Vol. 6, No. 1 2014-2015 ISSN 2153-0173 online

Teaching for Excellence and Equity in Mathematics

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Mathematics for ALL

An Affiliate Organization of the National Council of Teachers of Mathematics

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TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS
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The mission of TODOS: Mathematics for ALL is to advocate for equity and high quality mathematics education for all students — in particular, Latina/o students.

TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

IN THIS ISSUE

From the Editors

The *TEEM* editors are happy to present this issue of *TEEM* for the 2014-15 school year, the sixth issue of *TEEM*. Editor Marta Civil (now back at The University of Arizona) was joined by Luciana C. de Oliveira (University of Miami) as co-editor of *TEEM*, starting with this issue. The journal is a vehicle to provide a scholarly and pedagogical resource for mathematics educators, practitioners, leaders, and administrators at all levels. *TEEM* uses a rigorous double-blind review process to ensure that a paper is judged on its merits without the external reviewers (or even the Editorial Panelist coordinating that paper's external reviewers) knowing the identity of the author and vice-versa. For information on reviewing or writing for *TEEM,* please see page 6 of this issue or the *TEEM* webpage ***http://www.todos-math.org/teem.*** On that webpage, you will also find a link to a webinar on writing and reviewing for *TEEM*.

The current issue of *TEEM* includes an invited article and three externally peer-reviewed articles. The issue starts with Rachael Kenney and Luciana C. de Oliveira's article (accepted prior to Luciana's starting her work as co-editor) entitled "A Framework for Connecting Natural Language and Symbol Sense in Mathematical Word Problems for English Language Learners." Kenney and de Oliveira focus on the multiple semiotic systems of the language of mathematics and the challenges they present for English language learners (ELLs), offering a framework that connects mathematical word problem solving stages to multiple semiotic systems while providing elements of symbol sense that ELLs can develop in order to work with mathematical word problems.

The second article is by Linda Arnold and Patricia Davis-Wiley, "Preparing Teacher Candidates to Work with English Language Learners." This qualitative study, based on interviews with 16 instructors of mathematics methods courses for preservice teachers, examined instructors' reported classroom practices regarding helping teacher candidates learn to work with ELLs in mathematics. Larry Lesser's invited article "Learning Language: A Mathematics Educator's Reflection on Empathy and Privilege" describes his journey of cultivating empathy -- from personal perspective to professional development. It was submitted after he stepped down from being a co-Editor and was single-blind reviewed by three reviewers.

Then Ksenija Simic-Muller in "From 'Eye-opening' to Mathematical: Helping Preservice Teachers Look for Mathematics in Stories of Oppression" provides an analysis of preservice teachers' reflections about a visit to a campus event focused on injustice and oppression that they were required to attend as part of an assignment in a mathematics content course for preservice K-8 teachers. As is always the case if a paper's author has any *TEEM* affiliation, the review process was structured to keep the author completely out of the review and decision-making process.

TEEM gratefully acknowledges the support of all the leaders in our sponsoring organization, TODOS: Mathematics for ALL. We hope *TEEM* continues to serve the TODOS membership, and provides an inspiring pedagogical and scholarly resource for the broader mathematics education and education communities.

Marta Civil
The University of Arizona

Luciana C. de Oliveira
University of Miami

Call for Manuscripts

We encourage the submission of manuscripts that are aligned with the mission of TODOS: Mathematics for ALL (see p. 2). Manuscripts in applied or action research, literature surveys, thematic bibliographies, commentary on critical issues in the field, professional development strategies, and classroom activities and resources are encouraged and welcome.

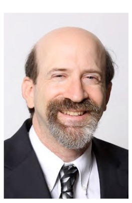
Please see **http://www.todos-math.org/teem** for guidelines and then submit complete manuscripts to **teem@todos-math.org.** *TEEM* Editors welcome query emails on the suitability of topics or approaches.

Call for Reviewers

Refereeing is not only a valuable experience and service to the profession, but is also an essential means to ensure that articles of high quality and relevance are published in a timely manner. To be eligible to be a reviewer (normally one manuscript per year), we invite you to send an email to **teem@todos-math.org** with the following information:

- Full name, affiliation, and contact information (including email, phone number, fax number, and mailing address);
- Grade levels (e.g., elementary, middle, secondary, college) where you have teaching or research experience; and
- Thematic areas with which you have particular interest and expertise, and any other pertinent professional information.

Your information will assist the editors in assigning papers to the various reviewers.



TODOS LIVE! Webinar Available: "Reviewing and Writing for TEEM"

On July 22, 2013, Lawrence Lesser conducted a live webinar that explored the big picture and process for reviewing and writing for *TEEM*. The target audience includes classroom teachers, coaches, administrators, curriculum coordinators, professional developers and university/college faculty. To access the recorded webinar, see ***http://www.todos-math.org/teem.***

A Framework for Connecting Natural Language and Symbol Sense in Mathematical Word Problems for English Language Learners

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Abstract

Working fluently within the multiple semiotic systems of the language of mathematics requires developing strong symbol sense and connecting meaning of symbols to meanings in natural language. Challenges can exist for English language learners (ELLs) when connecting natural language and symbolic representations, particularly in the context of a mathematical word problem. This article presents a framework that connects mathematical word problem solving stages to multiple semiotic systems while providing elements of symbol sense that ELLs can develop in order to work with mathematical word problems.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What challenges does mathematics language present to ELLs?
2. How is language used differently in mathematics than in other content areas?
3. What resources do ELLs bring to the classroom that can aid in their learning of mathematics?

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A Framework for Connecting Natural Language and Symbol Sense in Mathematical Word Problems for English Language Learners

Rachael H. Kenney, Purdue University

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Mathematics can be considered a language in itself, composed of natural language and a symbolic system of mathematical signs, graphs, and diagrams (Drouhard & Teppo, 2004). The learning of mathematics is heavily dependent on both the symbolic language of the discipline (including syntax and organization of symbols) and the natural language of instruction (including discourse practices specific to this discipline) (Crowhurst, 1994; Moschkovich, 2007). Halliday (1978) describes languages as *semiotic systems*, systemic resources for meaning-making. In a semiotic system, we understand what is being expressed based on prior experiences with that system. Working fluently within or between multiple semiotic systems such as natural and symbolic languages requires developing strong *symbol sense*, which includes having an awareness that one can successfully create symbolic relationships which represent written information; experiencing different roles played by symbols; and appreciating the power of symbols to display and explain relationships expressed in natural language (Arcavi, 1994, 2005).

Research and personal experiences tell us that the complexity of working in multiple semiotic systems in mathematics presents challenges for all learners. There are, however, additional linguistic demands for English Language Learners (ELLs) that make developing symbol sense and transitioning between the symbolic and natural language even more of a challenge, as they learn to filter their existing and developing knowledge of mathematical language through a second natural language (Brown, 2005). This paper focuses on these additional challenges for ELLs by analyzing the potential linguistic difficulties that may exist when connecting natural language and symbolic representations in mathematics, particularly in the context of a mathematical word problem.

Challenges in Mathematics for ELLs: The Case of Word Problems

An examination of ELL and non-ELL performance in mathematics shows small gaps for strictly computational problems, but large gaps on word problems and problems that contain linguistically complex terms (Abedi, 2004). An interplay between symbolic and natural language is clearly present when solving mathematical word problems where students must be able to decode not only the language of the question and the overlaying context, but must also have knowledge of and be able to represent words with the mathematical symbols needed to effectively answer the question. It is clear that many students (both ELLs and non-ELLs) encounter difficulties with this connection between words and mathematical symbols in word problems (e.g., Reed, 1999). Some studies, however, have discussed the additional complexity that exists for ELLs when working with mathematical word problems (Celedón-Pattichis; 2003; Martiniello, 2008). While trying to work within a second language of English, ELLs must negotiate the ways in which a "third language" of mathematics symbolically represents a given problem (Brown, 2005).

Some reasons suggested in the literature for added difficulties for ELLs on word problems include: a lack of built-in contextual clues found in literary narratives (Carey, Fennema, Carpenter, & Franks, 1995), unfamiliar cultural contexts and interpretations (Solano-Flores & Trumbull, 2003), reading comprehension issues (Schleppegrell, 2007), the artificial contexts of word problems (Wiest, 2001), and other issues (Celedón-Pattichis, 2003). Many suggestions have been offered for helping ELLs work with word problems, including helping students recognize and understand keywords (e.g., more than, take away, of, per, total, etc.), modifying the language complexity of the problem, and using manipulatives (Aguirre & Bunch, 2012). Another suggestion that has been found effective for helping ELLs in mathematics is to make use of the ideas and skills that they bring with them to the classroom. This can include assessing prior knowledge to determine an ELL's familiarity with a context, planning for the use of multiple tools and models (e.g., visuals, diagrams, gestures) by both the student and the teacher (Ramirez & Celedón-Pattichis, 2012), and using the language and cultural tools that an ELL brings to the classroom as resources for learning (Celedón-Pattichis & Ramirez, 2012).

These useful suggestions for helping ELLs may be easier to implement in some problems than in others. Consider, for example, the first part of a constructed response word problem selected from 6th grade sample items provided by the Indiana Statewide Testing for Educational Progress (ISTEP+) Grades 6 through 8 (Indiana Department of Education, 2012):

> Sue bought 4 rings for her mom. Each ring cost the same amount of money. The total cost was $31.What is the cost per ring?

This problem contains common keywords that might allow an ELL to recognize that the cost per ring is the total cost divided by the number of rings. The language is relatively simple, and knowing what "rings" are is not key to the solution of the problem. We need recognize only that we have four things that each cost the same and we spent \$31 in total.

Now consider another problem from the same set of sample items (Figure 1) (Indiana Department of Education, 2012). An examination of this problem shows that although some common keywords like "more" and "left" are found in the text, they are not as easily transferrable to mathematical symbols as they might be in the first example. We suggest that a problem like this requires a deeper understanding of how the larger sentence structure connects to mathematical symbols. We will revisit this problem in more detail below to look carefully at potential ways in which ELLs may struggle or succeed in working with it. We recommend that readers take a moment before continuing and solve this problem on their own, thinking carefully about the understanding of both the natural and symbolic languages needed to solve it.

Grade 7 Constructed Response Item

(Alg. & Functions/Problem Solving)

Irene spent half of her weekly allowance playing miniature golf. To earn more money, her parents let her wash the car for \$4.

Write an equation that can be used to determine Irene's weekly allowance (a) if she has \$12 left after washing the car.

Figure 1. ISTEP+ Mathematics Sample Grade 7 Item

A Framework for Looking at Word Problems

We know that when working with mathematical word problems, ELLs need to access the language of mathematics through multiple semiotic systems that fulfill different functions: (a) natural language introduces, contextualizes, and describes a mathematical problem; (b) symbolism is used for finding the solution of the problem; and (c) visual images deal with visualizing the problem graphically or diagrammatically (de Oliveira & Cheng, 2011; O'Halloran 2004, 2005). All of these systems may involve vocabulary, sentence structures, contexts, and representations that are new or unfamiliar to ELLs (Martiniello, 2008); we have chosen to focus primarily on the first two in this paper to examine difficulties with connecting natural language and symbolic representations in word problems. We also know that in most mathematical problem solving situations we can break the solving procedure down into different *stages* which may include: formulating the problem from a real-world application, solving the problem using some form of mathematical representation (symbolic, graphical, etc.), and interpreting and checking the solution in the context of the real-world situation. When working on a mathematical problem, learners call upon the semiotic systems in different ways at different stages of problem solving.

Multiple Semiotic Systems: Natural Language and Symbolism

Natural language use in mathematics is characterized by the dominance of relational processes presented through verbs that show relationships, such as *be, have,* and *represent*, and the frequent use of nominalizations, the expression as a noun or nominal group of what would in everyday language be a verb, adjective, or conjunction (e.g., *multiplication*, *exponent*). For example, in the following grade 6 ISTEP+ test item, "What is the area, in square feet, of a circle with a diameter of 8 feet? Use 3.14 for pi," the relational process is used in the question along with the nominalization *the area of a circle with a diameter of 8 feet.* (Indiana Department of Education, 2012).

Mathematical content is presented using natural language to carry forward the argument (O'Halloran, 2000). Making sense of the natural language in a word problem is something with which ELLs have commonly been seen to struggle (de Oliveira & Cheng, 2011; Martiniello, 2008).

Symbolism is used in mathematics for the solution process (O'Halloran, 2000). This semiotic system is often a cause of great confusion for all students due, in part, to the multiple ways in which symbols are used. For example, symbols name, label, signify, communicate, simplify, represent, reveal structure, and display relationships (Arcavi, 1994; Kinzel, 1999; Pimm, 1995; Stacey & MacGregor, 1999). For example, when stating the often used Pythagorean Theorem, instead of continually making the cumbersome statement "the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two adjacent sides," we label the sides of the triangle as *a, b*, and *c* (see Figure 2) and simply state $c^2 = a^2+b^2$. Here symbols make it much easier to quickly communicate and display the geometric relationship. Symbols also play multiple roles within a single mathematical statement, acting as generalized numbers, arguments of a function, parameters, unknown numbers, and variables (Usiskin, 1988). For example, in the symbolic representation for an equation of a circle, $x^2+y^2 = r^2$, r represents the radius of the circle and is a constant or parameter for the equation, while x and y are variables. In the equation $50 = 5x$, we can think of x as an unknown number rather than a variable because it can have only one value here. These numerous roles played by symbols make matters even more complicated as ELLs try to make connections between the language used to describe a mathematical problem and the symbols required to solve the problem. These symbols may exist across many different languages and understanding them is challenging regardless of one's native language, but ELLs need to draw on knowledge of a language they are still developing in order to use symbols when transitioning from words to symbols.

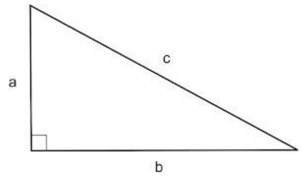

Figure 2. Using Symbols As Labels on a Triangle For the Pythagorean Theorem.

When working with mathematical word problems, the interaction with natural language occurs typically in the formulation and interpreting/checking stages of problem solving while the solving stage is heavily dependent on an interaction with and manipulation of symbols (including numbers and/or letters) or analysis of visual representations. Thus, the symbolic language and visual image systems (and the links between them) play a primary role in this stage.

A Focus on Symbol Sense

Within the natural language and symbolic semiotic systems, we can identify one common important element that we choose to focus on in this paper: *symbol sense*, which Arcavi (1994) describes as "a quick or accurate appreciation, understanding, or instinct regarding symbols" (p. 31) that is involved at all stages of mathematical problem solving. Kenney (2008) has used a symbol sense framework (constructed using adaptations of work by Pierce and Stacey (2001, 2002) and Arcavi (1994, 2005)), to investigate students' reasoning with mathematical symbols at different problem solving stages. In this paper, we have modified this framework to connect the problem solving stages to the semiotic systems and highlight the elements of symbol sense that ELLs may need to work with mathematical word problems (Table 1).

Applying Framework to a Standardized Test Item

In this section, we use a standardized test item from the ISTEP+ Grades 6 through 8 (Indiana Department of Education, 2012) to identify the potential challenges for ELLs. In the current era of teacher accountability, we know from experience that teachers are drawing heavily on sample items and practice tests from the end-of-year exams that their students will take to help prepare students for these tests. The problem we have chosen to discuss here (see Figure 1) was purposefully selected from a sample test bank

Table 1
A Framework for Symbol Sense for English Language Learners (ELLs)

Problem Solving Stage	**Semiotic Systems**	**Examples of Symbol Sense Required**
Formulation	Making sense of the natural language; Linking the natural language and symbolic systems	Knowing how and when to use symbols
		Knowing that symbols play different roles in different contexts
		Ability to select possible symbolic representations
		Knowing that chosen representation can be abandoned when they are not working
Solving	Working within the symbolism system	Recognizing conventions and basic properties Knowing meaning of symbols
		Knowing order of operations
		Knowing properties of operations
	Linking the symbolism and visual image systems	Knowing when to abandon symbols for other visual approaches
		Knowing meaning of symbols in a visual representation (e.g. labels)
		Linking key features
Interpretation and Checking	Linking back to the natural language system; Making meaningful sense of how the result connects to the original question	Linking symbol meanings to personal expectations Linking symbol meanings to the problem
		Using symbols to communicate results

because it represents a similar linguistic structure as textbook word problems that de Oliveira and Cheng (2011), as part of a larger study on the linguistic challenges of mathematics, found to be particularly difficult for ELLs in the classroom. In this paper, we apply our framework to show how the different semiotic systems connect within a mathematical problem of this type and how the resources of natural language and symbols are employed in its construction. The linguistic complexity of these types of word problems, as explained in de Oliveira (2012), makes them more likely to pose challenges for ELLs. We, therefore, explain some challenges that ELLs face in particular.

Analysis of Part 1 of the Task

An analysis of the word problem in Figure 1 shows that natural language and symbols are interconnected in students' possible solutions. This test item has two sets of tasks that students are to complete, one starting with *Irene spent half*... (we will refer to this as Part 1) and the other starting with *This week Irene used*.... (Part 2). In Table 2, we break down Part 1 and connect the framework for symbol sense to a linguistic analysis of the different clauses in the task.

Part 1 begins by introducing a context for the situation. The concept of weekly allowance is introduced in the first clause, which may cause difficulties for ELLs who may not be familiar with this concept and may not recognize it as an amount of money. In the same clause, we also see the word *half* which students have to connect to the symbolic representation ½. The second sentence begins with a clause that indicates purpose, *To earn more money,* so ELLs have to make the connection between earning more money and the following clause, *her parents let her wash the car for $4.* This clause structure is complex because ELLs have to understand that Irene would <u>receive</u> $4 <u>per</u> car wash and that she washes only one car; this is never stated in the problem but is implied in the construction of the clause. The task to complete is given in the clause *Write an equation that can be used to determine Irene's weekly allowance (a) if she has $12 left after washing the car*. This clause gives a command with the verb *write* and what it is that students are supposed to write, *an equation*. Further information is provided about *an equation* with an embedded clause *that can be used to determine Irene's weekly allowance (a) if she has $12 left after washing the car*. We notice here that the variable is provided through the symbol *(a)* referring to *Irene's weekly allowance,* which could present an additional challenge for ELLs. This symbol has to be used in the construction of the equation, as this test may be completed on a computer that would not recognize if another symbol, such as *(x),* were used instead. The conditional clause *if she has $12 left after washing the car* is another important piece of information for students to consider. *If* clauses are very common in mathematics and have been found to cause particular difficulties for ELLs (Fernandes, Anhalt, & Civil, 2009; Martiniello, 2008). Making sense of the phrase *$12 left* includes understanding that the word *left* means

Table 2
Application of Framework and Linguistic Analysis of Part 1 of the I-STEP Problem

Clause in Problem	Problem Solving Stage	Symbols Involved	Linguistic Analysis: What is required linguistically to work with the problem?
Irene spent half of her weekly allowance playing miniature golf	Formulation	1/2	Linking the natural language expressed in the word *half* to the numeric representation ½ Making sense of the concept of *weekly allowance* being an amount of money
To earn more money, her parents let her wash the car for $4.	Formulation	+4	Making sense of the concept of earning as meaning to receive more Linking receiving more to addition operation Making meaning by inferring that she will only wash the car once
Write an equation to determine Irene's weekly allowance, *a*, if she has $12 left after washing the car.	Formulation Interpretation	$a - \frac{1}{2}a + 4 = 12$ or $\frac{1}{2}a + 4 = 12$ $a = 2(12 - 4)$	Making sense of keywords in the phrase *$12 left* as indicating that this amount is what remains from allowance after washing the car Recognizing that she earned the extra 4 dollars *after* she had spent half (it was not $\frac{1}{2}(a+4)$). Interpreting the produced equation with the natural language to check the meaning of what they produced

remaining and understanding that the equation does not necessarily require a subtraction. For example, a student could write the equation (1/2)a + 4 = 12 directly if he or she interprets (1/2)a as already representing what is *left*.

Students may start to symbolize this problem by writing "a =…" since they are told to *Write an equation that can be used to determine Irene's weekly allowance.* However, determining the right-hand side of this equation involves a potentially complex interaction with the problem's natural language to mentally undo the actions on a, which adds to the complexity of a problem like this. That is, to come up with the equation $a = 2(12 - 4)$, the learner would need to work with the values in a different way than how they are presented in the problem. In this instance, it may be easier to think of a as part of what is being manipulated in the equation and not the result. This allows for a more direct translation from the natural language to the symbolic form, which lessens the challenges inherent in this translation process for ELLs.

We see in Table 2 that Part 1 asks students to engage mostly in the formulation stage of problem solving to move from the natural language system to the symbolism system. Some interpretation should also be used to check the meaning of the produced equation against the original natural language. The symbol sense for selecting appropriate symbols to use is done, in part, for the learner by telling them to use (a) for allowance. However, to set up the equation, students must fit this symbol into the larger symbolic representation. It is possible that students could determine the value of the allowance by doing mental computations, but

Table 3
Application of Framework and Linguistic Analysis of Part 2 of the I-STEP Problem

Clause in Problem	**Problem Solving Stage**	**Symbols Involved**	**Linguistic Analysis in relation to symbols sense: What is required to work with the problem?**
This week Irene used her allowance to buy each of her 5 friends a bracelet…	Formulation	5 friends 1 bracelet per friend	• Making meaning of the thing that is being bought versus the recipient of the action (Reading directly it looks like we could be buying friends instead of bracelets) • Linking the natural language expressed through the word *each* with the number 1
…and still had \$3 remaining.	Formulation	\$3	• Linking the word remaining to an idea that 3 dollars is what is left after spending (i.e., the result of buying bracelets). •
Each bracelet cost the same amount of money.	Formulation	Could let b = cost of one bracelet	• Linking the word each to understand that we can select one variable to represent any one of the bracelets
What was the cost of 1 bracelet?	Solving Formulation Solving Interpretation/Checking	1st solve ½ a +4 = 12 from Part 1 Create equation: $16 - 5b = 3$ Solve: $b = 13/5 = 2.6$	• Linking the word *cost* to the expectation that the result will be an amount of money • Recognizing an action that is not explicitly stated in the problem • Knowing that to find the cost of one bracelet, we have to find the total cost of all five bracelets and then divide. • Understanding that finding weekly allowance is not what the question is asking – need also to find a new amount of money, the cost of one bracelet by subtracting the cost of all bracelets from the total allowance and knowing that there will be \$3 left over • Interpreting the equation against the natural language to check the meaning of equation • Interpreting the result 2.6 as \$2.60 to connect to the context in the natural language

this problem is structured in a way that it requires them to be able to symbolize an equation using the letter *a*, so students must draw on multiple symbol sense elements to complete the task.

Analysis of Part 2 of the Task

In Table 3, we continue with a breakdown of Part 2 of this task. Part 2 begins by identifying the time of the next situation– *This week*. In the clause, *This week Irene used her allowance to buy each of her 5 friends a bracelet* we see how Irene used her allowance, but the construction in this clause may cause difficulties for ELLs because *each of her friends* is put before *a bracelet*. The next sentence, *Each bracelet cost the same amount of money* establishes an important piece of information for students to solve the problem. ELLs have to connect the word *each* with the numerical representation 1, and recognize that the same variable or letter can represent every bracelet. The question *What was the cost of 1 bracelet?* shows what students need to be able to calculate.

In Part 2, students are required to go through multiple problem solving stages, though not all are explicit in the problem itself. This may cause additional difficulties for ELLs. They must first determine, using their equation from the first part, the actual value of a typical week's (and therefore "this week's") allowance. This involves proceeding through the solving stage. Here students must know the order and properties of operations for "undoing" the equation to get *a* by itself. Once the value of *a* is identified to be \$16, however, students must know that the letter *a* is no longer necessary in their work. They need to know the meaning of this variable *a* as representing an unknown that, once determined, will not change again. Links may also be made back to natural language if the students try to interpret or check their solution.

Once they have found *a*, students need to be able to find the cost of one bracelet. The directions to *show all work* require the use of symbolization or visual images (i.e., mental computation will not suffice), so students must again engage in the formulation stage. Students may or may not choose to select a symbolic representation for the cost of a bracelet, such as *c* or *b*. Unlike the first part, they are not given directions on how to symbolize here. The symbolic representation $b = (16 - 3)/5$ can be used to find the solution, so only numbers are involved in the calculation. However, difficulties could arise, especially for ELLs, because the order in which the calculations need to occur is not the same order in which these values appear in the problem. This could potentially be problematic for ELLs, as they would have to figure out the order of the values by understanding the language that is expressing these values.

Implications for Classroom Teachers and Mathematics Teacher Educators

Teachers and teacher educators know well that the complexity of mathematics language presents challenges for *all* learners, and not just ELLs. As the example shows, however, being able to understand how different semiotic systems are used in the construction of a word problem and how to transition among these systems in solving a problem may present additional challenges for ELLs that are important for teachers to understand. There are additional linguistic demands for ELLs that make developing symbol sense and transitioning between the symbolic and natural language more of a challenge, as they learn to filter their existing and developing knowledge of mathematical language through a second natural language (Brown, 2005).

In particular, the symbolic system may cause great confusion for ELLs because of the ways in which it needs to interact directly with the natural language system throughout the problem solving process. Teachers need to be aware of these potential difficulties and provide opportunities for ELLs to engage with natural language and symbols and the links between them in the context of mathematics teaching. In other words, symbol sense cannot be fully developed in absence of natural language; thus, it is not sufficient to allow ELLs to avoid language issues by engaging them in mainly symbolic tasks. If we expect students to know how these semiotic systems interact in the construction of mathematics, they need experiences that help them build understandings of the multiple semiotic systems at work in mathematics word problems.

A major part of meaning making in mathematics word problems is in the connections between natural language and symbolic representations. As Tables 2 and 3 indicate, the formulation and interpretation stages, where these connections are key, do not just appear once at the beginning or end of the problem but repeatedly throughout the whole process. This suggests that teachers need to be aware of the additional challenges that ELLs developing their language proficiency may have throughout the entire problem. It is not just a matter of helping them remove words and create an equation – they need to develop meaning by constantly checking their symbol sense against the meanings in the natural language of the problem.

As teachers, we need to build a better awareness of the additional challenges that ELLs face with word problems and identify ways to help them use the understandings of language and mathematics that they bring to the table to overcome these challenges. It is critical for teachers to make use of ELLs' many existing skills, ideas and strategies. For example, all students bring with them language and cultural resources (Celedón-Pattichis & Ramirez, 2012) which mathematics teachers should use in authentic ways when constructing word problems to motivate interest and build relationships with and among students. Teachers must also be careful not to relate language fluency with academic competence (Gottlieb, 2006), but instead recognize that ELLs are often able to communicate sophisticated understanding of mathematics using multiple representations and draw on a range of resources to support their learning, including peers, family, and experiences (Aguirre et al., 2012). ELLs should have opportunities to make use of the tools and resources that work well for them as they build meaning from mathematical problems.

One way that is often recommended for helping ELLs build a connection between the natural language and mathematical symbols is to engage them with a third representation, visualizations. For example, in the problem analyzed here, a teacher may help an ELL to visualize the context by drawing five bracelets or five friends with one bracelet each. Students may also visually represent the money spent on bracelets ($16 - $3 = $13) with 13 dots on paper, which they may then partition out one at a time to each of the five friends. However, because 13 is not a multiple of 5, the answer is not a whole dollar amount and students may not find the visual representation useful. In this instance, a visual image may not be sufficient for helping ELLs develop meaning for all word problems, but may help students recognize the need for symbols, demonstrating again the need for development of strong symbol sense to secure ELLs' success in working with real world situations.

The framework presented in this paper can help teachers connect the problem solving stages to the semiotic systems while providing elements of symbol sense that students, in particular ELLs, can develop in order to work with mathematical word problems. This framework was designed and applied to word problems in middle school mathematics where students begin learning algebra. However, the framework can be adapted and used in other grade levels as well. We see this as one tool for helping teachers to think about new ways of helping ELLs work fluently within the multiple semiotic systems of mathematics in productive and meaningful ways.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. How do natural language, symbol sense, and visual representation relate to equity and excellence for ELLs?
2. Consider the test item in Figure 1. Would you consider this item reasonable for 7th-grade ELLs to know how to solve? Why or why not?
3. What does Table 1 reveal about the complexities of mathematics learning for ELLs?
4. How can we best prepare teachers to consider the multiple semiotic systems described in the article to address the needs of their current or future ELLs?
5. Use the framework described in this article to analyze the test item below (Indiana Department of Education, 2012):

    Test Item

    a) Linda sells video game systems at an electronics store. She earns $80 every week plus $7 for every video game system that she sells. Write an expression that represents Linda's weekly earnings given the number of video game systems (v) she sells.

    b) Linda has already saved $250. Her goal is to have a total of $600 after working two more weeks. What is the minimum number of video game systems Linda must sell in the next two weeks in order to reach her goal?

Preparing Teacher Candidates to Work with English Language Learners

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Abstract

We know little about how teacher candidates are prepared to work with English language learners (ELLs) in mathematics classes. This qualitative study, based on interviews with 16 instructors of mathematics methods courses for pre-service teachers, examined instructors' reported classroom practices regarding helping teacher candidates learn to work with ELLs in mathematics. Findings suggest that ELLs' needs may not be addressed in mathematics methods classes for varied reasons. This study has implications for mathematics teacher educators, PreK-12 mathematics teachers, and higher education and district level staff members who provide professional development for teachers.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What needs do ELLs have in the classroom in general and in the mathematics classroom, in particular?
2. What are some effective instructional strategies for teaching ELLs in the mathematics classroom?
3. Respond to the statement, "Mathematics is a universal language so ELLs should have little difficulty with it."
4. Reflect on the statement, "Good instruction for English learners views language as a resource rather than a deficiency."

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Preparing Teacher Candidates to Work with English Language Learners

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Mathematics Teacher Educators (MTEs) are tasked with helping their students learn to work with pupils with diverse needs (National Council of Teachers of Mathematics, 2000, 2014). This includes accommodating needs of English language learners (ELLs) enrolled in PreK-12 mathematics classes. What are MTEs doing in mathematics methods courses to prepare teacher candidates to work with ELLs? Are there constraints that may hinder them in seeking to address this topic? Our study, based on interviews with 16 MTEs who work with pre-service teachers (PSTs) in mathematics methods classes, addresses these questions.

Preparing Teachers to Work With ELLs

Teachers, administrators and other school staff need to understand issues related to the growing population of ELL students (Goldenberg, 2008). By 2025, one of four public school students will be an ELL (NCLEA, 2010 as cited in NEA Policy Brief, 2008, p. 1).

Lucas, Villegas and Freedson-Gonzalez (2008) report that, even though the number of ELLs is growing, "most mainstream classroom teachers have had little or no preparation for providing the types of assistance that such learners need to successfully learn academic content and skills through English while developing proficiency in English" (p. 1). It is not uncommon for pre-service mathematics teachers to enter the profession knowing little about the needs, resources and support that will be needed to teach mathematics effectively to ELLs (Chval & Pinnow, 2010). In fact, there is relatively little information concerning the knowledge of MTEs about ELLs (Arnold, 2013). Consequently, the authors wanted to learn what MTEs report about instructing teacher candidates on working with ELLs in mathematics methods classes. Such knowledge is indeed needed to inform continuing dialogue on how best to prepare mathematics teachers to work with ELLs.

Review of Literature

Opportunities for teachers to learn how best to educate ELLs has not kept up with ELLs' rapid growth (Samson & Collins, 2012). Though there is evidence that some teacher preparation programs are attempting to prepare candidates to teach ELLs, in general, most pre-service teacher educator programs, for various reasons, have "a long way to go" in developing necessary knowledge and skills among teacher candidates in this regard (Lucas, Villegas, & Freedson-Gonzalez, 2008, p. 2). According to a report from the U.S. Department of Education (2001), although 41% of K-12 teachers have ELL students in their classrooms, 72% of these teachers did not feel well prepared to work with ELLs. Even a decade later after these statistics were reported, Samson and Collins (2012) assert that mainstream teachers are still not able to fully meet the challenges of working with ELLs.

In mathematics in particular, many PreK-12 classroom mathematics teachers leave teacher preparation programs still holding misconceptions about ELLs and their needs (Costner, 2008; Moschkovich, 1999). Among them, the incorrect idea that having the ability to carry on a basic social conversation using *conversational language* (Cummins 2008) means the ELL should be able to understand and speak *academic language* (Cummins, 2008) in mathematics class. Other common myths are that a mathematics teacher is responsible for teaching only about numbers, as in "I'm not here to teach English – that's the ESOL teacher's job" (Costner, 2008, p. 31), that mathematics is its own language, independent of verbal speech, and that ELLs need help only with mathematics vocabulary and learning how to solve word problems (Costner, 2008).

The research literature on strategies for preparing teacher candidates is thin, but there are studies such as Fernandes' (2012) research on the process of PSTs doing task-based interviews in mathematics with ELL students. PSTs are sometimes shown research about working with ELLs that indicates the limitations of approaches to decoding words and word problems. Such approaches do not address the current increased emphasis on mathematical discourse and communication (Moschkovich, 2012; Vomvoridi-Ivanovic & Razfar, 2013).

Although "generalizing about the mathematical instructional needs of all students who are learning English is difficult" (Moschkovich, 2011, p. 18), two basic research-based principles suggest that good instruction for English learners views language as a resource rather than as a deficit and emphasizes academic achievement, not just learning English (Gándara & Contreras, 2009). ELLs should be encouraged to abstract, generalize, conjecture and engage in mathematical reasoning at the same level as native English speakers (Moschkovich, 2012). In order to accomplish this, classroom environments should furnish "abundant and diverse opportunities for speaking, listening, reading and writing" and "encourage students to take risks, construct meaning and seek reinterpretations of knowledge within compatible social contexts" (García & Gonzalez, 1995, p. 424). ELLs need to be involved in discourse within communities of practice, dialoguing about mathematics with peers, rather than be simply given individual help which, if used alone, only serves to isolate them from native English-speakers in the classroom (Chval & Pinnow, 2010).

ELLs require additional instructional support, beyond what is known to be effective for native English language-speaking mathematics students in general (Goldenberg, 2013). Additional support does not indicate lower expectations. It is essential that educators hold high expectations for the mathematics achievement of ELLs (Cady, Hodges, & Brown, 2010; Van de Walle, Karp, & Bay-Williams, 2012). Just because an ELL does not know as much English as his or her peers does not mean that the student should be expected to learn less mathematics.

Although there are several approaches to ELL instruction, one well-known general model is the SIOP or Sheltered Instruction Observation Protocol (Echevarria, Vogt & Short, 2012). This overarching model offers a framework of eight interrelated research-based components to support content area teachers. Goldenberg (2013) states that SIOP organizes instruction for ELLs and has empirical research-documented positive effects on student learning. Readily available materials, including entire books on teaching the subject of mathematics to English language learners, have been published using SIOP.

Moschkovich (2011) emphasizes focusing on mathematical reasoning, not just on accuracy in language. This can be accomplished by shifting to a focus on mathematical discourse practice, recognizing and supporting students to engage with the complexity of language, treating everyday language and experiences as resources (not obstacles), and uncovering the mathematics in what students say and do. Furthermore, in their guiding principles for teaching mathematics to ELLs, Ramirez and Celedón-Pattichis (2012) emphasize challenging mathematical tasks, providing a linguistically-sensitive social environment, supporting English while learning mathematics, using mathematical tools and modeling as instructional resources. Additionally, they suggest that MTEs identify ELLs' cultural and linguistic differences as potential resources, not obstacles, in the mathematics classroom.

Method

This was an exploratory study investigating what mathematics teacher educators (MTEs) report doing to prepare teacher candidates in methods courses to work with English language learners. The research question was: What do teachers of mathematics methods courses for pre-service teachers report doing to help teacher candidates prepare to work with English language learners in the mathematics classroom?

Description and Limitations of the Study

This was a qualitative study based upon in-depth individual interviews with volunteer participants. Limitations of the research were that: (1) data were collected from the 16 MTE volunteers who self-reported their own personal perspectives and classroom practices, (2) no actual classroom visitations were made by the principal investigators nor were the students of the teachers observed or interviewed, and (3) interviews were conducted via Skype and phone, not in person. Therefore, findings were limited to themes and ideas that arose from qualitative analysis of the verbatim transcripts of these one-time interviews.

Participants. Sixteen MTEs volunteered to participate in the study after being identified through a web search of U.S. mathematics teacher educators working at colleges and universities and invited via email. Those eligible were limited to those currently teaching mathematics methods courses for elementary and secondary teacher candidates in colleges and universities. All volunteers who met these criteria were accepted for the study. Fifteen held doctorates and one was in the dissertation-writing stage. There were 10 males and 6 females, and 13 of the 16 participants were native English speakers. The interviewees came from institutions of higher education that were diverse in terms of region, size, and public/private status.

Data collection. Each participant took part in one semi-structured interview (via Skype or phone) conducted by the first author. The interviews progressed organically following the initial question, "Could you describe what you are doing in your mathematics methods course to help pre-service teachers learn to work with English languages learners in the mathematics classroom?" Follow-up questions, depending on participants' responses, were open-ended and designed to guide but not dictate the flow of the interviews (Carspecken, 1996). Data were digitally recorded and transcribed verbatim for qualitative analysis.

Data analysis. Interview transcripts were read and re-read using a process of constant comparison (Strauss & Corbin, 1990), re-reading and constantly comparing data, after which manual coding began. As analysis continued, themes emerged related to awareness of issues dealing with accommodating ELLs and the participants' commitment to having teacher candidates in their mathematics methods classes engage with these issues. It appeared that most of the MTEs' approaches could be classified based on levels related to commitment, ranging from *quite limited* to *strong*. In no way does this indicate a claim that every MTE will fit perfectly into one of four levels– rather, the various levels of awareness may be conceptualized as part of a continuum. To be placed at a particular level, a substantial majority of individual participants' statements had to reflect that level. When all 16 participants' data were analyzed, data from two of the 16 participants could not be classified into any of the four levels identified for the other 14 and therefore, those two participants' data do not appear in Table 1 but still informed conclusions and recommendations.

The first author prepared four separate data summary sheets, one for each level shown in Table 1. Data were re-read and re-examined until findings emerged which led to conclusions concerning the participants' access, professional development, intended curricula, required curricula and cultural connections to English language learners.

Results

In general, it can be said that participants in this study were aware of the presence of ELLs in U.S. schools, yet the extent of their knowledge regarding how to accommodate ELLs varied. Due to a variety of self-reported challenges, the MTEs also expressed different levels of commitment to preparing pre-service teachers to work with ELLs. Data analysis identified the four classifications of responses presented in Table 1.

Level One

Half of the interviewees in this exploratory study were classified as Level One: having no plans to explicitly discuss the needs of ELLs with teacher candidates in mathematics methods classes. In some cases, participants felt challenged and perceived they had no time to address this topic in their classes. Thus, they omitted the topic and concentrated on others they deemed to be "more important," or about which they felt themselves to be more expert. In other cases, participants went so far as to express resistance to including the topic of accommodating ELLs in their mathematics methods classes. Reported reasons included: the belief that different approaches are not needed for accommodating ELL mathematics students, the belief that mathematics is its own language, the belief that the focus should be on "best practices for all" to help ELLs, a stated lack of training in how to teach mathematics to ELLs, time constraints, and a belief that other classes or professional development courses PSTs might eventually take provided everything necessary for PSTs to learn about the needs of ELLs.

Level Two

MTEs at Level Two had limited plans to address the needs of ELLs. One assigned readings on the topic, but did not plan on discussing them in class. Another told PSTs about using manipulatives as an effective way to help ELLs learn mathematics, but did not mention other ways to help ELLs. Participants here, as in Level One, were challenged either by lack of time to teach, lack of specific knowledge on meeting the needs of ELLs, or both. Additionally, these participants reported that discussing how to work with ELLs "arose naturally" at times, such as when working with manipulatives, but the MTEs had only very limited plans to discuss the topic if it did not arise.

Level Three

At Level Three, MTEs expressed some commitment to addressing the topic of ELLs in their classes, and talked about

Table 1
Classification Levels of Responses from Research Participants

Level and Fraction of Participants	Example of a Statement Classified at This Level
No commitment to present PSTs with research-based practices for ELLs in mathematics. 7/14	"I think your question really is, 'Do I take time out of teaching them how to teach math to focus on English language learners to the exclusion of other students?' The answer is no, I don't…"
Limited commitment to presenting PSTs with research-based practices for ELLs in mathematics. 2/14	"In the methods textbook I chose, there was a piece that had to do with English language learners, so they read about it, but we just couldn't find time to discuss that reading."
Some commitment to presenting PSTs with research-based practices for ELLs in mathematics. 2/14	"In their lesson plans they do have to address ESOL standards and write an ESOL objective. And I tell them in math that one of the best ones to do is a vocabulary objective for math."
Strong commitment to presenting PSTs with research-based practices for ELLs in mathematics. 3/14	"…I talk to (them) a lot about using content based language learning techniques, or those things that CEMELA and those folks have written about. And then SIOP materials (with) mathematics specific texts, so we can look at the lesson plans these folks have researched and used and talk about how to use them in their own classroom…"

explicit methods in their instructional repertoire. Also, at this level, the teaching of ELLs might be addressed because instructors were required to do so by their university or college, as part of the curriculum. For instance, one participant stated that she accommodated ELL needs by requiring teacher candidates to include a "vocabulary objective for math" in lesson plans. Others went beyond this. Specific accommodations mentioned included "printing off the Spanish version of the textbook and using that" and "checking with your textbook company for [second language] ... materials…. Don't put them [ELLs] behind twice, with language development *and* mathematical development."

Level Four

At this level, participants were strongly aware of the needs of ELLs. They shared knowledge of specifics, such as the SIOP model, and planned to explicitly address misconceptions that PSTs might hold about English language learners. They were familiar with research on the topic and presented teacher candidates with solid opportunities to learn about ELLs' needs, including the need for discourse, challenge and participation in learning communities. Some created their own curricular activities when necessary, and some showed activism in sharing their knowledge with fellow MTEs. The quote below illustrates how one MTE expressed her strong awareness and commitment to working with ELLs.

> You should always plan for students that have English as a second language or who may speak well but not really understand the mathematics terminology. We talk about how to modify and make accommodations with all of our materials in the classroom: giving things ahead of time, certain students realizing, online glossaries, text-to- speech options, using content-based language learning techniques or those things that CEMELA [Center for the Mathematics Education of Latinos/as; math.arizona.edu/~cemela/] and those folks have written about. And then also pulling SIOP materials mathematics specific, so that they can easily picture… how to do things in their own classroom.

Conclusions, Implications, and Recommendations

This was an exploratory study, and its small sample, although diverse (see participant descriptions), means that the authors cannot claim that results can be uniformly generalized. Additional studies with larger numbers of participants are recommended. However, data from a small, exploratory study (such as this one) may be found to be transferrable to a greater population, depending upon the situation (Hatch, 2002). Additionally, results gleaned from the qualitative analysis of the data can contribute to the dearth of published research on the topic of preparation of pre-service mathematics teachers to work with ELLs.

Several implications and recommendations would be consistent with our findings. First, the study provided examples of teacher preparation programs that have no explicit plans to specifically prepare future mathematics teachers to work with ELL students in mathematics. This correlated to research cited earlier indicating a deficit in PSTs' preparation to work with ELLs, even though large numbers would be asked to do so (Samson & Collins, 2012). We suggest that colleges and universities consider requiring instruction about ELL accommodations in mathematics methods courses, and not just in general education classes. They should also make specific plans to support MTEs in implementing these requirements.

It was not uncommon for participating MTEs to adhere to misconceptions, such as the idea that mathematics is a *universal language*; thus, ELLs will not have much trouble excelling at it. This study supports previous studies (Moschkovich, 2011) that have refuted such ideas and called for the continued need of professional development for MTEs. The literature reflects that such continued professional development should include how to support ELLs in their development of both oral and written academic language in the context of mathematics. Additionally, it should include information on the importance of holding continued high expectations for ELLs and how to encourage and nurture ELLs to feel comfortable in their oral language and ability to participate in classroom talk with peers. Data and information from the present research, indicating that many pre-service mathematics teachers may not have been prepared to work with ELLs in mathematics, may be of interest to school leaders, and to district level staff who provide professional development for new or even experienced teachers. Participants in the present study varied greatly in how they approached the topic of ELLs in mathematics. This level of variability can affect what mathematics teacher candidates learn, and can result in inconsistency of instructional quality for ELLs. Therefore, in addition to professional development for MTEs (already discussed), this study also supports recommendations for continued professional development for all PreK-12 content area teachers in school systems.

Half of the participating MTEs in the study reported that they did not intend to add explicit instruction about ELLs in the mathematics classroom to the curriculum. Some assumed the topic had been covered adequately in general education courses. This is consistent with the research of Chval and Pinnow (2010), who found that 63% of the PSTs in their study "did not focus on *their* own actions [in helping ELLs] but rather, on the actions of others" (p. 7). MTEs should not assume that instruction on the needs of ELLs in general education courses is sufficient to produce teachers who can help ELLs succeed in mathematics. MTEs should be aware of the importance of the needs of ELLs as they relate to mathematics instruction. We recommend that all MTEs should be certain that this topic is included in mathematics methods courses.

There has been little published information about how MTEs help prepare teachers and teacher candidates to work with English language learners. This study seeks to begin addressing this gap. All students deserve access to mathematical thinking and learning (NCTM, 2000, 2014). By looking at teacher preparation courses through the lens of the statements and reported practices of MTEs, various levels of commitment to the mathematics education of ELLs became apparent in the participants. Therefore, it is hoped that continued professional development and research will increase the level of commitment by all MTEs so that addressing the needs of ELLs in the mathematics classroom may become an explicit part of their mathematics teacher preparation courses.

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Discussion and Reflection Enhancement (DARE) Post-Reading Questions

1. What specific instructional strategies would you suggest for working with ELLs in the mathematics classroom?

2. How could the SIOP Model (Sheltered Instruction Observation Protocol; see Echevarria, Vogt & Short, 2012) be effectively used to teach ELLs in the mathematics classroom? Give specific instructional strategies and detailed student activities that are not already covered in question 1.

3. What type of accommodations do *you* feel comfortable implementing with ELLs in the mathematics classroom?

4. How can good instruction for English learners use language as a resource rather than as a deficiency (Moschkovich, 2011)?

5. What should the mathematics education community do to help support MTEs in addressing the needs of ELLs in the mathematics classroom?

6. Discuss this statement: A general commitment to diversity and equity on the part of MTEs is not sufficient to ensure that mathematics teacher candidates will be adequately prepared to work with ELLs.

Learning Language: A Mathematics Educator's Reflection on Empathy and Privilege

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Abstract

Some educators who are not English language learners (ELLs) do not fully appreciate the struggles and resources ELLs have. This paper, expanded from a reflection in the Spring 2013 newsletter of the North American Study Group on Ethnomathematics (NASGEm), shares a journey of cultivating empathy -- from personal perspective to professional development.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Is learning how to teach ELLs the responsibility of everyone or just specialists?
2. Have you been in a situation where you had to navigate a different language or culture? How did that feel and what strategies did you use?
3. Are you a member of a minority, non-mainstream, underrepresented, or non-dominant group?
4. If your answer to #3 is yes, how has this made you appreciate the distinctive struggles and resources of those in other such groups?

Acknowledgments: The author expresses appreciation to Dr. Lauren Davis and Project LEAP-UP (US Dept. of Education #T195N070132).

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Learning Language: A Mathematics Educator's Reflection on Empathy and Privilege

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Because I do not identify myself as an ELL (though I definitely keep learning more about the English language!), I am occasionally asked why I have made diversity and ELL issues a major part of my recent scholarship (e.g., Lesser, 2010, 2014; Lesser, Wagler, Esquinca, & Valenzuela, 2013; Lesser & Winsor, 2009; Lesser, 2010; Wagler & Lesser, 2011) and professional service. Part of my answer involves how ELL issues are becoming increasingly prominent, with the percentage of ELLs in U.S. K-12 schools projected to increase by 2030 to 25% (Goldenberg, 2008) or even 40% (Herrera & Murry, 2005). And because so much of the social order depends on having an educated public, I believe it is in everyone's interest to support education for all students, whether or not one is an ELL, whether or not one has a child in public school, etc.

Another part of my answer involves connections with the access-and-excellence mission of my university, which serves a regional population with a substantial fraction of ELLs. A further part of my answer is more personal: the values of my faith tradition (e.g., Jacobs, 2012; Schwarz, 2006) to "remember the stranger" and to be sensitive to the experiences of any minority group who lacks or has lacked equal opportunities for access in society. In this paper, I reflect on how my empathy for ELLs has inspired me to identify and implement strategies that may be effective in helping others cultivate empathy as well.

Making My Own Connections

Language and culture dynamics gained personal immediacy for me when I (as a not highly knowledgeable Jew) married into a Modern Orthodox Jewish family. This life change blessed me with a richness of meaningful experiences, though the denominational culture presented a much higher density and speed of Hebrew language and shorthand. My struggle to follow what was happening or said during times I did not recognize words or phrases gave me tangible empathy for the experience of students who are ELLs and what a difference support can make.

For example, a traditional Talmud class might use a text that lacked not only English translation but also markings for punctuation or vowels. A more accessible Talmud class might use a book (e.g., Shefa Foundation, 2012) which unpacks what is a very dense text by, for example, not only translating into English but also filling in referents and phrases that are often implied but unstated in the terse original Hebrew text.

I was also quite grateful when I found linguistic support at religious services. Some Orthodox synagogues regularly announce or display page numbers. Many congregations serve worshippers of varying Hebrew fluency by keeping on hand prayerbooks that include English translations at the paragraph level, linear (line by line), or even interlinear. An interlinear approach (e.g., Apisdorf, 2002) translates 1-3 words at a time, giving me the opportunity to understand the meaning of what I was pronouncing word by word, and incrementally build my vocabulary. ELLs in my mathematics content and pedagogy courses are usually quite appreciative when I make them aware of mathematics glossaries (e.g., COMAP, 2004), terms handbooks (e.g., COMAP, 2008; Dragt, 2009, Velázquez Press, 2010), or applets (e.g., http://nlvm.usu.edu/es/nav/ and http://www.eduteka.org/MI/master/interactivate/) that include their dominant language. And because I recall what it feels like to struggle to recover from temporarily falling behind or losing my place in a service or class, I try to incorporate periodic organizers, callbacks, and recaps into my teaching so that students can readily navigate or rejoin the flow of instruction even if there was something earlier that they did not understand.

Because I already knew how to pronounce Hebrew (aided by the fact that each letter is always pronounced the same way – like Spanish, but unlike English!), people sometimes overestimated my Hebrew vocabulary. Regularly reading the same passages from a prayerbook requires much less proficiency than, say, reading a Hebrew newspaper sans vowels. Later, I recognized the rough parallel that I had surely overestimated the academic language proficiency of many ELLs based on observing their solid proficiency with everyday English outside of formal academic discourse. Indeed, everyday language proficiency typically precedes academic language proficiency by several years (Cummins, 1992; Johnson, 2010).

A particular way my Hebrew proficiency has been overestimated is when someone with the well-meaning intention to make me feel included gives me the honor of publicly leading a Hebrew prayer that I lack the proficiency to read smoothly. Those awkward experiences have helped me remember to make sure that what I ask of ELLs in class is sufficiently scaffolded (e.g., using techniques such as sentence frames for students to complete: "a z-score is the number of ___ that a value is above the ____"; see Wagler & Lesser, 2011) so that they can keep their main focus on the content, not feel put on the spot, and feel that they belong to and can contribute to our classroom community. Another example is that a student asked when the mean could exceed the median may not be able to generate a phrase such as "a unimodal, right-skewed distribution", but I could allow them to use informal language or draw the shape to show me they understand the concept (Lesser & Winsor, 2009).

While there are many lexicons of Jewish terms (e.g., Eisenberg, 2008; Eisenberg & Scolnic, 2006; Olitzky & Isaacs, 1992), some researchers have gone further to include more comprehensive analysis and resources (e.g., Benor 2009, 2012; Weiser, 1995) that decode the distinctive cultural and linguistic patterns used by native-born Americans who are Orthodox Jews. In addition to rabbinic Hebrew words and phrases (e.g., *yotzei*, *assur*, etc.) often inserted in otherwise "regular English" constructions, this also includes Yiddish-influenced idiomatic use of English words, even a word as common as "by". Benor (2009) gives examples of the latter such as "Are you eating by [at the house of] Rabbi Fischer?" and "If you hold by [accept, believe in] Reb Aron.…" This made me more sensitive to the idea that my students could assume they knew each word of a phrase used in mathematics or statistics class (e.g., "in the long run", "expected value", "at least six") but yet not understand the particular way the phrase is being used as an intact whole. When such phrases come up, I make it a point to discuss with the class how the phrase works as an entity, rather than as the sum of its parts.

Ultimately, I had to navigate culture (Benor, 2012; Langman, 1999; Oppenheimer, 2013) as well as language. Knowing the content of the weekly Torah portion does not ensure a Jew can turn it into a short talk (*d'var Torah*) that will be effective for a particular audience. Likewise, knowing mathematics content does not guarantee an ELL understands the conventions of giving an academic or class presentation. In both cases, one has to know when to cite sources, when to make or avoid connections to personal perspectives or conjectures, when to use informal versus formal language, etc. There are also parallels in terms of whether one's identity as a newcomer (whether to the English language or to traditional Judaism) is permanent or is shed when one's knowledge or experience reaches a certain level, and how much one is able or wanting to keep the newcomer identity invisible (Benor, 2012). And in general, having now experienced Jewish congregations in almost every denomination, I better appreciate how ELLs (or Latinos, etc.) are likewise not a monolithic group and I try to avoid stereotypes (Lesser, 2014) or one-size-fits-all approaches.

Challenging in a different way is having one's capabilities *underestimated*. I have had people assume I could not handle more conceptually-rich discussions of Jewish text or ideas based on an assumption quickly formed from how I imperfectly used language or convention. This helps me remember that students can understand more of a language (including mathematics) than they can generate and not to assume that someone is incapable of higher-order thinking in mathematics just because they may struggle to express their understanding in academic English. More generally, this helps me remember to avoid the pitfalls of deficit models, and know that each person has knowledge and experiences in her/his background that can be a valuable resource (e.g., Khisty, 1997). Lesser et al. (2013) give examples of words (edifice, facile, felicity, pensive, confounded) that a Spanish-speaking ELL may be more likely than a native English speaker to recognize (e.g., because of cognates) certain English words or to relate better to the context of certain mathematics problems. And though I have been using the term ELL, I now better appreciate why some (e.g., Phakeng & Moschkovich, 2013) prefer the term "bilingual" because it emphasizes a resource rather than a deficit. In the case of an Orthodox Jewish study setting, I frequently am (or at least feel like) one of the few in the room whose education does not include Jewish day school or *yeshiva*, but I sometimes surprise people by how much I can nevertheless participate or contribute, drawing from my overall strength in logic and reasoning (thanks to degrees in mathematics and statistics), my university coursework in philosophy (including philosophy of religion), and my having studied connections (e.g., Lesser, 2006, 2013) between Judaism and my secular field of expertise.

Privilege

Like many people with much privilege (e.g., I am a member of society's historically privileged groups in terms of gender, gender identity, sexual orientation, and skin color, and was born in the U.S. to well-educated parents in the upper middle class), I found my privilege largely invisible to me until I finally found myself in contexts where I lacked it. I found it helpful to draw from positive and negative experiences as a religious minority (not to mention having non-native status within a minority subgroup of my minority religion!) to help sensitize myself more fully to the experiences of my university's students, whose modal ethnicity, gender, and religion differ from mine. My "privilege checklist" score (McIntosh, 1989) is certainly lower (though not as low as it would be if I were a person of color) when I view myself as a visibly-identified committed Jew compared to when I view myself more generically as a White person (e.g., Diamant, 2013; Killermann, 2012; Marcus, 2014). This nuanced concept of how one can be privileged in some ways and not in others is called *intersectionality*.

McIntosh's framework of unearned privilege can also be used to articulate *linguistic privilege* – an unearned asset I received simply because I happened to spend my best language-learning years (before I was old enough to make decisions about where I would live or attend school) in an environment where the dominant language is the one most widely used in documents, signs, websites, curriculum, commerce, research journals and conferences, international organizations, etc. I am now humbly aware how much I had taken for granted that I could understand virtually everything my teachers (or a standardized test) said, that my ability to communicate or interview for a job would not be hindered by an accent, that my use of my native language in school would not be held against me, etc. I now see that I also took for granted that the symbols and algorithms I learned in American elementary schools to do arithmetic were viewed as standard, even though the alternatives students in some other countries learn are equally mathematically valid (Moschkovich, 2013). Rather than let awareness of this privilege paralyze me with embarrassment or guilt, I have let it energize me to make my classroom a more level playing field and to seek effective gentle ways to raise the consciousness of other educators as well.

Motivating other Educators

Using a Different Language

Many educators have found that empathy for ELLs in the US can be cultivated with experiences such as a study abroad program (Marx & Pray, 2011) or sustained field experiences (e.g., Luft, 1999). Because most in-service or pre-service teachers may not have the opportunity, time, or money for such experiences, there is a need to identify opportunities of shorter duration that have a high bang-for-the-buck. Many educators speaking on ELL issues to broader audiences are finding it makes a memorable impact to open presentations by having the audience actively engage with some mathematics content in an unfamiliar language, to approximate an experience many ELL students have. For example, Asturias (2011) presented a PowerPoint slide with a mathematics word problem in Filipino (Tagalog) and invited participants to turn to their neighbors and try to solve the problem, or at least understand the question. Next, he showed a slide that simply added a picture. He then asked "How did it feel? Did you feel you had access to the problem?" Then he modeled strategies such as identifying cognates and then finally showed the problem in English.

Wagler, Lesser, Monárrez, and Salazar (2012) opened their presentation to statistics educators with some experiential examples for attendees. First, attendees were given a minute to try to understand what they could of a six-sentence excerpt (in German) from Sorto, White, and Lesser (2012), a translation of Sorto, White, and Lesser (2011). The excerpt (see Figure 1), reprinted here with permission, consists of a description (in German) of two tasks accompanied by a scatterplot with axes labeled in English, and the second of those tasks appears in Figure 1 below. Cognates were identified such as Kriterium (criterion), Studenten (students), Graphen (graphs), beste (best), and Daten (data). Attendees experienced how much or how little this enabled them to feel like they understood the entire excerpt, especially given that some words were false cognates, such as könnten meaning "compute," not "contain."

Aufgabe 2: Welches Kriterium haben Sie benutzt, um die Gerade festzulegen? Beschreiben Sie, was Sie gedacht haben, als Sie eine passende Gerade ausgewählt haben.

Aufgabe 3: Zwei Studenten haben unabhängig voneinander die ihrer Meinung nach beste Gerade gezeichnet. Die Graphen in Abbildung 2 zeigen ihre Geraden. Wenn Sie zu entscheiden hätten, welche der beiden Geraden besser zu den Daten passt, was wäre dann Ihre Wahl? Gibt es irgendetwas in diesem Zusammenhang, was Sie ausrechnen könnten?

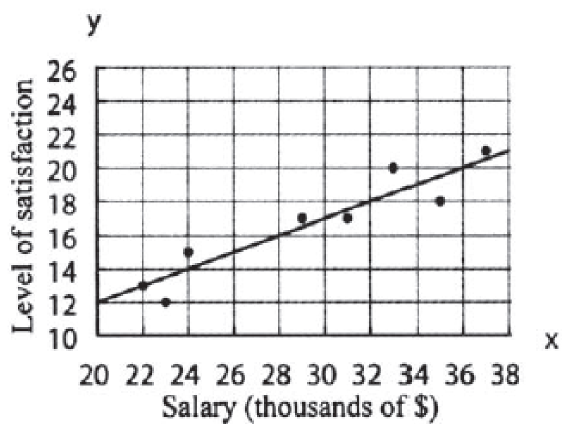

Figure 1. A description in German of two tasks accompanied by a scatterplot with axes labeled in English.

The impact of such demonstrations is arguably even greater when the chosen language does not use letters from the English alphabet. Washburn (2008) and Anhalt, Ondrus and Horak (2007) discuss the impact of an unannounced guest teacher giving a mathematics lesson in Chinese to pre-service teachers and middle school in-service teachers, respectively. In the post-lesson debriefing, students reported feeling confused, frustrated, lost, stupid, and overwhelmed during the lesson, even though they knew there were no consequences for not understanding. As a way to get the best of both approaches, bilingual educators (e.g., Giron & del Campo, 2009) have found it powerful to present the same piece of content using languages that get progressively "closer to English" such as Japanese, Croatian, German, Spanish, English. An audience experiencing such a sequence can feel how their level of comprehension and comfort increases with the emergence of cognates and other cues. Kubota, Gardner, Patten, Thatcher-Fettig, and Yoshida (2000) describe how a shock language experience (a 20-minute language arts lesson in Japanese, followed by debriefing in English) geared to ELLs' mainstream peers in elementary school affirmed diversity and encouraged peer collaboration.

Finally, the second language can also simply be the quirky language of an unfamiliar context. A recent example of this is Vomvoridi-Ivanović and Razfar (2013), who describe an innovative use of baseball to help pre-service teachers who are fluent in English but not in baseball gain empathy for students who are ELLs.

Filling in the Blanks

Another type of experiential example involves taking an excerpt from an English-language mathematics textbook, but with blanks inserted for each word that is not a "K1 word" (K1 words are words from the 1000 most commonly-used English word families; see West, 1953), adapting the idea of Nation (1990) cited at http://www.lextutor.ca/research/rationale.htm and perhaps viewable as a modified Cloze test (e.g., Gellert & Elbro, 2013). Rather than asking students to imagine being a second language learner themselves (as in the preceding examples), this approach asks students to imagine what an ELL in their class right now might experience. To illustrate, consider this not atypical exercise from a mainstream published statistics textbook (Larson & Farber, 2003, p. 387), which has been modified by replacing words that are not from K1 or K2 word families (i.e., words from the 2000 most commonly-used word families) by numbers in parentheses:

"A (1) (2) association believes that the mean (3) of fresh (1) fruits by people in the U.S. is at least 94 pounds per year. A (4) sample of 103 people in the U.S. has a mean (3) of fresh (1) fruits of 93.5 pounds per year and a standard (5) of 30 pounds. At $\alpha = 0.02$, can you (6) the association's claim that the mean (3) of fresh (1) fruits by people in the U.S. is at least 94 pounds per year?"

After reflecting on how comprehensible the above exercise was, reflect upon that same exercise below with the six distinct non-K1 or non-K2 words filled in using boldface and underline to denote words that are AWL (Academic Word List; see Coxhead, 2000) or Off-list words (i.e., not K1, K2, or AWL), respectively:

"A <u>citrus</u> <u>grower</u> association believes that the mean **consumption** of fresh <u>citrus</u> fruits by people in the U.S. is at least 94 pounds per year. A **random** sample of 103 people in the U.S. has a mean **consumption** of fresh <u>citrus</u> fruits of 93.5 pounds per year and a standard **deviation** of 30 pounds. At $\alpha = 0.02$, can you **reject** the association's claim that the mean **consumption** of fresh <u>citrus</u> fruits by people in the U.S. is at least 94 pounds per year?"

Note that this exercise includes two two-word phrases ("standard deviation" and "random sample") in which one word is "common" and the other is an AWL word, a situation which may make it difficult for a student to remember to treat the phrase as a single entity. Also challenging is the phrase "at least" (Nolan, 2002), which a student (especially an ELL) may use a "key word" approach (e.g., Clement & Bernhard, 2005) to operationalize "at least" incorrectly as "less than." Other issues are created by the fact that the words "mean" and "association" each are K1 words that can also be used as statistics terms, but in this particular exercise, "mean" is used as a statistical term (i.e., average), while "association" is not (i.e., it uses the everyday meaning of "a group of people" rather than statistical correlation). Finally, we note that the off-list words ("citrus" and "grower") may make it difficult for students to feel they sufficiently understand the real-world context for the exercise. Here is a rewrite of the opening sentence that preserves the mathematics but stays completely within K1 (except for the K2 word "oranges"): "An organization of farmers who grow oranges believes that people in the U.S. eat a mean of at least 94 pounds of oranges per year." When I am unsure that my lecture notes, test problems, or worksheets have avoided unnecessarily complicated language, I simply paste the text into the LexTutor VocabProfile window (http://www.lextutor.ca/vp/eng/) or generate Readability Statistics, an option in MSWord (Wagler & Lesser, 2015).

Concluding Thoughts

By having had my own concrete experiences with navigating culture and language, I have increased awareness and understanding of some dynamics faced by my ELL students and have increased motivation to give other educators experiences that will evoke further empathy in them as well. As Howard (1999, p. 2) notes, "Diversity [of the students we teach] is not a choice, but our responses to it certainly are." More generally, I believe that cultivating empathy for this significant subgroup of my students has been humanizing and has increased my desire and ability to connect with other subgroups as well. And because language diversity can be (at least initially) invisible, it is a humble reminder how the students I teach may have still other hidden diversity that impacts how they experience content. Plank and Rohdieck (2007) give the example of two white women looking at unemployment data among military spouses, but having very different reactions because one is a military spouse herself and the other is gay "and thus [at the time] legally excluded from both marriage and the military."

Almost all of my students are preservice or inservice teachers, and they have (or certainly will have) ELLs among their students in this part of the country, and some of my students are (or have been) ELLs as well. This is not surprising because my university's population reflects the population of the Paso del Norte region and UTEP is the largest university (and the only doctoral research university) in the country with a majority Mexican-American student population. Therefore, even when I teach "content classes," I try to share support resources and model ELL-friendly best practices for instruction (e.g., Lesser, 2011), and empower all students to find their voice (Reyes, 2012). And so, I continue making transfer to my professional role as a mathematics educator from my own personal experiences as a minority. My journey of empathy is ongoing, continuing to evolve over my lifetime. And empathy is a way to contribute to the healing process needed in our increasingly diverse society (Howard, 1999).

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What are some reasons it is helpful for educators to cultivate empathy for ELLs?

2. What are some techniques or tools you might use or adapt to cultivate empathy for ELLs?

3. Of the examples generated by the preceding question, which is the most powerful for you? Why?

4. Taking into account ideas in Noddings (2010), discuss the concept of empathy and how it differs from sympathy.

5. See D'Ambrosio et al. (2013) for a conversation about positioning oneself (i.e., discussing one's frameworks, ideologies, identities, etc.) in one's mathematics education work. How does the current *TEEM* paper succeed -- or fall short -- in this?

From "Eye-opening" to Mathematical: Helping Preservice Teachers Look for Mathematics in Stories of Oppression

Ksenija Simic-Muller
Pacific Lutheran University

Abstract

This article analyzes preservice teachers' reflections about a visit to a campus event focused on injustice and oppression that they were required to attend as part of an assignment in a mathematics content course for preservice K-8 teachers. Prior to the assignment, the preservice teachers had had limited exposure to social justice-based mathematics contexts and extracting mathematics from the real-world. Their reflections provide valuable information about the types of social justice contexts preservice teachers find relevant, and the mathematical possibilities they see, on their own, in events such as this one.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What real-world contexts are appropriate and relevant for preservice teachers to investigate in a mathematics content course?
2. Should oppression be discussed in teacher education? If so, how?
3. Should oppression be discussed in a mathematics class? If so, how?
4. How can campus and community events be brought into the mathematics classroom?

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From "Eye-opening" to Mathematical: Helping Preservice Teachers Look for Mathematics in Stories of Oppression

Ksenija Simic-Muller
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Real-world mathematics contexts and social justice issues, highly relevant but not necessarily well-known to preservice K-8 teachers (PST), are closely connected through a pedagogical approach usually referred to as *teaching mathematics for social justice*. This approach is not uniformly defined in literature (Gonzalez, 2009), and in fact some authors prefer to leave the term undefined, highlighting its complexity and nonlinearity (Bartell, 2013; Wager & Stinson, 2012), but most authors agree that an essential component is "the use of mathematics as a critical tool for understanding social life; one's position in society; and issues of power, agency, and oppression" (Gonzalez, 2009).

The scholarship of teaching mathematics for social justice has significantly grown in the recent years (e.g. Bartell, 2013; Gonzalez, 2009; Gutstein, 2006; Gustein & Peterson, 2013; Wager & Stinson, 2012), as more educators see the potential for empowerment that social justice-based curriculum brings. My work is particularly influenced by Gutstein (2006) and Frankenstein (1998; 2009), who in turn build on the work of Freire (1985) when they write about "reading the world with mathematics." Reading the world with mathematics is a widely-encompassing term, which implies, among other things, using authentic real-world mathematical contexts to understand the social and political forces that shape our lives and our societies. Mathematics content courses for preservice K-8 teachers provide a convenient setting for reading the world with mathematics, since the mathematics content often used to analyze social justice issues, such as algebraic reasoning, rational numbers, proportional reasoning, probability, and statistics, is part of the standard curriculum in these courses.

This manuscript provides an analysis of PSTs' reactions to a campus event featuring a variety of social justice topics through a mathematical lens. The PSTs' reflections about the event offer insights relevant to mathematics teacher educators, about social justice contexts PSTs consider relevant and about their interpretations of mathematics connected to these contexts.

Context and Data Collection

I teach a two-part mathematics content course for preservice K-8 teachers at a medium-sized liberal arts university. The majority of students intend to teach lower elementary school. They fit the profile of a typical student in a teacher education program: the vast majority are white females from suburban and rural areas (National Center for Education Statistics, 2011; Zumwalt & Craig, 2005), and, likely because I teach at a private university, many come from wealthier backgrounds. Research shows that PSTs who fit this profile are often unaware of issues faced by a large and growing number of public school students who deal with racism and poverty in their daily lives, are food insecure, have incarcerated parents, have no access to resources, or are homeless (Hollins & Guzman, 2005; Milner, 2006). In fact, PSTs may even have deficit views of these students, and believe them to be at fault for their circumstances (Castro, 2010; Gay, 2010; Ladson Billings, 2006; Villegas, 2007). These views should be addressed in every aspect of an education program, including in mathematics courses. In particular, a mathematics content course can raise PSTs' awareness of issues that affect their future students and their communities, while showing them the power of reading the world with mathematics.

Because the primary focus of the course is developing mathematical knowledge needed for teaching (Ball, Thames, & Phelps, 2010), I am unable to give the social justice content a central place in the curriculum. Instead, I create occasional problems, lessons, and assignments about topics that vary from semester to semester, often following current and campus events, and have included sweatshops, homelessness, income inequality, incarceration, and sustainability as topics. Due to past instances of resistance to social justice contexts, I align assignments as much as possible with the social justice efforts taking place at the university. In particular, I have created an assignment around an event titled *The Tunnel of Oppression* (referred to as "the Tunnel" throughout). The Tunnel is an annual, student-created event that takes place on college campuses nationally, and on our campus every spring, featuring a tunnel-like setting with scenes portraying different forms of oppression or injustice, through spoken, written, or acted-out information. I require PSTs to attend this event; write a reflection about their experience; and create a report, based on additional research, about a scene of interest (the detailed instructions are included in the Appendix). I have chosen the Tunnel as a topic for an assignment because of its inherently mathematical nature, as almost every scene features numerical and graphic data, typically obtained through mathematical processes. For example, some facts that have occurred in the Tunnel in the past have included the percentages of rapes in the military that are reported and prosecuted, the difference in employment rates between Black and White college graduates, or differences in graduation rates between school districts.

While the reports are much more structured, incorporating mathematical content addressed in the course, and requiring PSTs to submit multiple drafts, the reflections are deliberately open-ended. The prompts require PSTs to describe their reactions to the scenes and the mathematics they saw in the event, and to give examples of situations in which mathematics was absent but could have been beneficial. Their purpose is for PSTs to look for mathematics in new places, and for me to better understand what they see as mathematical.

I have used the Tunnel assignment for three consecutive years. The work collected includes reflections, reports, responses to the assignment, and some problems posed by PSTs, who are aware that their work is the focus of my research. This manuscript focuses solely on the reflections, 109 collected over three years. My purpose in analyzing the reflections is to investigate what mathematics PSTs notice on their own, having had little prior experience with similar assignments; and it is in no way intended as criticism of PSTs' perceived limitations. Prior to this assignment, PSTs will have encountered a few problems or assignments that deal with social justice, but this is the first assignment that requires that they look for mathematics or pose mathematical questions.

All the reflections were coded for themes after being collected, using a method most closely resembling the constant comparison method (Merriam, 1998): reflections were repeatedly read, and new codes were added in the process, uncovering themes related to mathematics and oppression, which became the categories in the data analysis. For this manuscript I focused on the codes related to the contexts PSTs found relevant, the mathematics they recognized in the event, and the mathematics they thought was missing.

Results

Oppression: Moving, but Is It Relevant?

In the reflections, 75% of the PSTs have used an emotional term such as "moving" or "shocking" to describe their experience at the event, and some variation of the term "eye-opening" is used in the reflections 48 times; only eight PSTs expressed disappointment at the event, and only one had a strong negative reaction to the Tunnel, which clashed with her personal beliefs. Assuming PSTs were truly moved by the event as they claim, an important question to ask is which contexts had the greatest impact on them, and why.

The Tunnel topics change every year, but issues related to education and youth are always present in some form, including scenes about high school graduation rates or negative stereotyping of local youth. Many other scenes have addressed issues relevant to large numbers of public school students, including ones about racial profiling, immigration, poverty, or access to health care. Since elementary PSTs overwhelmingly choose teaching as a career because they care about children (DeLong, 1987; Reif & Warring, 2001), I expected PSTs to make connections between Tunnel and teaching, but this was evident only in about 12% of the reflections.

One scene explicitly addressed the disparity in graduation rates between different races and ethnicities, and while many reflections identify this scene as mathematical, only one PST connected it with her/his future career, noting that it "offered a jumping off point for me in particular because it really made me look towards my future and the future of those who I will be teaching." Some reflections show awareness of the relationship that exists between problems in society and problems that children face, like the one noting that "when I teach, all of these [social justice issues] are things that I am going to see and experience," and motivation to make a difference, but none discussed systemic roots of inequitable educational outcomes, instead only describing their future individual efforts to help all children.

More PSTs were moved by issues that resonated with them personally: women were affected by scenes that involved sexual violence; parents commented on scenes that dealt with victimizing children; and almost all seemed impacted by local scenes, such as human trafficking in our state and the strained relationship between our campus and the local community. This is not surprising, but serves as a reminder that, if we want PSTs to be open to the social justice contexts we expose them to, we should begin with topics personally relevant to them. We should also not assume that PSTs will pay attention to education contexts just because they are offered, nor that they will be able, on their own, to go beyond feeling compassion for youth who do not graduate and instead begin to understand the systemic forces that marginalize them in schools and result in inequitable outcomes (Willey & Drake, 2013).

Mathematics: Omnipresent, but Is It Deep?

The simpler mathematics in the Tunnel is easily seen, usually in the form of numbers or percentages. The underlying mathematics, which includes methods of counting people or measuring impacts, is not as obvious, especially for those PSTs who have an uncomfortable relationship with mathematics, and limited prior experiences with it.

What mathematics is there? The vast majority of PSTs see mathematics in the Tunnel, expressing the belief that numbers help convey the seriousness of the issue being highlighted. One PST wrote:

"Originally, I consciously questioned how math could take part and really make an impact in this type of event. Believe it or not, I felt that one of the strongest aspects of the whole event was the statistics provided to help support at the different scenes. Statistics are obviously a part of math and helped the viewers of this event really see and compare the tragedy of what was happening."

Reactions like this one are uncommon. There have been only five dissenting voices, almost all lamenting that no mathematics was seen in the Tunnel other than statistics and percentages, which raises the question of which topics PSTs consider legitimate mathematics if some are willing to dismiss ones most frequently encountered in our daily lives.

At the other extreme are PSTs who only see mathematics in statistics and percentages, which are overwhelmingly the most commonly mentioned topics. The reflections especially refer to numbers, lauding their impact on the understanding of a particular issue. While important in conveying information, numbers are not the only mathematical concepts present in the Tunnel. Take for example a scene that asked participants to find a wheelchair-accessible route between two parts of campus that could be traveled in a certain amount of time. Only three PSTs identified this scene as containing mathematical information, possibly because it featured non-numerical mathematical content.

Other topics that were successfully identified by PSTs as mathematical were life-expectancy, pre-existing conditions for health insurance, taxes, "budgeting with WIC," or "showing oppressed families unable to pay for their loved one's medical expenses." Though rates and ratios are occasionally mentioned, only two PSTs explicitly noted the role of proportional reasoning in understanding large numbers. One in particular noted "the power of comparing a more abstract concept to something more familiar to a general audience," in response to a scene that scaled down the numbers related to sexual assault in the military to the size of our university's student body. The PST found this "really impressive because it immediately made the numbers real and tangible and was incredibly effective." However, comments like this one are rare, and the PSTs' descriptions of mathematics are often vague, repeating the numbers seen and heard, but seldom asking about how they were obtained, or commenting on their meaning. The following comment is an exception:

"The … scenes also offered data on rates that a particular issue or injustice effected [sic] people but … they rarely went into depth about where this data was found or if it was exclusive to a particular group of people."

Ideally, such comments should be more common. Content courses need to provide multiple opportunities for PSTs to develop a broad understanding of mathematics, one that goes beyond numbers and counting. For example, all PSTs, not just one or two should learn to appreciate the use of proportional reasoning to understand large numbers. Similarly, considering how data are obtained and understanding the limitations of almost any type of survey or census, is essential to reading the world with mathematics, and PSTs need more experience with this practice.

What mathematics was missing? Just as they praised numerical information for helping better understand certain issues, PSTs also critiqued scenes that were devoid of numbers. This was especially evident in a scene that presented opinions the local youth and university students hold of each other. Because the scene only featured words, PSTs found it hard to form an opinion about the issue. One wrote,

It would have been easier to understand the peers I share classes with if there was a number or percent of students on campus surveyed … because I'm assuming a large number when there might have only been a small amount (changes perception).

Along similar lines, another PST called for a campus survey, because "if people were surveyed on their views, the ability to see actual percents to go along with the phrases presented, this scene would have stood out more."

While calls for more numbers and data are prevalent in the reflections, few concrete mathematical questions are asked. Some questions are unrealistic, like wanting to know how many people use certain derogatory words, or how many people in the world are oppressed daily. More successful questions highlight comparisons. For example, when discussing pricing of health care, a PST wanted to know "how much it could actually cost in comparison to how much an average low income family makes with x amount of kids." This topic raised more interesting questions, such as "about how many families in the United States have to live without health care or how much families have to pay for the most simple health situations when they don't have proper health care." Finally, one PST who found the mathematics in the Tunnel lacking proposed the following comparisons:

"[C]ompare how these numbers have changed (in any area) to show that either things are getting worse; which would make a bigger impact on those learning, or that they are getting better; which would show that people are becoming educated and doing something to change the situation."

The PSTs' positive reactions to the use of numbers as a tool for shedding light on social justice issues are promising; but to become mathematically literate citizens fluent at reading the world, they need mathematical tools in addition to openness.

Recommendations

It would be unreasonable to expect that PSTs will be successful in identifying complex mathematics, posing relevant mathematical questions, and noticing the sociopolitical complexities in the Tunnel scenes (or another context) entirely on their own. Instead, instructors need to take concrete steps to help PSTs begin to develop these competencies.

First, PSTs, like K-12 students, respond better to a curriculum that is personally or culturally relevant to them (Gay, 2000; Ladson-Billings, 1995). Therefore, their initial contact with social justice mathematics should be through contexts that are familiar and relevant, and preferably not too uncomfortable. For my students, sustainability is a much easier initial topic than sweatshops or homelessness. I also require PSTs to pose mathematical questions about issues that are of concern to them, which helps them see that they already possess social justice concerns, even though they may be different from mine. Once PSTs are comfortable with the idea of social justice in a mathematics course, more complex topics can be introduced. It is especially important for PSTs to understand issues related to education, and in particular we can present them with data sets, questions, and assignments that can help them question the current rhetoric around standardized testing, "achievement gap," "failing schools," and others. For example, PISA results, graduation rates, or suspension rates, when segregated by race, ethnicity, and socioeconomic status, all provide a powerful starting point for a conversation about the inequities in the U.S. educational system.

PSTs should be provided with multiple opportunities to read the world with mathematics, to stretch their understanding of mathematics beyond the obvious uses of numbers, and encompass the messy mathematics that helps better understand oppression in particular and the world in general. Through the Common Core Standard for Mathematical Practice of modeling (CCSSI, 2010), introducing these messy contexts in content courses is easier now than it has been in the past. Also, because PSTs will not ask these questions themselves, it is important not only to use real-world data, but also to consider how they are obtained.

Finally, a stronger focus on problem posing is needed. Because I typically require PSTs to pose problems only once or twice in a semester, they are unable to ask the deep and meaningful questions I hoped for in the Tunnel assignment. In the future, I intend to make problem posing a larger component of the course, and in particular to scaffold the assignments for PSTs to be able to progress from straightforward to important and complex questions about their world. The instructor should explicitly model this process at the beginning of the semester, providing multiple opportunities, ample feedback, and, as already suggested, relevant contexts, to help PSTs grow in this process. The end-result should be, in the words of one PST, "being able to teach in a way that not only explains algorithms but explains and looks for solutions to social justice issues."

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Appendix: Tunnel of Oppression assignment

1. Attend the event. Allow yourself at least 45 minutes (most people take 50 minutes to an hour) to walk through it. If possible, go to a debriefing session available when you finish walking through the tunnel. Be warned that some of the scenes may make you uncomfortable. If you believe you will not be able to handle the emotional stress of going through the tunnel, please speak to me beforehand.

2. Write a ½-1 page reflection on the event. Describe what you saw and your reaction to the scenes. In addition, respond to the following questions:
   - How, if at all, did mathematics come up in the scenes?
   - How, if at all, did mathematics come up in the debriefing session (if applicable)?
   - Were there any situations in which mathematics was absent but would have been beneficial for understanding the issue?

3. Do some additional research about one or more of the scenes. Do some fact checking and data collection on the content of the scene(s). Then write a 1-2 page report to the Tunnel organizers. Your report should contain the following components:
   - At least three mathematical facts that were not in the Tunnel, from at least two different outside sources (site sources);
   - At least one application of addition/subtraction/multiplication/division;
   - At least one fraction;
   - At least one use of percents;
   - At least one ratio or proportion;
   - At least one mathematical argument combining the numbers you found, as in the examples given at the end of the document;
   - An explanation to the organizers about how more mathematics could have strengthened their argument, with concrete examples of mathematics that could have been used.

4. Feedback on reports and an opportunity to revise. In particular, feedback will be given on making arguments mathematical based your research.

5. Revised reports due.

I would like to share your reports with the students who created the Tunnel scenes, as well as with the Tunnel organizers. If there is interest, we can organize a session during which you can present your findings and conclusions to them. In the past years, the Tunnel organizers have taken into consideration feedback given by students, and in particular have included more mathematical data in the scenes as a consequence.

This assignment is somewhat open-ended, and there is no one right way to do it. One of my primary objectives is to show you how mathematics can strengthen ethical and philosophical arguments, and to teach you to "mathematize" the world around you. The main thing to keep in mind is that the assignment needs to be as mathematical as possible, while also keeping in mind the issues addressed in the Tunnel.

Below are excerpts from some successful reports from previous years:

"Let us take a look at one statistic and expand on it, that the more than half a billion bottles of water are purchased in the United States every year can circle the globe more than five times. If the average height of a water bottle is 9 inches, or approximately 0.75 feet, and the circumference of the globe is 24,900 miles, then the approximate 600,000,000 bottles times 0.75 feet is 450,000,000 feet or, divided by 5280 feet in a mile, 85,227.27 miles, is the distance of the bottles purchased in the US. Divide 85,227.27 miles by the circumference of the earth and you get the bottles circling the earth 3.42 times. Now, this does not match up to the originally stated amount of five times, but perhaps they were calculating with a different bottle height and more exact number of bottles, as I have to work with generalizations and approximations."

"Out of the 57.7 million [people living with depression[1]], only 4 million will receive any treatment for their anxiety, and only 400,000 receive the proper treatment for their illness. Using math to find the percentage that is only 6.9% of the total who will receive any treatment at all and only .07% of people who will receive the correct treatment for their particular illness. By using those percentages we can figure out that 93.1% of the 57.7 million go without any treatment at all and 99.93% go without the proper treatment. This leaves many untreated individuals vulnerable and even suicidal."

"According to the tunnel, Americans consume an average of 23 pounds of pizza each year, which is about 46 slices. I think I definitely exceed that amount. With the Unites States population currently at 313,286,647, an average of 7,205,592,881 pounds of pizza is consumed each year. This got me thinking. With all of the pizza being consumed, where are all of the pizza boxes going. Although the pizza boxes are recyclable, you can't recycle the parts of the box that have been soiled by the food. That is at least half of the box. So, most people just end up throwing the whole box away. According to the company Good News Reuse, enough pizza boxes are thrown away each year to circle the earth 26 times (goodnewsreuse.com)."

"I was most upset by the statistics concerning the trafficking of children. The fact that the *mean* age of girls coerced into the sex industry is 13, according to law-enforcement leaders is extremely disturbing (Seattle Times). This means that while there are girls both older and younger in the industry, 13 is the average age. The total estimated amount of children in the sex trade each year in the United States is 300,000. That's roughly 100 times the amount of students enrolled at PLU. If the aforementioned number of people estimated to be trafficked into the U.S. per year is 17,500, this is only 5.83% of the annual amount of child prostitutes in the United States. Furthermore, there are 39 total counties in Washington, and trafficking has taken place in at least 18 of them. This means that the *probability* of living in a Washington county where trafficking occurs is 46.15% - almost half. It is my personal belief that if this particular scene had utilized more mathematical applications, it would have been even more effective. In my own research, I found a variety of discrepancies in statistics - for example, some sources said there are 100,000 children in the U.S. sex trade annually, while others said 300,000. This shows the crucial importance of fact checking (these statistics are all estimates because it is impossible to acquire exact numbers)."

[1]http://www.anxietycentre.com/anxiety-statistics-information.shtml

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What are the benefits of learning and teaching mathematics through contexts related to oppression? What are the disadvantages?
2. How could K-5, 6-8, 9-12 students learn mathematics through contexts related to oppression?
3. What teaching approaches can help facilitate students' and preservice teachers' ability to uncover more complex mathematics in the real world?
4. What teaching approaches can help facilitate students' and preservice teachers' ability to gain a deeper understanding of social justice contexts?
5. What teaching approaches can help facilitate both uncovering complex mathematics and deeper understanding of social justice issues?
6. How can mathematics instruction for preservice teachers be responsive to their experiences and interests, while also addressing issues that will be relevant to them as teachers?

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