attend to these words because ELLs need to learn these words as they become fluent in mathematical discourse (Cobb, Stephan, McClain, & Gravemeijer, 2001; Khisty & Chval, 2002; Vomvoridi-Ivanovic & Razfar, 2013).

Moschkovich (1999) suggested that focusing on correcting vocabulary or grammatical errors obscures the mathematical content in what ELLs communicate mathematically. Henry's misinterpretation of the clause impacted his answer to the task, but not his reasoning. Thus, the teacher in this study did not address the unexpected misinterpretation of the clause immediately. This implies the teacher focused initially on Henry's mathematical discourse rather than on his language misinterpretation. If ELLs experience difficulty solving a task because they are not able to make sense of the problem statement, teachers should intervene and help them understand the situation embedded in the problem (I, 2015). However, when the misinterpretation does not affect their core mathematical process, teachers can be flexible, especially during assessments. When teachers stop listening to ELLs' mathematical thinking, both parties may lose sight of the mathematical goals. We encourage teachers to consider each instance individually in attending to the unique needs of ELLs.

References

Boaler, J., & Humphreys, C. (2005). *Connecting mathematical ideas: Middle school video cases to support teaching and learning*. Portsmouth, NH: Heinemann.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *Journal of the Learning Sciences*, *10*(1 & 2), 113–164.

Coggins, D., Kravin, D., Coates, G. D., & Carroll, M. D. (Eds.). (2007). *English language learners in the mathematics classroom*. Thousand Oaks, CA: Corwin Press.

Echevarria, J., Vogt, M., & Short, D. (2010). *the SIOP model for teaching mathematics to English learners*. Boston, MA: Pearson.

I, J. (2015). *Preservice teacher learning for supporting English language learners to make sense of mathematics* (Unpublished doctoral dissertation). University of Missouri, Columbia, MO. Retrieved from https://mospace.umsystem.edu/xmlui/handle/10355/47120

Khisty, L. L., & Chval, K. B. (2002). Pedagogic discourse and equity in mathematics: When teachers' talk matters. *Mathematics Education Research Journal*, *14*(3), 154–168.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics*, *19*(1), 11–19.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning*, *4*(2&3), 189–212.

Moschkovich, J. N. (Ed.). (2010). *Language and mathematics education: multiple perspectives and directions for research*. Charlotte, NC: Information Age Pub.

Ramdass, D., & Zimmerman, B. J. (2008). Effects of self-correction strategy training on middle school students' self-efficacy, self-evaluation, and mathematics division learning. *Journal of Advanced Academics*, *20*(1), 18–41. https://doi.org/10.4219/jaa-2008-869

Schoenfeld, A. H. (2011). Toward professional development for teachers grounded in a theory of decision making. *ZDM*, *43*(4), 457–469. https://doi.org/10.1007/s11858-011-0307-8

Sorto, M. A., Mejia Colindres, C. A., & Wilson, A. T. (2014). Uncovering and eliciting perception in linguistically diverse classrooms. *Mathematics Teaching in The Middle School*, *20*(2), 72–77.

Vomvoridi-Ivanovic, E., & Razfar, A. (2013). In the shoes of English language learners: Using baseball to help pre-service teachers understand some complexities of language in mathematics instruction. *Teaching for Excellence and Equity in Mathematics*, *5*(1), 7–15.

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Consider a moment when you noticed your student made a misinterpretation either mathematically or linguistically. How did you react/interact in that situation? Are there any different decisions you could have made? How might each option have impacted the outcome?

2. What supports or opportunities would be helpful for teachers when enacting complicated mathematics word problems with ELLs?

3. What are some words that might impede students' mathematical reasoning or problem solving if they do not know the definition of the words? How would you build meaning for those words in teaching mathematics?

4. How can you create mathematical tasks that minimize possibilities of students' misinterpretations?

5. In what ways, if any, do you think the teacher's approach may have differed if she were teaching an entire class rather than one student?

A Framework for Modifying Mathematics Tasks for Accessibility

Walter G. Secada, Edwing Medina, and Mary A. Avalos
University of Miami

Abstract

For our work in *Language in Mathematics*, we developed a framework for analyzing mathematics tasks along lines of mathematics concepts, mathematics practices, contexts, and language demands. By referencing these features, we worked across our distinct academic specializations of mathematics education and language/literacy education more easily. They also helped us to draw important distinctions between task characteristics (concepts and practices) that cannot be modified without changing what is being assessed mathematically; and those that can be changed (context and language demands) as long as the changes are done with care. We share our framework, which can be used for curricular and instructional purposes, in hopes it can help other educators to work cross disciplinary areas for improving the accessibility of mathematics tasks more generally.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What is an example when the real-world context of a mathematics problem seemed to affect that task's accessibility for English language learners in your classroom?

2. What is an example when the language of a mathematics problem seemed to affect that task's accessibility for English language learners in your classroom?

3. What does the CCSSM say about #1?

4. What does the CCSSM say about #2?

Walter G. Secada (wsecada@miami.edu) is Professor of Teaching and Learning and Senior Associate Dean in the School of Education and Human Development at the University of Miami, Florida. As a mathematics educator, he has written about issues of equity in education and the education of this nation's English language learners.

Edwing Medina (e.medina4@umiami.edu) is a Ph.D. student in the Department of Teaching and Learning at the University of Miami, Florida. His dissertation is focused on first-and-second grade children's use of visual-graphic representational systems while solving addition and subtraction word problems.

Mary A. Avalos (mavalos@miami.edu) is a Research Associate Professor in the Department of Teaching and Learning at the University of Miami (FL). Her research interests include language and literacy education for K-12 emergent bilinguals, teacher preparation, and professional development.

A Framework for Modifying Mathematics Tasks for Accessibility

Walter G. Secada, Edwing Medina, and Mary A. Avalos

This article provides an illustrative summary of a four-dimensional framework that we used in creating an assessment of academic language in mathematics for *Language in Math* (*LiM*), a research and development project funded by the Institute for Educational Sciences (IES). *LiM* aimed to combine what we know about how upper-elementary and middle-school students learn mathematics with what we know about how students who speak Spanish as their first language acquire English as a later language. In *LiM*, we worked with certified grade 4-8 teachers who taught mathematics to self-contained classes that included students who had been identified by the school as "limited English proficient." At the time that *LiM* was implemented, "LEP" was Florida's terminology, though we prefer the term English-language learner (ELL). Almost all the students in our study were at the intermediate or advanced proficiency stage of learning English. The assessment of academic language in mathematics was meant to give us a sense of how changes in mathematics-relevant language would affect ELL students' performance on and reasoning about tasks that are typically administered in mathematics tests.

The Framework

Mathematical content, *mathematical practices*, *context*, and *language demands* are terms that gloss over some important distinctions found in the research literature. Yet these terms provided a good starting point for us to communicate ideas among ourselves, and now to teachers and other colleagues, without getting too bogged down in details.

Mathematical Content

A task's mathematical content is the mathematical idea(s) or concept(s) that an individual must call upon in order to solve that task. *On the Shoulders of Giants* (Steen, 1990) and the domains found in the Common Core State Standards in Mathematics (CCSSM; National Governors Association and Council of Chief State School Officers, 2010) provide ways of describing the "big ideas" of mathematics.

Mathematical content can also entail somewhat smaller-sized ideas such as place value, fraction equivalence, or linear expressions. Mathematics content may become even more narrowly focused as in "knowing that fractions, percents, and decimals are all different ways of expressing the same number" and/or "adding two fractions with the same denominator."

Mathematical Practices

A task's mathematical practices are the social and conceptual processes that an individual must often call upon to solve tasks; these may differ depending upon the task's content and context. Heuristics described in *How to Solve It* (Pólya, 1957) reflect practices one may use when solving problems, and the eight cross-cutting practices found in the Common Core State Standards in Mathematics (NGA & CCSSO, 2010) provide examples of practices being promoted for school-mathematics. The CCSSM practices have social and psychological aspects: (1) make sense of problems and persevere in solving them; (2) reason abstractly and quantitatively; (3) construct viable arguments and critique the reasoning of others; (4) model with mathematics; (5) use appropriate tools strategically; (6) attend to precision; (7) look for and make use of structure and (8) look for and express regularity in repeated reasoning.

Context

Context refers to the setting within which a mathematics task is found and which gives rise to that mathematical problem. Tasks vary in how much support their contexts provide: familiar or even personally-interesting contexts might motivate and help someone to access mathematics concepts and to engage in the mathematical practices that are needed to solve the task because they know and understand the context in which the task is embedded. Unfortunately, too many tasks incorporate contexts that create barriers and/or have no meaning for students who become confused and unmotivated (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). Computational tasks are often said to have "no contexts", though computations can also be thought of as purely "mathematical contexts."

Language Demands

Language demands refer to the language-processing demands that are placed on the reader of a task. Because we were focused on text-based assessment tasks (that include printed words, graphic representations, symbols, and numbers) that had to be seen, this dimension excluded the demands of certain language modalities (speaking, listening, writing). When extending this framework to instruction, all five language modalities (reading, writing, listening, speaking, and viewing and representing) need to be considered. For example, in Avalos, Medina and Secada (2015), we included attention to oral and graphical forms of communication in our presentation on how teachers might use visual graphics to help multilingual students access algebraic word problems.

An Example

Our goal was to create or revise assessment tasks to be more accessible to English language learners and so that we could better understand how modifications in a task's non-core-mathematical features (i.e., context and language demands) affect student performance. By comparing performance on tasks with relatively higher to relatively lower language demands, we hoped to better understand how students' academic language proficiency in mathematics affects their performance.

The following example is drawn from the Florida Comprehensive Assessment Test 2.0 (FCAT; Florida Department of Education, 2011). According to the Test Book and Answer Key, this task corresponds to Benchmark Code MA.8.A.1.1 (FLDOE, 2011), which in turn refers to Mathematics, 8th grade, Big Idea 1 (Analyze and represent linear functions, and solve linear equations and systems of linear equations), Sub-idea 1 (Create and interpret tables, graphs, and models to represent, analyze, and solve problems related to linear equations, including analysis of domain, range, and the difference between discrete and continuous data) (CPALMS, 2008).

Sami installed a 6-foot-tall cylindrical storage tank to collect rainwater from the roof of her house. She used the rainwater to water the lawn and garden during dry spells. Sami recorded the rise in the water level in her storage tank after each of 3 rainstorms. Her results are shown in the table below.

Rainfall (in inches)	Rise of Water Level in Storage Tank (in inches)
1.5	24
0.5	8
2.5	40

Which is the best prediction of the rise of the water level, in inches, in her tank after a storm produced 2.25 inches of rain?

A. 16 inches B. 28 inches
C. 32 inches D. 36 inches

Figure 1. Sample Task 1 (FCAT; Florida Department of Education, 2011)

Mathematical Content and Mathematical Practices

From sub-idea 1, above, this task requires students to "… interpret tables, … and models to … solve problems related to linear equations …." This task does not include specific mathematical practices in the sense that students are not required to demonstrate or to use them as a condition of being scored right. Any changes in either content or practice would undermine this task's validity when it is used for assessment. Hence, we kept these features of the task, while changing other task features in an effort to make it more accessible.

Context

The context for Sample Task 1 entails a large container filling with rain water for use in gardening. In Southeast Florida, rainy weather is common; water pooling in pots or containers to use for other purposes could link to the problem's context of water collection. Hence ELL students may be familiar with a task entailing water collection. On the other hand, tasks like this one are criticized for failing to motivate a need for storing rainwater in the first place and for predicting the amount of rainwater rise in a tank based on predicted rainfall.

We tried to increase the likelihood that students would be familiar with the problem context by referencing a school's garden, something that many middle schools are planting and that need to be watered regularly. We hypothesized that students would also find this setting more motivating than a context involving an unknown individual. Another alternative is simply to strip away all contexts, thereby converting this task into something that is purely symbolic.

Language Demands

Among the features that make text more difficult to read and understand are the unnecessary inclusion and/or use of:

- extraneous information, such as the location of water storage cylinder on the roof;
- overly long sentences, such as the problem question;
- technical vocabulary, such as stating that the storage container is cylindrical.[1]

In revising this task, we addressed the above language demand concerns. Also, we did not use a picture to try to reduce language load because we were not sure that ELL students would understand how the picture referred to what had been written, which is necessary for the picture to be helpful.

Result: Two Revised Tasks

Informed by our analysis, we created two revised tasks (see Figures 2 and 3).

A 6-foot-tall storage tank is used to collect rainwater which is then used to water the school's garden during dry spells. Sami recorded how much the water level rises in the storage tank after each rainstorm. Her results for 3 rainstorms are in the table below.

Rainfall (in inches)	Rise of Water Level in Storage Tank (in inches)
1.5	24
0.5	8
2.5	40

A recent storm produced 2.25 inches of rain. How much did the water in the tank rise?

A. 16 inches C. 32 inches
B. 28 inches D. 36 inches

Figure 2. Alternative Wording

Given the following relationship between x and y:

x	y
1.5	24
0.5	8
2.5	40

If $x = 2.25$, then $y = ?$

A. 16 C. 32
B. 28 D. 36

Figure 3. Purely symbolic

[1] We understand that the cistern's placement on the roof allows gravity to empty it; but so does its being placed anywhere above the ground. That the cistern is a cylinder may explain why there is a continuously linear relationship between the amount of rainfall and the rising water levels; yet the same would be true for a cube or rectangular polyhedron. Hence while correct, this information is not central to the problem's statement.

Figures 2 and 3 maintain the original task's mathematical content and non-specification of mathematical practices. Figure 2 modifies the context to be more motivating; and it modifies the language demands so that the resulting task would be more accessible to English language learners. Figure 3 removes all context and strips language demands to a minimum in case purely symbolic problems are, in fact, more accessible to ELL students.

Extension to Curriculum and Teaching

Concerns for construct validity in assessment limit our ability to modify a task's mathematical content and mathematical practice. However, no such constraints limit our ability to modify the tasks that comprise students' mathematics curriculum and its teaching.

This task could be modified by changing some combination of its mathematics content, mathematics practices, context, and language demands for purposes of curriculum and teaching. Some modifications might work alone or in tandem to make the task more accessible to ELL students; others, to make it more difficult. The changes would depend on teachers' instructional goals.

Mathematical Content

Changes in the shape of the water container could motivate exploration of non-linear functions. Instead of being cylindrical, the container could be spherical (as in the case of some containers that sit atop water towers) or even a series of pyramids and polyhedra (as in the case of swimming pools). The resulting tables would represent non-linear functions.

Switching over to real-world water containment systems, such as lakes or ponds, would require the use of some combination of shapes to approximate their volume. The resulting tables relating rainfall to the rise in the containers' water levels would be quite complex as is the case for functions that are piece-wise linear or non-linear.

For these examples, changes in context would lead to changes in mathematical content. Furthermore, the shapes of the water containers would actually matter; and hence, the task's language demands would also be affected.

Mathematical Practices

Two mathematics practices found in the Common Core are implicit.

- If students drew a picture to represent the container and sketched it filled at various levels, they could be making sense of the problem (practice #1);
- If students reorganized the table so that rainfall amounts and corresponding rises in water level were ordered from lowest to highest, they are looking for structure (practice #7);
- If students halved the amount of rainwater rise corresponding to 0.5 inches and then, either (a) built up from 1.5 inches of rain to 2.25 inches by adding the amounts of rainwater rise corresponding to 0.5 and 0.25 inches or (b) reduced the rainwater rise corresponding to 2.5 inches of rain by the amount corresponding to 0.25 inches, they are making use of structure (practice #7).

Alternate strategies for solving this task could include (a) plotting the graph (using a graphing calculator, if appropriate) corresponding to the table presented above and interpolating between 1.5 and 2.50 inches to see how much the rainwater rises in the containment structure when 2.25 inches of rain falls; (b) computing the amount of rainwater that the container rises per inch and then multiplying that by 2.25; and/or (c) deriving an equation from the table and "plugging in" 2.25 for x.

Very often, these sorts of tasks are used to teach eighth graders about the rise-over-run method of computing slope. However, an open-ended in-class discussion of how students made sense of and solved this task would allow students to engage the Common Core practices numbers 1 and 7. In addition, if classroom norms permitted, in-class discussion would encourage students to construct viable arguments and critique the reasoning of others (practice #3). If the original task were extended to the use of different shaped containments, students would have to model with mathematics (practice #4).

Context

As noted in our discussion about assessment, one could argue that the original task lacks relevance for the students and may not seem to provide a compelling reason for being solved. On the other hand, flooding devastation is one of many reasons why we might want to predict when a natural or human-made water containment structure might overflow. Yet even in the case of more localized water collection system as in the case of watering a garden, the subsequent cleanup to the container's flooding can be time-consuming and messy.

The threat of hurricanes in Florida often leads to the draining of water from Lake Okeechobee in an effort to stave off flooding (Reid et al., 2016). Similarly, floods caused by snow melt or thunderstorms take place throughout much of United States and students see the resulting damage in the news. Even though drought has plagued much of the Western United States (Pacific Institute, 2017), including Lake Mead (Worland 2016), increased rainfall has ameliorated many of those concerns and seems to be motivating questions about the impact of too much rainfall on surrounding areas and the possibility of draining some water to avoid flooding. These real-world contexts of how too much rainfall can lead to rising waters and flooding are too complex to be incorporated into students' mathematics curriculum without modification. But if modified tasks were presented in conjunction with science and social studies lessons on the environment, it may be possible to use such settings to motivate sets of tasks that, individually, are accessible to ELL students and that, in the aggregate, lead to a more sophisticated set of mathematical understandings.

Language Demands

Assessment tasks should be as easy to read and understand as possible because students must read the texts by themselves and solve the resulting problems without the social processes that provide support during instruction. Also, busy teachers cannot revise every task found in their students' mathematics books. However, it is possible for teachers to scaffold a task's language demands in anticipation of when students first read them and to be sensitive to those demands during the rapid give-and-take of a mathematics lesson. For example, teachers can discuss a text's technical vocabulary, its cultural references, and other features as part of instruction and students can create and maintain their own glossaries of unfamiliar terminology. The glossary may be further refined by similar or different uses of a particular term in other disciplines and contexts.

In planning language-focused class discussions, teachers should remember that, as general rules of thumb, for ELLs:

- passive voice is more difficult to understand than active voice;
- past and complex tenses are more difficult to understand than present tense;
- longer sentences are more difficult to understand than shorter sentences when a student has limited knowledge of the mathematical concept(s) within the text;
- when precision is required and students have some prior understanding of the concepts that are involved, technical vocabulary may actually help them to understand what is being asked because of its precision;
- when technical vocabulary provides false precision or when a student has not encountered the basic conceptual underpinnings of that terminology, technical vocabulary may render a task more difficult;
- a picture may be "worth a thousand words", but students have to understand what the various components of mathematical illustrations refer to in order to make use of them;
- mathematical symbols place their own unique demands on someone's ability to read and to understand the information that a task provides and what is being asked.

Concluding Comments

Figure 4 provides a visual summary of our Framework and some of the salient issues that arise when thinking about its utility for classroom instruction.

Dimension → Application ↓	Mathematical Concepts	Mathematical Practices	(Mathematical) Context	Language Demands (Math, Academic, and Everyday)
Classroom Assessment	Cannot be modified without raising validity issues	Cannot be modified without raising validity issues	Can be modified, with care, to motivate and to increase text comprehensibility	Can be modified with care to increase text comprehensibility; mathematics terminology can provide precision
Curriculum and Teaching	Can be modified with care to related concepts, depending on specificity	Can be modified with care to related practices, depending on specificity	Can be modified, with care, to motivate, to increase text comprehensibility, and/or to extend to new concepts or new practices	Can be modified with care to increase text comprehensibility; mathematics terminology can provide precision

Figure 4. Annotated Framework

From the *Language in Math* project we learned a lot about the challenges of meaningfully teaching mathematics to ELL students in ways that allow students to understand mathematics and that are consistent with the standards set out in the CCSSM. Being from different disciplines -- mathematics education and language and literacy education -- we learned that we had to develop ways of communicating with one another so that we were talking about the same things; for example, what mathematics educators mean by semantic structures of arithmetic word problems (Secada & Carey, 1990) is quite different from what language and literacy educators mean. We created this framework as a first step in organizing our own work around complexity of mathematical language found in tasks, and of fostering communication among ourselves. Through this article, we are taking some first steps in sharing that framework with teachers and other educators in the hopes that they, too, find this helpful.

Acknowledgment

This work was made possible by Award Number R305A100862 from The Institute of Educational Sciences. The content of this chapter does not necessarily reflect the views or policies of IES or the U.S. Department of Education nor does mention of trade names, commercial products, or organizations imply endorsement by the U.S. Government. One author received funding from a University of Miami Provost Research Award. We acknowledge the collaborative efforts of our student-colleagues (Naomi Iuhasz, Kristen Doorn, Alain Bengochea, and Michael Kucharczyk) and of district personnel (teachers, students, and administrators) who contributed to this project.

References

Avalos, M. A., Medina, E. & Secada, W. G. (2015) Planning for instruction: Increasing multilingual learner's access to algebraic word problems and visual graphics. In A. Bright, H. Hansen-Thomas, & L. de Oliveira (Eds.). *Common Core State Standards in Math, High school* (Chater 2, pp. 5 – 23). Annapolis Junction, MD: TESOL Press.

CPALMS (2008). *Mathematics Standards* (Florida mathematics benchmarks and access points). http://www.cpalms.org [retrieved January 2010]

Florida Department of Education (2011). *FCAT 2.0*. Tallahassee, FL: Author. http://www.fldoe.org/accountability/assessments/k-12-student-assessment/history-of-fls-statewide-assessment/fcat-2-0/sample-questions-answer-key.stml [retrieved January 31, 2017]

Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. *Journal for Research in Mathematics Education, 44*(4), 646-682.

National Governors Association (Center for Best Practices) and Council of Chief State School Officers (2010). *The common core standards-mathematics*. Washington, DC: Authors. http://www.corestandards.org/Math/ [retrieved January 31, 2017.]

Pacific Institute (2017). *California drought: Impacts and solutions*. http://www.californiadrought.org/ [retrieved January 31, 2017)

Pólya, G. (1957). *How to solve it.* Garden City, NY: Doubleday.

Reid, A., Uraizee, I., and Zhu, Y. (2016, March 11). *A draining problem: How the release of Lake Okeechobee floodwaters is dirtying Florida's coastline* . Sun Sentinel http://interactive.sun-sentinel.com/lake-okeechobee-flooding/ (retrieved January 31, 2017)

Secada, W. G., & Carey, D. A. (1990). *Teaching mathematics with understanding to limited English proficient students* (Urban Diversity Series No. 101). New York: ERIC Clearinghouse on Urban Education.

Steen, L. A. (Ed.) (1990). *On the shoulders of giants: New approaches to numeracy.* Washington, DC: The National Academies Press.

Worland, J. (2016, May 23). Largest U.S. water reservoir shrinks to record low amid major drought. *Time*. http://time.com/4344983/lake-mead-water-reservoir-drought/ (retrieved January 31, 2017)

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Pick a specific mathematics problem and discuss how the eight mathematical practices of CCSSM might play out in its exploration.

2. What is an example of a mathematics problem that you needed to revise (either before or after using it with students)? Describe the process or nature of your revision.

3. How would eighth-graders in the city in which you teach relate to the tasks found in Figures 1, 2, and 3 for assessment purposes? For instructional purposes?

Taking a Stand for Humanity

TODOS: Mathematics for ALL is an organization that seeks to create a more just, humanizing and equitable mathematics education experience for all. Regardless of your political views, we cannot let our differences overshadow our humanity toward each other. We recognize that the current political climate may affect how we move forward as a people that value democracy and justice for all. We must find strength and resolve to reach out to people hurting, scared and uncertain of their futures. We must find ways to support educators to hold space for listening, emotions, and deeper understanding. We have much work to do.

We reiterate here our TODOS mission and goals. In the present political climate, we interpret these as including the following:

- Respecting and incorporating into our mathematics programs, the role language and culture play in teaching and learning mathematics.
- Supporting teachers who need help navigating the political and emotional situations occurring daily in their classrooms.
- Generating and disseminating knowledge that supports our mission of advocacy for all students.
- Informing the public and influencing educational policies that protect our students and enhance the educational experiences of all of our students. Informing families about the opportunities available to their children and working continuously and ardently to enable these children to become mathematically proficient.

As mathematics educators, we will continue to stand with our students and their families, advocate for them and affirm their futures. [Published November 2016 on TODOS Website]

Taking a Stand for Humanity … Continued

TODOS in its "Taking a Stand for Humanity" statement after the presidential election presented our belief that we must work to create a more just, humanizing, and equitable mathematics education experience for all. Education happens within a context. One such context is the political climate in which children exist. Regardless of political views, we cannot stand by and watch while the civil rights guaranteed by the Constitution of the United States are swept aside by executive orders. To reiterate, "We must find strength and resolve to reach out to people hurting, scared, and uncertain of their futures. We must find ways to support educators to hold space for listening, emotions, and deeper understanding. We have much work to do."

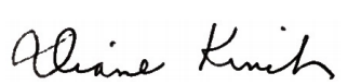
We at TODOS believe in the following:

- Human Rights. We support the rights of immigrants and others who are being attacked.
- Mutual Respect. We support the rights for all and denounce rudeness, divisiveness, and spite that are becoming the norm.
- Science. We believe in the laws of science that are being attacked by powerful people.
- Social Justice. We challenge "the roles power, privilege, and oppression play" in our society. (From the TODOS and NCSM joint position statement on Social Justice in Mathematics.)

As Robert Frost said in his poem, Mending Wall: *"Something there is that doesn't love a wall, that wants it down."*

We at TODOS pledge to continue our work to support all children and to tear down the walls that prevent them from reaching their full potential. We will continue to speak out when prejudice outweighs justice.

[Published February 2017 on TODOS Website]



Diane Kinch
President, TODOS: Mathematics for ***ALL***

A Call for a Collective Action to Develop Awareness: *Equity and Social Justice in Mathematics Education*

During the 2016-2017 school year, educators from around the country and possibly the world were engaged in *A Call for a Collective Action to Develop Awareness: Equity and Social Justice in Mathematics Education*.

Purpose: A year dedicated to building our collective knowledge and understanding of topics and issues related to Equity and Social Justice in Mathematics Education

1. Monthly readings
   a. Each organization will identify a key reading (book, collection of published articles, white papers) for ALL to read
   b. Group will develop a guiding set of questions to focus the year of reading
   c. Start reading September
2. Quarterly webinars (Dates and Times will be available by October 1)
   a. One-hour webinar – November, February, May, and August
   b. 15-20 minutes overview/key take-away(s), considerations, and/or questions of reading for each of the previous months
3. Face-to-Face informal conversations
   a. As organizations hold their national conferences/meetings - one morning or evening hour be set aside for those to gather and talk

Contributing Organizations

Association of Mathematics Teacher Educators (AMTE) https://www.amte.net/
Association of State Supervisors of Mathematics (ASSM) http://www.statemathleaders.org/
Benjamin Banneker Association, Inc (BBA) http://bannekermath.org/
California Mathematics Council-South (CMC-South) http://www.cmc-south.org
Journal of Urban Mathematics Education (JUME) http://education.gsu.edu/JUME
National Council of Teachers of Mathematics (NCTM) http://www.nctm.org/
National Council of Supervisors of Mathematics (NCSM) http://www.mathedleadership.org/
North American Study Group on Ethnomathematics (NASGEm) http://nasgem.rpi.edu
TODOS: Mathematics for ALL http://www.todos-math.org/
Women and Mathematics Education (WME) http://www.wme-usa.org
Robert Berry, University of Virginia

For information on the monthly readings and webinars, go to ***http://www.todos-math.org/a-call-for-collective-action***.

TODOS Resources for Educators

Teaching for Excellence and Equity in Mathematics Journal
Refereed Journal
(2009, 2010, 2011, 2012, 2013, 2014- 2015, 2016, 2017)

TODOS Research Monographs
(2008, 2010, 2014)

Bibliography of Diversity and Equity in Mathematics Education
1st Edition (2004), 2nd Edition (2007)

NOTICIAS de TODOS—Semiannual Newsletters (electronic)
(Spring 2005 - present)

TODOS: Mathematics for ALL Electronic News (Enews)
(Monthly since 2008)

TODOS Live!
An Interactive Webinar Series

Mathematics Education Through the Lens of Social Justice:
Acknowledgement, Actions, and Accountability
A Joint Position Statement with the National Council of Supervisors of Mathematics (2016)

TODOS 2014 Conference: Beyond Awareness ~ Equity, Access and Achievement for ALL
Presentation slides, handouts, and program book on website

TODOS 2016 Conference: Ensuring Equity and Excellence in Mathematics for ALL
Access to presentation slides, handouts, and program book

See ***www.todos-math.org*** to join and have access to all resources.

2016-17
ELECTED LEADERSHIP

APPOINTED LEADERSHIP

TODOS 2018 CONFERENCE

Save the Date!

Next year!

June 21 - 23, 2018
Scottsdale Plaza Resort
Phoenix Metropolitan Area

TODOS
Mathematics for ALL

It's ALL about ALL Students Learning Quality Mathematics: Advocating for Equity and Social Justice

TODOS
Mathematics for ALL

A Mathematics Equity Organization

Equitable, rigorous, and coherent mathematics!

http://www.todos-math.org

www.todos-math.org

The publications of *TODOS* present a variety of viewpoints. The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official position of *TODOS*. *TODOS* is a 501(c)(3) non-profit organization.