Vol. 5, No. 1 Fall 2013 ISSN 2153-0173 online

Teaching for Excellence and Equity in Mathematics

TODOS
Mathematics for ALL

An Affiliate Organization of the National Council of Teachers of Mathematics

TODOS
Mathematics for ALL

PEARSON

TODOS and the Editors of
TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS
express deep appreciation to PEARSON EDUCATION
for its generous sponsorship of this issue.

The mission of TODOS: Mathematics for ALL is to advocate for an equitable and high quality mathematics education for all students — in particular, Hispanic/Latino students — by increasing the equity awareness of educators and their ability to foster students' proficiency in rigorous and coherent mathematics.

Teaching for Excellence and Equity in Mathematics

IN THIS ISSUE

From the Editors

Five years ago, Cynthia Anhalt, Michael Matthews, Lawrence Lesser and Miriam Leiva (at the time, the first three were co-editing TODOS' semi-annual newsletter *Noticias de TODOS*) drafted a proposal to start a refereed journal for TODOS: Mathematics for ALL and submitted it on October 20, 2008 to the TODOS Board. The Board passed it unanimously and the first issue was published one year later. There have been issues in 2009, 2010, 2011, 2012, and now in 2013. Now that *TEEM* has spanned 5 years (and 20 articles), it is an appropriate time to reflect on the journal's trajectory, both within TODOS and in the field at large.

Several features of *TEEM* have been distinctive from the beginning. For example, there was an inclusive targeting of the readership to provide a scholarly and pedagogical resource for mathematics educators, practitioners, leaders, and administrators at all levels. (That said, the Editors would like to have more success in getting "From the Classroom" submissions written or co-written by classroom teachers for teachers.) Also, there was the bookending of each article with DARE (Discussion And Reflection Enhancement) questions, to facilitate their being not just read but actively used in settings such as professional development workshops, department meetings, regular courses, etc. (To this end, *TEEM* invites readers to "Share the DARE" and send us short (100-500 words) reports of how they have successfully used a particular *TEEM* paper.)

The first issue of *TEEM* was atypical, with its articles selected by a committee (consisting of the Editors as well as other TODOS leaders) from past issues of *Noticias de TODOS*. Since then, a rigorous double-blind review process has been used which ensures that a paper is judged on its merits without the external reviewers (or even the Editorial Panelist coordinating that paper's external reviewers) knowing the identity of the author and vice-versa. The acceptance rate to date for refereed articles is approximately 35-40%. *TEEM* issues also have a very high standard for proofreading, layout and art/aesthetics, despite no professionals employed to do these tasks. We also gratefully acknowledge assistance with the journal's database management provided by John Burdick (former preservice teacher, University of Arizona, and currently a high school teacher) and Bryan Fede (graduate student at University of North Carolina at Chapel Hill).

Certain *TEEM* policies have evolved over the years. Originally, submissions were solicited only two months each year, but now are accepted year-round. Originally, a TODOS membership was required to access any issue, but starting in October 2011, this requirement was limited to only the most current issue. This latter change struck a balance between encouraging TODOS membership while increasing the visibility and impact of *TEEM* in the broader mathematics education and education communities. Indeed, *TEEM* articles have already been cited in other respected refereed journals.

By having only one issue each year (so far), this journal has been able to take a more hands-on approach in which authors can get feedback from the journal at multiple stages of writing, even before submission (we welcome query emails if you have an idea to bounce off of us), to maximize the chance of producing a paper that will meet the standards of review and serve the readership well. For example, in the current issue, one Editor particularly enjoyed getting to apply personal knowledge of baseball to suggest some additional examples (that the authors incorporated) where baseball involves nonstandard uses of mathematics or language.

Now that the fifth issue has been published, Luciana de Oliveira (Teachers College, Columbia University) will join Marta Civil as a co-Editor, while *TEEM* co-founders Cynthia Anhalt and Lawrence Lesser retire as co-Editors but remain available in more limited roles as Associate Editors. Editors depend upon quality work from external reviewers and this issue gratefully acknowledges those who have reviewed since the publication of our fall 2011 issue. For information on reviewing or writing for TEEM, please see page 6 of this issue or the TEEM webpage **http://www.todos-math.org/teem** (which allows you to access a newly-archived webinar).

The current issue of *TEEM* includes two externally peer-reviewed articles. The issue leads off (so to speak) with ~~an~~ baseball-based intervention Eugenia Vomviridi-Ivanovic and Aria Razfar designed to engender empathy among preservice teachers about language issues. Then Marlene Kliman, Nuria Jaumot-Pascual and Valerie Martin describe the impact that can be made with informal mathematics education in libraries. Our third paper, invited and reviewed by the Editors, is Judit Moschkovich's adaptation of the Iris Carl Equity Address she gave at the 2012 meeting of the National Council of Teachers of Mathematics. With the publication of this (fifth) issue, the first four issues are now available to ALL. Enjoy our 2013 issue.

Lawrence M. Lesser
The University of Texas
at El Paso

Cynthia O. Anhalt
The University of Arizona

Marta Civil
The University of North Carolina
at Chapel Hill

Call for Manuscripts

We encourage the submission of manuscripts that are aligned with the mission of TODOS: Mathematics for ALL (see p. 2). Manuscripts in applied or action research, literature surveys, thematic bibliographies, commentary on critical issues in the field, professional development strategies, and classroom activities and resources are encouraged and welcome.

Please see **http://www.todos-math.org/teem** for guidelines and then submit complete manuscripts to **teem@todos-math.org.** The *TEEM* Editors welcome query emails about the suitability of proposed topics or approaches.

Call for Reviewers

Refereeing is not only a valuable experience and service to the profession, but is also an essential means to ensure that articles of high quality and relevance are published in a timely manner. To be eligible to be a reviewer (normally one manuscript per year), we invite you to send an email to **teem@todos-math.org** with the following information:

- Full name, affiliation, and contact information (including email, phone number, fax number, and mailing address);
- Grade levels (e.g., elementary, middle, secondary, college) where you have teaching or research experience; and
- Thematic areas with which you have particular interest and expertise, and any other pertinent professional information.

Your information will assist the editors in assigning papers to the various reviewers.

New TODOS LIVE! Webinar Available: "Reviewing and Writing for TEEM"

On July 22, 2013, Lawrence Lesser conducted a live webinar that explored the big picture and process for reviewing and writing for TEEM. The target audience includes classroom teachers, coaches, administrators, curriculum coordinators, professional developers and university/college faculty. To access the recorded webinar, see **http://www.todos-math.org/teem**.

In the Shoes of English Language Learners: Using Baseball to Help Pre-service Teachers Understand Some Complexities of Language in Mathematics Instruction

Eugenia Vomvoridi-Ivanović and Aria Razfar

Abstract

This article discusses qualitative data collected from elementary pre-service teachers (PSTs) who participated in an activity that uses the context of baseball to highlight some of the complexities of language in mathematics instruction. Through this activity, PSTs moved from a more discrete vocabulary orientation for teaching mathematics towards an embedded discourse approach, broadened their views on whom they classify as an English language learner (ELL), and developed empathy for the meaning-making struggles of ELLs in mathematics classrooms.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Who do you consider as an ELL? What are the characteristics of an ELL?
2. Imagine a student who speaks English fluently and does not have a "foreign" accent. Could this student be an ELL?

Eugenia Vomvoridi-Ivanović (eugeniav@usf.edu) is an Assistant Professor in Mathematics Education at the University of South Florida. Her research interests include the mathematics education of language minority students, mathematics teacher preparation for diverse student populations, teacher development in informal mathematics learning contexts, and culturally responsive mathematics teacher education.

Aria Razfar (arazfar@uic.edu) is an Associate Professor of Literacy, Language and Culture at the University of Illinois at Chicago. His research is grounded in sociocultural and critical theories of language, learning, and development. He teaches courses on linguistics for teachers and directs several nationally funded projects aimed at training teachers of English learners to develop academic literacy practices through mathematics, science, and action research.

In the Shoes of English Language Learners: Using Baseball to Help Pre-service Teachers Understand Some Complexities of Language in Mathematics Instruction

Eugenia Vomvoridi-Ivanović and Aria Razfar

Pre-service teachers (PSTs) enter the profession with little knowledge about the needs, resources, and support required to teach mathematics effectively to English Language Learners (ELLs) (Chval & Pinnow, 2010). They often have very simplistic notions about the language demands that are present in mathematics classrooms (Vomvoridi-Ivanović & Khisty, 2007). Their notion of mathematical language is reduced to mathematical terminology and they assume that simply providing their students with a vocabulary list or definitions of mathematical terminology will suffice. In addition, many of the PSTs and in-service teachers we have encountered tend to think of ELLs as students who have difficulty speaking in English or speak with a "foreign" accent. They would not view as an ELL a child who speaks English fluently or has a native-like American-English accent.

Activities that are often used with (monolingual) PSTs or inservice teachers to have them experience what ELLs experience in the classroom, and to perhaps model strategies that can be used to accommodate ELLs, are in a language that none of the PSTs speak (see Anhalt, Ondrus, & Horak, 2007). Such an activity, for example, is a mathematics problem written in a language the pre-service teachers are not familiar with, or a health video giving instructions in Farsi language (e.g., Harding-DeKam, 2007). While these activities can be useful to have pre-service teachers experience what it feels like to be an ELL who has recently moved to the US and speaks no English, the majority of the ELLs that pre-service teachers will be teaching will not fall into that category. In fact, most ELLs have some level of conversational fluency in English, and many of them might not have an easily detectable foreign accent, which makes it tricky for teachers who, mistakenly, do not classify them as ELLs. According to Cummins (1981), conversational fluency in English is acquired within 2 years, while it takes 5-7 years to acquire academic (including mathematical) fluency in English. This is an important distinction that teachers need to be aware of and understand its implications for teaching mathematics to ELLs. Pre-service teachers are often taught this distinction in their coursework but do not necessarily make connections with what this means for teaching mathematics to ELLs (Vomvoridi-Ivanović & Khisty, 2007).

This article discusses an activity intended to help PSTs and other educators understand some of the complexities of language in mathematics instruction and experience what many ELLs who have conversational fluency in English may experience in the mathematics classroom.

The importance of distinguishing "language" from "discourse" is central to teacher education, including professional development activities. Typically, the notion of language refers to the structural aspects of language (i.e., code) and/or the use of national languages (e.g., Spanish, English). In contrast, "discourse" refers to the specialized and situated language of mathematics that is generally more quantitative and symbolic.

In addition, the concept of discourse more explicitly includes performative, semiotic, and critical dimensions of language use (Gee & Green, 1998; Razfar, 2012). These dimensions account for language use in real communicative situations, focus on actual meaning-making, and how people draw on contextual cues and relationships when they purposefully use language to solve problems. Performative aspects of language use consist of paralinguistic and non-verbal dimensions of language like tone, intonation, loudness, pitch, gestures, facial expressions, and rhythm. Semiotic and critical dimensions of language use focus our attention on the less apparent aspects of communication, namely intentionality, meaning, values, histories, world-views, power relations, and *language ideologies.*

These aspects require in-depth ethnographic relationships in order to gain closer approximation of how people make sense of the complex web of interpersonal, institutional, and ideological relations that inform their lives. While consideration of semiotic and critical dimensions of discourse complicate our understanding of language use, they constitute a more complete view of language that is essential for understanding how language mediates mathematical problem-solving (Razfar, 2012).

As part of this activity, PSTs discuss and solve a mathematics problem in the context of baseball. The mathematical skills required to solve the problem are at elementary level. However, for someone who is not familiar with baseball and/or baseball discourse it is impossible for him/her to make sense of the problem. The purpose of the activity was to move PSTs from a *discrete vocabulary orientation* for the language used in teaching mathematics towards an *embedded discourse approach*.

An embedded discourse approach frames mathematical terminology as nested within activity systems mediated by cultural rules and concrete goals. In contrast, a discrete vocabulary orientation treats mathematical terminology as decontextualized entities abstracted from cultural practices and activity systems. In this orientation, mathematical terms are provided with absolute, fixed, and universal definitions often in the form of flash cards, word lists, and skill-based worksheets. Data collected from PSTs participating in this activity suggest that PSTs moved from a discrete vocabulary orientation for the language used in teaching mathematics towards an embedded discourse approach, developed a greater empathy for the meaning-making efforts of ELLs in a mathematics classroom, and broadened their views on ELLs to include those who have conversational (but not academic) fluency in English.

The Baseball Activity

The baseball activity is an adaptation of a version the authors created and used at a week-long seminar for the Center of Mathematics Education for Latinas/os (CEMELA) and also at the middle of the first author's mathematics methods course, after students completed relevant readings: Bresser (2003), Coggins, Kravin, Coates, and Carroll (2007), and Moschkovich (1999). At the beginning of the activity, the PSTs are asked to imagine an ELL and write a description of that ELL on a piece of paper. Then they are prompted to describe what the ELL they imagined looks like, sounds like, etc. The instructor then provides additional experiences that distinguish between language and discourse to foster empathy for the challenges of doing mathematics not only as a language learner, but also as a discourse learner.

First, PSTs are informed that they will solve a baseball problem working with peers who have similar knowledge about baseball. PSTs are asked to self-identify as being a member of one of three groups, the latter two of which are labeled as "Baseball Language Learners" (BLLs):

- Group 1: Baseball experts, PSTs who are very knowledgeable about baseball and can fluently talk about the sport.
- Group 2: Baseball novices, PSTs who have only basic knowledge about baseball.
- Group 3: Foreign to baseball, PSTs with no or almost no knowledge of baseball, other than that it is a sport. They could not explain how the game is played.

Second, PSTs work in their groups to define the following baseball terms: slug, bat, batting three hundred, ball, strike, diamond, base, steal, stealing home, hit and run, Triple Crown, run, out, balk, and save. Groups are asked to share their definitions with the rest of the class starting with Group 3, then Group 2, and then Group 1. As might be expected, the baseball experts are easily able to define all these terms. However, the BLLs, struggle to accurately make sense of the terms within a baseball context. The point of the activity becomes clear as many of the BLLs express that they experience what it is like to be an ELL since, although they all speak English, they do not speak baseball.

As a result, the word 'bat' conjures a flying mammal rather than a stick and is viewed only as a noun as opposed to both noun and verb. 'Ball' represents a spherical object instead of a pitch that is not good to hit. To 'strike' something is equivalent to hitting it whereas in baseball, a batter that swings the bat and completely misses the ball gets charged with a strike. Once the class reaches consensus on each definition, the definitions are displayed on poster boards.

Third, PSTs are informed that they are to collaborate with their group members to solve the following baseball problem (see Appendix for a solution) that includes some of the baseball terms they previously defined:

> *Barry Bonds, one of the most prolific home run hitters of the modern era, slugged over "eight-hundred" in one season. If he had six hundred at bats, how many total bases did he get?*

As each group attempts to solve the problem, the "BLLs" are asked to monitor the process of trying to understand the problem and to note the resources that they draw from to make sense and solve the problem. At the same time, the "baseball experts" are asked to think about how they would help the BLL groups make sense of the problem and its solution.

Fourth, BLLs are asked to share their solutions and the processes they went through as they tried to make sense of and solve the problem. Members from the BLL groups become increasingly frustrated as they go back and forth from the definitions to the problem text and still fail to make meaning of the text. Some BLLs make parallels between what they experience through this process and trying to make sense of a text written in a foreign language that they do not understand with the aid of a dictionary. All agree that it is almost impossible to understand a foreign text this way. This is perhaps the first time that many of them realize that discrete dictionary definitions of words do not provide sufficient context to engage in embodied meaning making. This leads to a whole class discussion on *register* (Halliday, 1978) and on why it did not suffice simply to go over key baseball terms (vocabulary) at the beginning of the activity.

Fifth, the groups are rearranged so that there is at least one "baseball expert" in each new group. The "baseball experts" are asked to assist the BLLs in understanding and solving the problem. Typically, "baseball experts" try to explain the problem using various visual aids and/or physical representations, such as drawings, diagrams, and acting out aspects of a baseball game. Despite the experts' efforts, the BLLs, especially those in Group 3, typically do not fully understand the problem and its solution and express that they would not be able to solve another similar baseball problem. A whole class discussion follows as various groups share their solutions and the methods the "experts" employ to assist the BLLs.

Sixth, baseball experts are asked to "talk baseball" and to argue about a baseball related issue. After listening to the experts "talk baseball" the BLLs quickly feel at a loss and experience what it is like not to be a member of a particular discourse community.

Finally, "lessons learned" are discussed from this experience, particularly as they relate to ELLs. In this discussion, PSTs are expected to make connections with relevant readings. These prompts generate very rich discussions around language, context, mathematics, and ELLs. BLLs, for example, may interpret a slugging average of "eight hundred" as '800' rather than 0.800, which is what it means within baseball. Others interpret the slugging average as a percentage, thus, they set up the slugging percentage as 800%, which is incorrect also. This brings up the point that mathematical meaning is situated in the context and common language is shared and understood by a community, such as this case, a baseball community. BLLs often use the two numbers 800 (given in the problem) and 4 (the number of bases implied by a homerun) and perform some mathematical operation such as total bases equals 4×800 or 800/4. This is characteristic of how students generally approach a problem: if they can't make sense of the problem, they focus instead on key words and numbers.

PSTs' Post-Activity Reflections

As a course assignment, PSTs write a reflection with the following prompt: "What kinds of insights have you gained from this experience that relate to the teaching and learning of mathematics for ELLs?" A total of 129 PSTs' reflections were collected: 105 from the three undergraduate level sections and 24 from the one master's level section that the first author taught. These reflections were completed by all the PSTs who participated in this activity and, since they were a course assignment, were graded. To protect PST's identities, the first author removed all identifying information from the reflections prior to data analysis. A grounded theory methodology (Strauss & Corbin, 1990) was used to identify recurring themes in the PSTs' reflections. Our coding scheme aimed to characterize the nature and content of the PSTs' written comments so as to identify patterns in the insights that PSTs gained through the baseball activity. From the analysis across all 129 PSTs' reflections, three major insights (discussed in the following three subsections) emerged with respect to teaching mathematics to ELLs.

From Vocabulary to Discourse

The vast majority of PSTs (123 out of 129, or 95%) realized that knowing vocabulary and having definitions of terms available were not sufficient for understanding text. Many of the comments suggest a change from a more discrete vocabulary orientation to an embedded discourse approach, such as this PST's reflection:

I used to think that just having a dictionary with all the math terms along with the math book would be enough for an ELL to understand what the book says. When we did the activity in class and we went over the terms of baseball before being given the problem, I still was not able to understand exactly what the question was asking even though it was in English.

Another PST noted:

> Before this activity, I assumed that providing an ELL with a list of vocabulary and definitions, giving a lot of visuals, and sitting them with a more fluent English speaker would be enough to help them understand conceptually. Now I see there is more to it and I need to make sure my ELLs have opportunities to develop the language of mathematics in English.

In their reflections, many baseball experts expressed how difficult it was to modify their talk in order to explain the problem to the BLLs who did not "talk baseball" and were not competent members of the baseball discourse community. One PST, for example, wrote:

It was so hard to explain to BLLs how this problem is solved without using baseball language. I think it is the same way when teaching math to ELLs, we need to be conscientious of the language we are using and modify it so that they understand what we are teaching them and at the same time learn the new language.

Baseball experts also shared how they became so knowledgeable about baseball and how they came to be members of the baseball discourse community by playing baseball, watching baseball games, interacting with "baseball experts," forming and expressing opinions about baseball, discussing baseball-related issues, etc. They noted that just as one does not become a member of the baseball discourse community simply by learning definitions of baseball terms, the same is true with mathematics. As one "baseball expert" commented, "Why don't we learn math just how we learned baseball… watching, playing, listening, and talking about baseball? With math it's mostly watching and listening to the teacher." Baseball experts expressed that showing BLLs how to solve the problem did not lead to BLLs understanding the language of baseball, especially to those who were not familiar with baseball. For example, one "baseball expert" wrote:

It would have been very easy to give BLLs the formula and assume they understand the problem because they can solve it using the formula. But that doesn't mean they know baseball or understand the language used in baseball or can solve a related problem. Mathematically they might be able to do the problem, but they may (will) not understand it because of the language, even if they have a mathematics dictionary available.

Along the same lines, a BLL wrote:

> Just because I was shown how to solve this problem, it doesn't mean I really understand. Like if you give me the same exact problem with different numbers, sure I can solve it, but if it is worded differently or if there is another baseball math problem, I would be lost. So it's the same way with math. We can't assume that because our students solve something because they memorized a formula that they really understand it.

In the Shoes of ELLs

For 112 out of the 129 PSTs (87%), this activity provided a context to develop empathy for the meaning-making struggles of ELLs, even those who have conversational fluency in English. While the PSTs knew the mathematics content necessary to solve the problem, they had never experienced 'language barriers.' As one of the pre-service teachers commented:

> I really didn't have an insight to how ELLs personally must feel in math class until we did the baseball activity in class. I felt like I had a basic knowledge of baseball, I actually thought it was better than most people. When it came time to do the problem, though, my confidence lessened. It was remarkable to me how lost I was, particularly since math has always been a strong subject for me. The problem seemed so hard and I could not even start the problem, let alone figure it out. What made me even more surprised was when the problem's formula was written out on the board and it was such a simple algebraic problem, that I know I could have figured out.

Many of the PSTs observed that most of their methods courses, even English as a Second Language (ESL) courses, did not provide activities that engender this type of empathy for ELLs. The following comment illustrates this point:

> If you are not an ELL then it is hard for you to understand what it feels like to be one! Sure we can attend ESL classes and learn how it is best to teach them and that is helpful. But it is so hard to actually understand what it is like to have to learn in school when you cannot understand what is being said. I found the baseball activity to be a good demonstration of what it feels like to be an ELL student.

BLLs noted that they particularly felt at a loss when listening to the "baseball experts" talk and argue about baseball. As one BLL commented: "They (baseball experts) were talking and talking and I could just catch some words that I understood, and it was so hard to follow them. I guess that's what it might feel like to be an ELL in a math class during math discussions."

Rethinking Who is an ELL

This activity also helped PSTs broaden their views on whom they classify as an ELL. About three-quarters (98 out of the 129) of the PSTs expressed that although they had initially described an ELL as a student who "does not speak English very well" and/or who "has an accent," this activity changed their views to include all students who speak another language at home and whose first language is not English. Many students are not typically classified as ELLs because they are conversationally fluent in English but they are not 'discourse fluent.' This is an important realization not only for math educators of ELLs, but also for all language minority students, as exemplified by this comment:

> It also made me realize one more thing. There is a student in the class I am observing who is from Peru and has a slight accent but speaks English very well. After doing this activity I realized that maybe he is only fluent in everyday English but might feel the same way I did today during the baseball activity. I really had no clue he could be going through this in class! I need to look into it.

Of course, it will be important for teacher educators to ensure that PSTs do not fall into making "deficit assumptions" about students such as this example of a Peruvian student, or assuming that everybody who speaks another language at home or whose first language is not English is automatically an ELL.

Final Thoughts

This paper described an activity in the context of baseball in which PSTs gained insights on some of the complexities of language in mathematics teaching. Those PSTs who were baseball novices experienced what it is like to have conversational fluency and know the mathematics content but not be able to solve a problem because of lack of specialized baseball language and not being part of the baseball discourse community. Those PSTs who were very knowledgeable about baseball and its language realized how difficult it is to have the rest of the PSTs who are not part of the baseball discourse community make sense of the problem without having experience with baseball. Through this activity, PSTs broadened their views on whom they classify as an ELL and developed empathy for the meaning-making struggles of ELLs in mathematics classrooms.

Baseball has proven to be a strategic context for elementary PSTs, as only a few consider themselves baseball experts. This has allowed for the formation of both baseball expert and BLL groups. A problem in the context of soccer may yield a similar formation of groups in populations who are likely more familiar with baseball than soccer. It would be worthwhile, however, to do parts of the baseball activity even with teacher populations who are very familiar with baseball. Such a population might not experience what it is like to be an ELL but may be able to explore further how mathematical meaning and procedures are distinctively situated in a particular context. In baseball, for example, if a batter gets 1 hit in 3 at bats and then the next day gets 2 hits in 5 at bats, her overall accumulated batting average is computed as (1+2)/(3+5) = 3/8 = .375, a very different result from what is obtained from (performing the common denominator algorithm for) adding the fractions 1/3 and 2/5.

Also, statistics for how many innings a pitcher pitches are often represented as decimals in nonstandard ways. Since an inning (technically, a half-inning) has three outs, the only possible "fractions" of an inning for a pitcher to pitch are 1/3 or 2/3, but those fractions are usually represented in baseball statistical summaries as .1 and .2, respectively. Furthermore, the infield is referred to as a "diamond," which could suggest a non-square rhombus rather than the square that it is.

Although the baseball activity has been presented as a context (others might include medicine, the sport of cricket, etc.) that can help educators get 'in the shoes' of ELLs and gain insights about teaching mathematics to them, what is most important is not whether English is the language that a student started with or hears at home. The important issue is that students have had the opportunities to develop the academic language and discourse skills needed to be successful in mathematics when it is taught in English. In contrast to other aspects of language, which are very robust in society (for example, storytelling), the language for mathematics is developed mainly in school. Thus, future teachers need to realize that what this article describes also applies to native speakers of English. Just because students are native speakers does not mean they have the language and discourse skills they need to be successful in mathematics. In this sense, all children are MLLs (mathematics language learners, analogous to the Mathematics as a Second Language designation in Winsor (2007)), whether they are labeled as ELL or not. Thus, what teachers develop to help ELL students become more proficient with mathematical language may often be very beneficial for many other students as well.

In order to move towards preparing teachers to teach mathematics to ELLs, mathematics teacher education programs and professional development opportunities need to be improved so that they develop teacher knowledge related to teaching ELLs. The development of this knowledge should not be deferred to additional certification programs or professional development, but rather needs to be initiated early in the preparation process (Chval & Pinnow, 2010). It is hoped that readers will find ways to modify or expand the baseball activity discussed in this article for use in various teacher preparation and professional development contexts.

References

Albert, J. (2003). *Teaching statistics using baseball*. Washington, DC: Mathematical Association of America.

Anhalt, C., Ondrus, M., & Horak, V. (2007). Issues of language: Teacher insights from mathematics lessons in Chinese. *Mathematics Teaching in the Middle School, 13*(1), 18-23.

Bresser, R. (2003). Helping English language learners develop computational fluency. *Teaching Children Mathematics*, *9*(6), 294-299.

Chval, K. B., & Pinnow, R. (2010). Preservice teachers' assumptions about Latino/a English language learners. *Teaching for Excellence and Equity in Mathematics*, *2* (1), 6-12.

Coggins, D., Kravin, D., Coates, G. D., & Carroll, M. D. (2007). *English language learners in the mathematics classroom.* Thousand Oaks, CA: Corwin Press.

Cummins, J. (1981). The role of primary language development in promoting educational success for language minority students. In California State Department of

Education, *Schooling and language minority students: A theoretical framework* (pp. 3-49). Los Angeles: California State University, Evaluation, Dissemination, and Assessment Center.

Gee, J. P., & Green, J. (1998). Discourse analysis, learning, and social practice: A methodological study. *Review of Research in Education*, *23*(1), 119-169.

Halliday, M. A. K. (1978). *Language as social semiotic: The social interpretation of language and meaning.* London: Edward Arnold.

Harding-DeKam, J. L. (2007). Foundations in ethnomathematics for prospective elementary teachers. *Journal of Mathematics and Culture, 2*(1), 1-19.

Moschkovich, J. (1999). Understanding the needs of Latino students in reform-oriented mathematics classrooms. In L. Ortiz-Franco, N. G. Hernández & Y. De La Cruz (Eds.), *Changing the faces of mathematics: Perspectives on Latinos* (pp. 23-33). Reston, VA: National Council of Teachers of Mathematics.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning*, *4*(2/3), 189-212.

Moschkovich, J. (2007). Examining mathematical discourse practices. *For the Learning of Mathematics, 27* (1), 24-30.

Quinn, R. J. (1996). Having fun with baseball. *Mathematics Teaching in the Middle School, 1(*10), 780-785.

Razfar, A. (2012). Discoursing mathematically: Using discourse analysis to develop a sociocritical perspective of mathematics education. *The Mathematics Educator, 22*(1), 39-62.

Ron, P. (1999). Spanish-English language issues in the mathematics classroom. In L. Ortiz-Franco, N. G. Hernández & Y. De La Cruz (Eds.), *Changing the faces of mathematics: Perspectives on Latinos* (pp. 23-33). Reston, VA: National Council of Teachers of Mathematics.

Vomvoridi-Ivanović, E., & Khisty, L. L. (2007). The multidimensionality of language in mathematics: The case of five prospective teachers. *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 991-998). Stateline, NV: University of Nevada, Reno.

Weist, L. R. (2008). Problem-solving support for English language learners. *Teaching Children Mathematics, 14* (8), 479-484.

Winsor, M. S. (2007). Bridging the language barrier in mathematics. *Mathematics Teacher*, *101*(5), 372-378.

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Revisit the two pre-reading DARE questions and discuss how your answers may have changed.
2. How has your thinking regarding teaching mathematics to ELLs changed after reading this article?
3. What kinds of experiences would the BLLs need to make sense of the problem? How do those experiences relate to the teaching and learning of mathematics?
4. What is the difference between conversational language and the language needed to participate in a mathematics discourse community?
5. What background information or examples from Albert (2003) or Quinn (1996) might be useful to add to the baseball activity?

Appendix

Baseball problem:

Barry Bonds, one of the most prolific home run hitters of the modern era, slugged over "eight-hundred" in one season. If he had six hundred at bats, how many total bases did he get?

Solution to Baseball problem:

First, someone with knowledge of the context of baseball recognizes that "slugged over eight-hundred" (which means .800 in baseball language spoken as a three-digit whole number) does not mean "slugged over 800 home runs" (the record for a season is well under 100), but refers to the value for his slugging average. Slugging average measures the power of a hitter by dividing the total number of bases attained by the total number of official at bats, where walks do not count as official at bats and each single, double, triple, and home run generates 1, 2, 3, and 4 bases, respectively. An ideal player who gets a home run on every at bat would attain the maximum possible slugging average of 4.00.

Bonds slugging over .800 in one season means that his slugging average over the course of that season can be computed the following way:

Slugging Average = Total Bases /At Bats

0.800 = Total Bases / 600

Total Bases = 600 * 0.800 = 480 bases

Since Barry Bonds slugged over .800, he got at least 480 bases that season from some combination of singles, doubles, triples and home runs. Readers can look up online to see that Bonds had slugging averages over .800 in two seasons: 2001 (.863) and 2004 (.812), and .863 is the highest value of any player in history.

Editor's Notes: Slugging average (the mean number of bases obtained per official at bat) is a number from 0 to 4.000, which is not consistent with a percentage. However, readers should know that these words are sometimes used in baseball as if they were interchangeable. For example, on the Major League Baseball (MLB) page *mlb.com/stats/*, hovering over the column heading SLG reveals the term "slugging percentage", while the Baseball Almanac page *http://www.baseball-almanac.com/hitting/hislug2.shtml*, uses the term "slugging average." The Baseball Almanac site remarks, "In 1920, Babe Ruth set the all time single season record when he hit fifty-four (54) home runs in four-hundred fifty-eight (458) at bats (plus his other extra base hits) giving him an unbelievable slugging average that year of .847. So unbelievable that when Barry Bonds crushed the record in 2001 he secured his place in baseball immortality."

TEEM readers should also be aware that Bonds was convicted in 2011 on obstruction of justice during the U.S. Government's investigation into the use of steroids. Additionally, keep in mind that baseball statistics sites such as the ones mentioned above can be used to create mathematics problems using any players' statistics.

Designing for Diversity: Strategies for Embedding Mathematics in Out-of-school Programs for Children in the Elementary Grades

Marlene Kliman, Nuria Jaumot-Pascual, and Valerie Martin

Abstract

Informal (out-of-school) education, with emphasis on local community and resources, can be particularly beneficial to children from non-dominant cultures. To support integration of more mathematics into such programs, we worked with informal educators based in public libraries (including librarians and after-school educators) to create and make available English and Spanish mathematics activities that they could embed in their daily work with children. We discuss self-reported impacts on informal educators' math-related attitudes, beliefs, and professional practices.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Do you think that mathematics is culture-bound and contextual? Why or why not?
2. Are there circumstances in which it seems appropriate to view mathematics as devoid of context and culture? Explain.
3. What do you think is an appropriate role for mathematics in out-of-school programs such as after-school and library programs?

Marlene Kliman (marlene_kliman@terc.edu) is a Senior Scientist at TERC, a non-profit STEM education organization in Cambridge, MA. Her development and research focus on collaborating with a wide range of informal educators to explore ways to integrate interdisciplinary math into their work with a broad range of children and families.

Nuria Jaumot-Pascual (Nuria_Jaumot-Pascual@terc.edu) is a Senior Research Associate at TERC. Her main research interests and expertise lay in the confluence of informal education, professional development, and diversity.

Valerie Martin (Valerie_martin@terc.edu) is a Lead Designer at TERC, where she works with formal and informal educators to ensure that math-related images and materials formats engage a diverse audience.

Acknowledgements The authors are grateful to the National Science Foundation for its support of MATH OFF THE SHELF (MOTS) (Grant DRL-0714537). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. The authors also are grateful to TERC colleagues Martha Merson and Lily Ko for partnering with us on this project, to Cindy Char (Char Associates) for an insightful approach to gathering and analyzing impact data, and to the many library-based informal educators who shaped the development of the MOTS activities.

Designing for Diversity: Strategies for Embedding Mathematics in Out-of-school Programs for Children in the Elementary Grades

Marlene Kliman, Nuria Jaumot-Pascual, and Valerie Martin

Imagine the chart in Figure 1 is posted near the checkout desk of a public library branch in a large city. A group of children ages 8-11, having put their own information on the chart, eagerly watches the data set evolve as passers-by contribute. They silently root for even more stickers in the columns for the #5 and #23 buses, which are slated for elimination. The city Transportation Department has proposed cutting these bus routes in order to close a budget shortfall; the potential cuts have particular impact on low-income neighborhoods—including the area in which the library branch is located. Like many in the neighborhood, the children and their families rely heavily on the 5 and 23 buses for transportation to jobs, to shops, and to the library, which lies right along the bus routes.

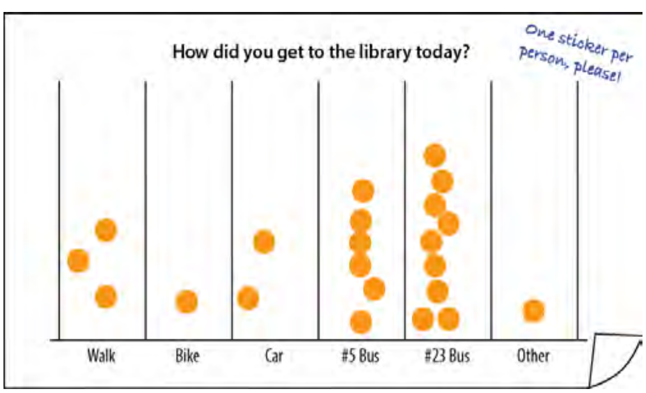


Figure 1. Collecting data on an issue of community concern.

Lupe, the children's after-school group leader, encourages them to think of how they can demonstrate to the Transportation Department the extent to which cutting service will impact their community. The group decides to gather data to see how many library patrons arrive by bus. While not a foolproof method of proving the value of the bus service (e.g., some people might be able to walk or drive if bus service were cut), it is an approach that yields a benchmark.

As the dots on the chart accumulate, children reflect on the growing patterns of responses: Marcel notes that the majority of respondents arrived by bus; Sofia observes that the number who took the 23 bus alone is greater than the number of non-bus arrivals; Ximena, reflecting on prior group conversations about characteristics of different neighborhoods in the city, wonders whether there would be so many bus users if they collected data at a library branch in Rosedale, a wealthy neighborhood across town. Lupe guides the group to consider what constitutes a sufficient sample for presentation to the Transportation Department. They decide to gather data for a week, requesting only one response per person.

At the end of the week, the group has 201 responses, with 158 indicating arrival by the #5 or #23 bus. They make a plan to write to the Transportation Department to ask them to reconsider their plan to cut the bus routes because, according to their data, the large majority of library patrons—158 out of 201, or 79%—arrive by bus that week. In their letter, they will underscore the fact that without a way to get to the library, many residents no longer have access to free computers, ELL classes, and after-school programs. These children are learning to use math as a way to explore and potentially make a case for addressing what they see as an injustice.

Each week, as Lupe plans programs for and with the children, she finds ways to engage them in collecting and analyzing data to enhance the topic: sometimes data serves as a way to spark discussion of a social issue; sometimes as a springboard for investigating commonalities and differences among the group; and sometimes, as in this instance, as a basis for taking action.

Role of Mathematics in an Out-of-school Program

Although topics that form the core of out-of-school programming for the elementary grades are replete with mathematics opportunities, scenes such as the above—in which children use mathematics in a way that resonates with their interests—are rare. One obstacle to integration of mathematics in such programs is that many adults lack mathematical comfort and confidence. Informal educators, from after-school providers to librarians to parents, care deeply about children's mathematical success, but they often are math-avoidant themselves and thus shy away from mathematics with children (Gasbarra & Johnson, 2008; Intel, 2009). Another obstacle is a widespread conception of mathematics as devoid of context. In everyday life, adults estimate, measure, and navigate, but they do not typically think of these as mathematics and do not share strategies with children (Esmonde, 2013; Kliman, 2006; Lange & Meaney, 2011). Rather, they are likely to view mathematics as a set of culturally neutral facts and algorithms (Allexsaht-Snider, 2006; Martin, 2009a, 2009b).

When informal educators do integrate mathematics into their programs, children stand to benefit in several ways. For all children, participating in out-of-school activities that embed mathematics—from tithing to card playing to shopping—bolsters skill development, appreciation of the relevance of mathematics, and mathematics attitudes (Guberman, 2004; Harris, 2011; Nasir, Hand, & Taylor, 2008). Combining mathematics and social justice in an after-school mathematics project can also lead to children to develop mathematical understandings needed to explore and address injustices (Turner, Varley Gutiérrez, & Díez-Palomar, 2011; Turner, Varley Gutiérrez, Simic-Muller, & Díez-Palomar, 2009; Simic-Muller, Turner & Varley, 2009).

For children least likely to succeed during the school day, including children from non-dominant cultures and children from low-income families, out-of-school programs confer particular benefits, in part because discontinuities between home/community and school cultures are substantially mitigated (Noam, Biancarosa, & Dechausay, 2003). For instance, unlike in public school, where 83% of teachers are White (Cowan, 2010), after-school staff members usually reflect the diversity of the enrolled children, and many have strong local community ties. The emphasis on local resources and community-based activities prevalent in elementary-grades out-of-school programs can promote belonging and self-esteem, which are important ingredients in learning (Miller, 2003).

Although informal educators rarely embed mathematics in their offerings, they *could*: they typically have substantial autonomy in programming, unlike school teachers, who may be limited in efforts to integrate children's everyday experiences in mathematics class because of required testing and curriculum (McCulloch & Marshall, 2012; Wager, 2012). To engage children, particularly those who flourish in out-of-school settings but struggle in school, a critical first step is to engage informal educators (especially those who may be math avoidant themselves). This paper describes impacts of a project intended to spark a new mathematical reality for out-of-school programs and the informal educators who lead them.

MATH OFF THE SHELF Project Background

TERC (originally known as Technical Education Research Centers) initiated MATH OFF THE SHELF (MOTS) to investigate strategies for bolstering the presence of mathematics in a wide range of library-based programs for elementary-grade children. Public libraries exist in virtually every community in the nation, and increasingly, families rely on them as a free, safe place for children to spend time in the absence of other out-of-school care. Library-Based Informal Educators (LBIEs) —including after-school providers, children's librarians, and youth workers— offer programs such as story and craft times, summer reading events, and drop-in after-school activities.

Design Phase

This two-year phase began by collaborating with LBIEs to create interdisciplinary English and Spanish mathematics activities that they could embed in the projects, activities, and conversations that form the core of their daily work with children. We employed an iterative design process in conjunction with several dozen LBIEs in four regions: Queens, NY, Westchester County, NY, and several communities in MA and CT. The majority of our LBIE partners were based in urban areas with significant low-income Latino/a or African-American population. First, we solicited from LBIEs upcoming programming themes (e.g., poetry month), special events (e.g., El día de los niños/El día de los libros), and needs (e.g., games children can play quietly while waiting around, activities in

English and Spanish that allow family members of all ages to participate together). For instance, two of the many family activities created include "Say It with Shapes / Jugando con Geometría" described below, and an activity involving creating towers from recycled materials.

After about 75 initial LBIE partners chose among, implemented, and gave feedback on the activities, we revised and then invited a wider group to try them. During this period, we gathered feedback via dozens of observations, primarily at local sites, and hundreds of phone conversations with LBIEs at distant sites. We abandoned or conducted substantial revisions to activities that the majority of LBIEs reported were not engaging to children, or which LBIEs chose not to implement. If most LBIEs described leading an activity in a didactic manner or noted that children seemed to be doing little or no mathematics during the activity, we typically revised to incorporate questions and prompts LBIEs could use to draw out the mathematics as they interacted with children.

Our process continued until we had a varied bank of vetted activities, including dozens each of projects, games, and short activities (http://mixinginmath.terc.edu) that connected with NCTM Standards for the elementary grades (NCTM, 2000). The activities were developed prior to but address many key K-5 topics in the *Common Core State Standards for Mathematics* (NGA, 2010). Some examples follow:

Using Mathematics to Spark Exploration of a Community Issue

The data collection activity described above is based on "Quick Questions" (see Appendix), in which children collect data, explore range, mode, and overall data shape, and gather and analyze samples from different populations. LBIEs and children have used this activity to investigate a variety of topics, such as local demographics (e.g., language(s) library patrons speak at home, country of birth).

Using Mathematics for Self-expression

Crafts, projects, and development of personal voice are integral to many library-based out-of-school programs for the elementary grades. "Say It with Shapes" offers an opportunity for self-expression with patterns and words. Children select from among a set of shapes, each imprinted with an English or Spanish word, to create a poem such that each line follows a visual pattern. Children may use any pattern, as long as they can describe it, whether with words ("triángulo, triángulo, cuadrado"), letters ("AAB"), or in some other way. The activity includes blanks, so that children or LBIEs can contribute words in any language.

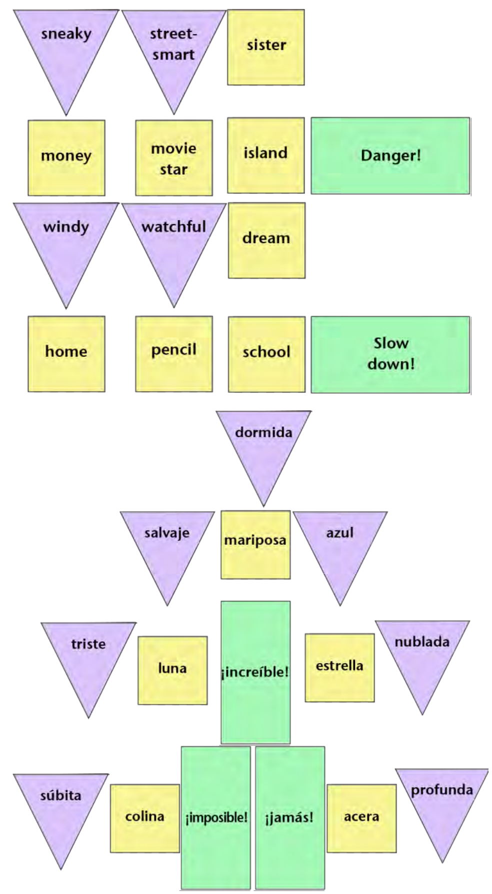


Figure 2. Creating poetry with patterns.

Using Mathematics to Engage Children in Assuming Responsibility in Daily Activities

At some library-based programs, children help prepare daily snack; with "Double or More," children also do mathematics. They start with a recipe for one or just a couple of people, and they work together to adjust the recipe for the group. Younger children might determine the increased quantities by repeated measurement, older ones by repeated addition or by multiplication. Even those who measure with handfuls and pinches, common across many cultures, need to attend to keeping amounts in proportion as they adjust the recipe. Unlike with textbook word problems about cooking, with "Double or More" children choose the recipe or bring it from home, decide how much to increment it, and then go on to make and eat the food.

Figure 3. Staying in proportion.

Extend-and-evaluate Phase

At the start of this three-year phase, some selected activities from MOTS were made available on a public website (http://mixinginmath.terc.edu). For evaluation purposes, we promoted the activities to groups of LBIEs in San Jose (CA), St. Louis (MO), selected low-income communities in Arizona and Florida, and additional sites in the original four regions. At each site, we connected with a library leader or library-based after-school leader, who in turn, encouraged LBIEs to review our website and use any of activities they wished, and to provide us with information on what they used and how. We used their input as a basis for final revisions and for development of activities to meet additional LBIE needs.

During this phase, our independent evaluator, Char Associates, surveyed LBIEs on impact of exposure to MOTS activities on a variety of math-related topics, including their attitudes and beliefs, incorporation of mathematics into their work with children, and reasons for using mathematics. Baseline data was gathered on a subset of these issues at the beginning of the project by the independent evaluator. Data reported in this paper are drawn from evaluator reports (Char & Foote, 2009; Char & Berube, 2010; Char & Clark, 2011) and are hereafter referred to only by year.

Evaluation of the Project

For each of three years, LBIEs exposed to MOTS activities in the aforementioned regions were invited via e-mail to click on a link to a survey containing about 40 multiple choice and open-response items. (We use "exposed" to mean learned of the MOTS activities and website at least four months earlier.) Participants received the survey link from the evaluators or from their supervisors, but raw survey data was accessible only to evaluators. Each year, the response rate was approximately 50%. Within each year, each LBIE generally came from a different library. From year to year, the LBIEs included some different people (due to turnover of LBIEs) but from the same regions.

Table 1
Number of LBIEs Surveyed

Survey year	2008 (baseline)	2009	2010	2011
Number of LBIEs	67	28	83	148

Each year, LBIE respondents reflected a range of involvement (e.g., from participant in the design phase to awareness of the project through a regional e-mail), number of months/years since initial exposure to MOTS activities, professional role, and use of MOTS (from those who chose to use it daily to those who reported never using it). In this paper, we provide data in aggregate for each year, as disaggregation and tracking individuals over time was beyond budgetary constraints. Survey questions varied to some extent from year to year.

Change in Frequency of Mathematics Activities Offered in Library Settings

The LBIEs that participated in the project offered activities such as story times, library orientations, craft projects, and activities children can do independently. At baseline, approximately 10% of LBIEs surveyed ever used mathematics in *any* of these contexts (2009). Because of exposure to MOTS activities, the majority reported providing a variety of math-related offerings at least monthly (see Table 2).

Many LBIEs included mathematics weekly or even daily: 28% reported including mathematics in craft activities with children weekly, and 3% daily; 20% of those surveyed now led math-related activities for families at least

Table 2

Changes in Incorporation of Mathematics into Offerings

Year	Percent of LBIEs from that year (see Table 1)	"Because of exposure to MOTS, I now *at least monthly...*"
2009	74%	... combine mathematics and craft activities
2009	53%	... incorporate mathematics in story times (e.g., with choice of books, conversations about books, questions I ask when reading to children etc.)
2009	26%	... fold mathematics into library orientations
2010	53%	... create my own mathematics-related activities (apart from using MOTS activities)
2010	53%	... offer mathematics-related independent activities

monthly, compared to 2% baseline (2010).

In explaining how they implemented their more frequent math offerings, LBIEs emphasized that participation in the project enabled them to find ways to integrate mathematics into what they are already doing. For instance, one LBIE who regularly led story times noted that now, "I'm more apt to bring in a ruler or different sized objects to illustrate part of a story" (2010). As a result of MOTS activities, this LBIE incorporated a mathematical lens into story times, drawing out sizes and measurements in the stories.

Change in Communication about Mathematics

In addition to *doing* more mathematics with children, LBIEs reported *talking about* mathematics a great deal more: 61% noted that because of MOTS, the nature of their communication with children changed; 54% noted changes in regular communications with parents and caregivers (2011). A full 50% reported ability to explain how mathematics for elementary grades fit the library mission, compared to only 5% at baseline (2009, 2011). One key topic was the role of mathematics in everyday life. At baseline, only 11% reported ever discussing this with children (2009), but this percentage was found to be much higher (see Table 3) after exposure to MOTS.

LBIEs reflected that accompanying this change in practice was a change in their own abilities. At baseline, 5% reported being able to explain how mathematics offerings for grades K-6 fit the library mission, compared to 50% after exposure to MOTS activities (2009, 2011).

Table 3

Changes in Communication about Mathematics in Everyday Life with Children (2010)

Percent of total respondents (*N*=83)	Because of exposure to MOTS, I now talk about mathematics in everyday life with children....
8%	at least once daily
30%	about weekly
19%	about monthly
27%	less frequently than monthly
13%	never
2%	(no response given)

Note: Percentages do not add to 100 because of rounding.

Some LBIEs noted that they communicated more about mathematics because they now had a way to do so that meshed with their library work. For instance, assessing changes in communication about mathematics, an LBIE noted, "I [now] can talk about math without feeling/acting like a classroom teacher." (2009). Traditional classroom teaching can be a pervasive model for how mathematics should be discussed, but one that may seem unconnected to the library; MOTS provided a relevant alternative.

Change in Beliefs and Attitudes about Mathematics

As Table 4 indicates, participating LBIEs reported considerable changes about the role of mathematics in the library when asked to compare their views before exposure to MOTS to their present views. In addition, 90% reported gaining a more positive attitude toward mathematics because of their participation in MOTS activities (2011).

Table 4
Post-MOTS Agreement about LBIEs

	"Because of exposure to MOTS, I...." item used in a particular year's survey
91%	believe that librarians should learn more about integrating mathematics into programming for children (2010)
90%	believe that including more mathematics in their offerings is a strong priority (2010)
70%	have undergone a substantial change in ability to explain to children how mathematics is relevant to library use (2009)

Sustained Impacts of Exposure to MOTS Activities

Overall impacts sustained over the three years of surveys, with mathematics becoming integral to LBIEs' programs. Each year, about 90% stated that including more mathematics in offerings for the elementary grades is a strong priory; 50% reported going beyond MOTS activities to create similar mathematics activities; and just over 50% reported discussing the role mathematics in everyday life with children on a regular basis (2009, 2010, 2011). LBIEs surveyed did not *have* to use MOTS; they chose what to implement. Yet, despite almost no use of mathematics before using MOTS activities, a large majority, once exposed to MOTS activities, chose to integrate mathematics on a regular basis. When asked which factors contribute to their sustained use, the top two reasons each year were *their own commitment to offer mathematics to children*, and *interest/demand from children* (2010, 2011).

Discussion and Conclusion

MOTS offered various types of activities that engaged children with mathematics that is contextual, relevant, and accessible in out-of-school informal programs. LBIEs particularly valued the fact that they could integrate mathematics into their existing areas of strength and expertise, drawing upon the themes, projects, and ways of interacting with children that they have developed over time to address local interests and needs. If these LBIEs are representative of informal educators as a whole, they may have felt strongly from the start that children should succeed in mathematics. However, before encountering MOTS activities, LBIEs may not have seen themselves as capable of playing a role in helping children to realize that success.

MOTS activities enabled a very broad range of LBIEs to incorporate substantially more mathematics in their offerings and to model enthusiasm and positive attitudes toward mathematics; to support them in creating even richer learning environments for children, we had initially planned to offer ongoing professional development on mathematics content and pedagogy. Early in the project, we encountered several insurmountable obstacles. The most pervasive involved LBIE availability: even with the offer of release time covered by project funds, many LBIEs were not able to take time away from their library duties during their work hours; some worked under contracts that prevented job-related professional development outside of work hours. Those who were granted time for professional development often had to make hard choices.

For many LBIEs, offering activities to children is but one job component; they also develop book collections, provide reference services, catalog, maintain records, and stay current with the latest technology available at the library. Professional development relating to children's programming, never mind mathematics programming, was not always their priority. Thus, while the project succeeded in changing LBIEs' attitudes and behaviors, we were not able to explore supporting LBIEs in deepening their mathematics knowledge.

The MOTS project findings suggest that even without mathematics professional development for staff, out-of-school programs offer tremendous potential for engaging a diverse range of children in doing, discussing, and enjoying mathematics. With increasing numbers of children from non-dominant groups participating in out-of-school programs (Afterschool Alliance, 2012), more efforts are needed in order to identify strategies for leading informal educators to embrace mathematics, so that they can then pass on their enthusiasm to children.

With that in mind, we conclude by summarizing strategies that emerged from the MOTS project and could be promising for the development of resources for other out-of-school realms:

Ground activities in authentic situations that informal educators find compelling. Activities should be designed to honor informal educators' areas of comfort, expertise, and passion—whether certain topics, types of programs, or ways of interacting with children.

Ground activities in what children find compelling. Children typically choose whether to participate in out-of-school activities. If children enjoy the offerings, informal educators will provide more. If mathematics is embedded in what children love, they are poised to appreciate mathematics,

Start with mathematics that informal educators know. Instead of asking informal educators to undergo a mathematics refresher course or require professional development that may not be feasible for them, support them in becoming more aware of mathematics they already do in everyday life and in making this mathematics more explicit for children.

Let informal educators lead. Informal educators, often from the same demographic as the children with whom they work, serve as role models and mentors. If they engage in mathematics activities and conversations directly with children, they demonstrate that mathematics is for everyone.

References

Afterschool Alliance (2012). *Fact Sheets*. Retrieved on April 12, 2012 from http://www.afterschoolalliance.org/researchFactSheets.cfm

Allexsaht-Snider, M. (2006). Urban parents' perspectives on children's mathematics learning and issues of equity in mathematics education. *Mathematical Thinking and Learning, 8*(3), 187-195.

Burns, M. (2004). *Math and literature: Grades 2-3*. Sausalito, CA: Math Solutions.

Char, C., & Clark, L. (2011). *Annual report for MATH OFF THE SHELF Year 4*. Montpelier VT: Char Associates. Retrieved from http://mixinginmath.terc.edu/aboutMiM/researchReports.cfm

Char, C., & Berube, S. (2010). *Annual report for MATH OFF THE SHELF Year 3.* Montpelier, VT: Char Associates. Retrieved from http://mixinginmath.terc.edu/aboutMiM/researchReports.cfm

Char, C., & Foote, M. (2009). *Annual report for MATH OFF THE SHELF Year 2*. Montpelier, VT: Char Associates. Retrieved from http://mixinginmath.terc.edu/aboutMiM/researchReports.cfm

Cowan, K. (2010). *Harvard Education Letter*, *26*(5). Retrieved from http://www.hepg.org/hel/article/478

Esmonde, E., Blair, K., Goldman, S., Martin, L., Jimenez, O., & Pea, R. (2013). Math I am: What we learn from stories that people tell about mathematics in their lives. In B. Bevan, P. Bell, R. Stevens, & A. Razfar (Eds.), *LOST opportunities: Learning in out-of-school time* (pp. 7-27). New York: Springer.

Gasbarra, P., & Johnson, J. (2008). Out before the game begins: Hispanic leaders talk about what's needed to bring more Hispanic youngsters into science, technology and mathematics professions. Retrieved from http://www.publicagenda.org/citizen/researchstudies/education

Guberman, S. (2004). Out-of-school activities and arithmetical achievements. *Journal for Research in Mathematics Education*, *35*(2), 117-150.

Harris Interactive (2011). STEM perceptions: Student & parent study. Retrieved from http://www.harrisinteractive.com

Intel (2009). Parents more comfortable talking drugs than science. Retrieved from http://www.intel.com/pressroom/archive/releases/2009/20091021edu.htm

Kliman, M. (2006). Mathematics out of school: Families' math game playing at home. *The School Community Journal, 16*(2), 69-90.

Lange, T., & Meaney, T. (2011). I actually started to scream: Emotional and mathematical trauma from doing school mathematics homework. *Educational Studies in Mathematics, 77*(1), 35-51.

Martin, D. (2009a). Researching race in mathematics education. *Teachers College Record*, *111*(2), 295-338.

Martin, D. (Ed.) (2009b). *Mathematics teaching, learning, and liberation in the lives of Black children*. New York: Routledge.

McCulloch, A., & Marshall, P. (2012). K-2 teachers' attempts to connect out-of-school experiences to in-school mathematics learning. *Journal of Urban Mathematics Education*, *4*(2), 44-66.

Miller, B. (2003). *Critical hours: After-school programs and educational success.* Quincy, MA: Nellie May Education Foundation.

Nasir, N., Hand, V., & Taylor, E. (2008). Culture and mathematics in school: Boundaries between 'cultural' and 'domain' knowledge in the mathematics classroom and beyond. *Review of Research in Education*, *32*(1), 187-240.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Governors Association (NGA). (2010). *Common core state standards for mathematics*. Washington, DC: Author.

Noam, G., Biancarosa, G., & Dechausay, N. (2003). *After-school education: Approaches to an emerging field*. Cambridge, MA: Harvard Education Press.

Simic-Muller, K., Turner, E., & Varley, M. (2009). Math club problem posing. *Teaching Children Mathematics*, *16*(4), 206-212.

Thiessen, D. (2004). *Exploring mathematics through literature*. Reston, VA: National Council of Teachers of Mathematics.

Turner, E., Varley Gutiérrez, M., Simic-Muller, K., & Díez-Palomar, J. (2009). "Everything is math in the whole world": Integrating critical and community knowledge in authentic mathematical investigations with elementary Latina/o students. *Mathematical Thinking and Learning, 11*(3), 136-157.

Turner, E., Varley Gutiérrez, M., & Díez-Palomar, J. (2011). Latino/a bilingual elementary students pose and investigate problems grounded in community settings. In K. Téllez, J. Moschkovich, & M. Civil (Eds.), Latinos and mathematics education: Research on learning and teaching in classrooms and communities (pp. 149-174). Charlotte, NC: Information Age Publishing.

Wager, A. (2012). Incorporating out-of-school mathematics: From cultural context to embedded practice. *Journal of Mathematics Teacher Education, 15*(1), 9-23.

Whitin, D. J., & Whitin, P. (2004). *New visions for linking literature and mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. The approach described in this paper is designed for informal education. Which aspects would work well in a school setting as well? Which would be less successful in most classrooms? Why?
2. Some educators attempt to engage children of different backgrounds in mathematics through use of word problems that incorporate names of children in the class (e.g., see Wager, 2012). How does this approach differ?
3. Why might informal educators who care deeply about children and want them to succeed academically avoid doing mathematics with children? What comparisons, if any, can you make between informal educators' attitudes toward mathematics and those of elementary classroom teachers?
4. What features of MATH OFF THE SHELF materials enabled informal educators to incorporate more mathematics?
5. Choose an activity from the project website (http://mixinginmath.terc.edu) and describe how you would use it with a group of children from many different backgrounds. How might you adapt the activity? Why do you think it would engage children? If you work with children, try out the activity.

Discussion And Reflection Enhancement (DARE) Post-Reading Questions (continued)

6. Find a different mathematics resource designed for after-school programs serving elementary grade children (for instance, look at http://www.sedl.org/cgi-bin/mysql/afterschool/curriculum-choice.cgi?location=by_grade&subj=m for some options). How does your chosen resource engage children from diverse backgrounds and/or non-dominant groups? How might you adapt the resource to draw on children's home cultures?
7. Browse a resource (e.g., Burns, 2004; Thiessen, 2004; Whitin & Whitin, 2004) designed to support integration of children's literature and mathematics and choose one idea from it. How is it alike and different from the approach described in this paper? In what ways does it draw in children from different backgrounds?

Appendix

mixing in MATH

Quick Questions

Goal: Collect data to learn about the people around you

Grades: K-6+

Minimum number of participants: 4

Suggested grouping: individual

Time: 10 minutes or less

Math: analyzing data

Materials:

large sheet of paper

markers or stickers

Prerequisites: some reading and writing

Books about people:

Children Just Like Me. Kindersley, Barnabas and Anabel. NY: DK Publishing, 1995.

A Life Like Mine: How Children Live Around the World. DK and UNICEF. New York: DK Publishing, 2002.

Before beginning

- Think up a multiple choice question children will enjoy answering. Write the question at the top of a large sheet of paper.
- Put possible answers along one side of the paper.

1 Everyone responds

Children use markers or stickers to show their answers.

If the question is posted in a public area, encourage others to respond as well.

2 Explore the answers

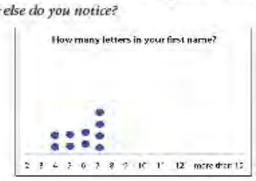
Ask children to look over the answers and say what they notice.

What's the most common answer? the least common?

What's the lowest number? the highest number?

What else do you notice?

Talk About...



Variations

Yes or no (easier). Pose a question with just two answers. For instance, "Were you born in the US?" "Did you eat any fruit today? Children predict whether the most common answer will be "yes" or "no," then they try it and see.

Ask two groups (harder). Provide two color markers or two color stick-on dots for different groups to respond. For instance, those under 10 use blue; those 10 and over use red. Then, compare responses of each group.

Human Graph (same as main activity). Children make a human "bar graph"—everyone with the same answer stands together.

For more math ideas, visit http://mixinginmath.terc.edu

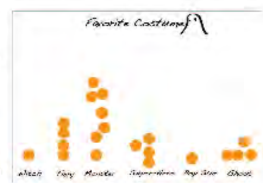
Quick Questions

Bar Graphs

Early elementary grades: Count and compare to find most and least

Which choice got the most votes? Which got the least?

Let children contribute to the graphs, even if it's a little messy. Give children a chance to count the dots, so they learn to distinguish how many dots from how much space they take up.



Middle elementary grades: Compare responses of different groups

Survey results can vary depending on who responds. Color-coding can help children compare two groups. In one library, everyone under 10 used red dots: Those 10 and up used blue.

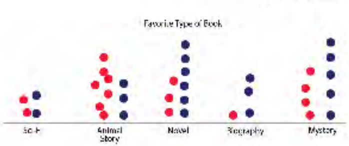
Do people under 10 like the same kinds of books as people 10 and up? How do you know?

Why might librarians want to know this?

Upper elementary grades: Predict based on a sample

So far, what are the most and least popular types of books overall and for each age group? How could librarians use this information?

Do you think the results will change by the end of the week? Why or why not?



Older children can make predictions based on responses so far. If it's afternoon and a lot of teens visit the library in the evening, results might change once they respond. On the other hand, if many people of all ages have already voted and there is a clear favorite, another 50 or even 500 responses might not change the final results.

"DARE to Reach ALL Students!"

Equitable Practices in Mathematics Classrooms: Research-Based Recommendations

Judit Moschkovich

Abstract

This paper is based on the Iris M. Carl Equity Address the author delivered at the 2012 annual meeting of the National Council of Teachers of Mathematics. That invited keynote considered the question of equitable teaching practices in mathematics classrooms for students from non-dominant communities. Although research cannot provide quick answers to this question nor can it provide a recipe for equitable teaching practices, there are research-based recommendations that can guide researchers, teachers, and administrators in developing their own approaches to supporting equitable practices in mathematics classrooms. Several resources are provided for considering this question: a definition of equity, a definition of equitable practices, a framework for organizing research findings relevant to equitable practices, and questions to consider when designing equitable mathematics instruction. This discussion is informed by a sociocultural and situated perspective on mathematical thinking, on language, and on bilingual learners (for details of that framework, see Moschkovich, 2002, 2010).

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What dimensions would you include in a definition of equity?
2. What recommendations for equity in mathematics classrooms have you come across? Are you aware of research that supports these recommendations?
3. What do you think are characteristics of mathematics classrooms that support academic achievement for students from non-dominant communities?
4. What do you know about mathematics learners who are bilingual or learning English?

Judit Moschkovich (jmoschko@ucsc.edu) is professor of mathematics education at the University of California Santa Cruz. Her research uses sociocultural approaches to study mathematical thinking and learning, mathematical discourse, and mathematics learners who are bilingual and/or learning English.

Equitable Practices in Mathematics Classrooms: Research-Based Recommendations

Judit Moschkovich

Equity

How should we approach (if not define) equity? Gutiérrez (2009; 2012) proposes that there are four dimensions reflected in research addressing equity: access, achievement, identity, and power. In Gutiérrez's view, *access* relates to the tangible resources that students have available to them to participate in mathematics, including quality teachers, adequate technology and supplies, rigorous curriculum, classroom environment that invites participation, reasonable class sizes, tutoring, etc. *Achievement* focuses on tangible results for students at all levels of mathematics. Achievement involves course taking patterns, standardized test scores, and participation in mathematics courses at different academic levels (from elementary to graduate school). Studies focusing on *identity* examine whether students find mathematics meaningful to their lives and have opportunities to draw upon their cultural and linguistic resources (e.g., other languages and dialects, algorithms from other countries, different frames of reference). This dimension pays attention to whose perspectives and practices are valued. The *power* dimension can involve examining voice in the classroom, for example who gets to talk and how contributions are taken up (or not).

One way to summarize this approach to equity is to say that students from non-dominant communities need *access* to curricula, instruction, and teachers shown to be effective in supporting the *academic achievement, identities,* and *practices* of these students. I use the phrase "students from non-dominant communities" not to describe students who are in the majority or minority in terms of numbers, but instead to describe students who are not from the culturally dominant communities (middle-class, white, Anglo, English speaking). This phrase thus refers to poor and working class students; in U.S. schools these students are predominantly students of color and many are English learners. The issue is not numbers (majority or minority) but instead dominant and non-dominant cultural practices (Gutiérrez & Orellana, 2006).

How can curricula, instruction, and teachers support the academic *achievement, identities, and practices* of these students? First, students need access to important mathematics. Curriculum policies should follow the guidelines for traditionally underserved students (AERA, 2006), such as instituting systems that broaden course-taking options and avoiding systems of tracking students that limit their opportunities to learn and delay their exposure to college-preparatory mathematics coursework. Second, students need access to environments that have been documented as supporting the academic achievement of students from non-dominant communities.

The general characteristics of such environments in the United States are that curricula provide "abundant and diverse opportunities for speaking, listening, reading, and writing" and that instruction "encourage students to take risks, construct meaning, and seek reinterpretations of knowledge within compatible social contexts" (García & Gonzalez, 1995, p. 424). And third, students need access to teachers who have been documented as being successful with students from non-dominant communities. Some of the characteristics of such teachers are: a) a high commitment to students' academic success and to student-home communication, b) high expectations for all students, c) the autonomy to change curriculum and instruction to meet the specific needs of students, and d) a rejection of models of their students as intellectually disadvantaged (García & Gonzalez, 1995).

Equitable Teaching Practices

I define equitable teaching practices for students from non-dominant communities in mathematics classrooms as those practices that: (a) support mathematical reasoning and mathematical discourse---because we know these lead to conceptual understanding and learning mathematics, and (b) broaden participation for students from non-dominant communities---because we know that participation is connected to opportunities to learn.

To support mathematical reasoning, classroom practices need to provide opportunities for students to participate in different kinds of mathematical practices and use multiple resources to do and learn mathematics. To broaden participation, classroom practices need to provide opportunities for students to use multiple resources to participate in classroom work. Equitable classroom practices, then, are fundamentally focused on honoring student resources, in particular the "repertoires of practices" (Gutiérrez & Rogoff, 2003) that students from non-dominant communities bring to the classroom. Lee (2003) argues that we should "neither attribute static qualities to cultural communities nor assume that each individual within such communities shares in similar ways those practices that have evolved over generations (p. 4)."

To avoid reducing cultural practices to individual traits that are static or that all members of a group share, Gutiérrez and Rogoff (2003) propose that we focus not on what an individual does or says but, instead, on what they call "repertoires of practice." These are a collection of multiple and varied practices because learners have access to multiple practices. Any individual is likely to have had multiple experiences with different practices from many different communities, not only their families, but also through their friends, school, sports, mass media, etc. These "repertoires of practice" are not static because individuals develop and communities change.

An example of a linguistic practice that is important in the classroom is intonation. For example, intonation patterns vary, not only across national languages (e.g., English and Spanish), but also among varieties of a national language (e.g., Spanish). In the case of Chicano English:

> "Perhaps the most prominent feature distinguishing Chicano English from other varieties of American English is its use of certain intonation patterns. These intonation patterns often strike other English speakers as uncertain or hesitant" (Finegan & Besnier, 1989, p. 407).

In order to honor the resources that students bring to the classroom, teachers need to learn what practices (cultural, linguistic, mathematical, etc.) are common among students from non-dominant communities, including students who are bilingual, and/or learning English, or use non-dominant language varieties (although these are usually called "dialects," I use the phrase "language varieties" because the label "dialect" can reflect a deficit view of those language varieties). There are many ways to learn about the practices that are common in students' home communities. Getting to know the local communities, attending local events, or visiting students' homes are arguably the best windows into students' lives outside of school. Reading (fiction, non-fiction, books on multicultural approaches to education, articles on social justice approaches to mathematics teaching, etc.) is a more indirect way to learn about students' home practices.

Framework for Equitable Classroom Practices

To frame the many connections among language, culture, and mathematics learning/teaching, I will use Brenner's (1998) framework for cultural relevance for instruction and curriculum (see also Nelson-Barber & Moschkovich, 2009). This framework identifies three areas central to ensuring that curricula and instructional practice are culturally relevant for students: *cultural content*, *social organization*, and *cognitive resources*. Brenner's three-part framework can be used as a broad guide for designing curricula, instruction, and assessments. The three dimensions can be used to organize the results of relevant research.

1. CULTURAL CONTENT
- Do mathematical activities connect to those in local community?

2. SOCIAL ORGANIZATION
- Do classroom practices facilitate comfortable and productive student participation?
- Do classroom practices fit with learners' communication practices in home/community?

3. COGNITIVE RESOURCES
- Does instruction enable children to use their prior knowledge and experiences as resources?

Figure 1. Dimensions for equitable classroom practices (Brenner, 1998).

Brenner's (1998) framework includes the following questions: Do mathematical activities connect to those in local community? Do classroom practices facilitate comfortable and productive student participation? Do roles and responsibilities fit with learners' communication practices? Does instruction enable children to build on their existing knowledge and experiences as resources? These questions for each dimension are useful for considering the complexity in what constitutes comfortable and productive participation for learners, as well as the multiple communication practices that students have experienced, both at home and in school.

As Brenner sees it, examining materials and instructional techniques for their *cultural content* can reveal the extent to which mathematical activities utilized in instruction relate to mathematical activities operating in local community practices, no matter what communities students come from. Similarly, ensuring that classroom *social organization* takes into account a variety of possible roles, responsibilities, and communication styles and includes multiple and hybrid repertoires of practice (Gutiérrez & Rogoff, 2003) will more likely support comfortable and productive student participation.

Classrooms that make use of the *cognitive resources* students bring from previous instruction and from home—a variety of ways of thinking used in their communities to solve problems—make the most of students' existing knowledge and lived experiences (Moll & González, 2004). Language is one such cognitive resource. Teachers' ability to recognize and appreciate students' particular cognitive resources ultimately has a bearing on how they interpret student talk and activity in the classroom.

Connecting Mathematics to Local Communities

The central question for this dimension is whether mathematical activities in the classroom connect to the local community. Connecting school mathematics with children's own experiences and intuitive knowledge has been an important theme in efforts to improve formal mathematics instruction (e.g., Lipka, Webster, & Yanez, 2005; Trumbull, Nelson-Barber & Mitchell, 2002). Several projects in mathematics education have focused on documenting community mathematical activities in different settings. For example, publications from the following projects provide the details of mathematical activities in different communities: "Funds of Knowledge" (Civil, 2002, 2007; González et al., 2001), "El Mercado" (Fuson et al., 1997), and work in Alaska (Lipka, 1998; Lipka & Adams, 2004; Lipka, Webster, & Yanez, 2005) documented local mathematical activities. Even when teachers are working in communities where researchers have not yet documented the local mathematical activities, these publications provide ways to learn about students and their communities through home visits, reports form students, conversations with parents, and other approaches (González et al., 2001). Work in mathematics for social justice (Gutstein, 2003; Gutstein & Peterson, 2005; Powell & Frankenstein, 1997) also provides mathematical tasks that can be readily connected to students and their communities.

When working with students who are immigrants it is important to consider differences in symbols and algorithms (Orey, 2004; Perkins & Flores, 2002; Secada, 1983). For example, in some countries a period is used for marking the thousands place, not for decimals as in the United States (writing 1.234 instead of 1,234), and the comma is used to mark decimals (writing 10,03 not 10.03). Mathematics educators have also documented algorithms common among immigrant students, for example the "Rule of three" or "Regla de tres" to solve proportion problems, and several different approaches to long division (Civil & Planas, 2010; Corey, 2004; Perkins & Flores, 2002):

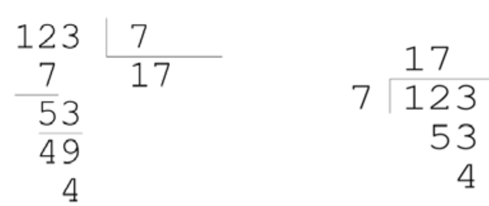


Figure 2. Alternative algorithms for dividing 123 by 7.

Social Organization of Classroom Practices

The central question for this dimension is whether classroom practices facilitate comfortable and productive student participation and fit (as much as possible) with learners' communication practices at home or in their communities. To address this dimension, teachers need to understand children's home language practices. Teachers can learn to value and build on student's linguistic skills while also explicitly modelling the discourse styles expected in school. The rules about who can talk when, about what, and how, and communication routines are established in every classroom. The practice of incorporating students' own ways of using language into the classroom is recognized as one aspect of the success of some classrooms. For example, one successful approach to integrating community language practices that resulted in gains in reading scores is the Kamehameha school integration of "talk story" style of overlapping participation into native Hawaiian children's classrooms (Au, 1980). Another example is Lee's work with African-American high school students' ways of talking (Lee, 1993).

The question to ask about language practices in the classroom is whether a classroom facilitates participation for students from non-dominant communities in terms of the roles, responsibilities, and styles of learners' communication practices. Answering this question means having substantial information about and deep understanding of children's home practices and the local community (Moschkovich & Nelson-Barber, 2009). This entails knowing not only local activities that may be used in the mathematics classroom but also students' language practices at home and other community settings. It is important to remember that there may be differences between home and school participation structures. For example, a participation structure common in many homes of students from traditional communities, "intent participation," is a style that involves lots of watching and little talk (Rogoff, Paradise, Arauz, Correa-Chavez, & Angelillo, 2003), in contrast to school instruction that involves large amounts of talk.

What are typical communication practices for students who use two languages? Common practices among mathematics students who are bilingual or learning English include using arithmetic facts in first language, doing arithmetic computation in their first language and then translating the answer, and code-switching, using two languages during one conversation. The social organization of the classroom should include these language practices and these practices should be seen as cognitive resources for doing mathematics in the classroom.

Children's Prior Knowledge and Experiences as Cognitive Resources

The central question for this dimension is whether instruction enables children to build on their prior knowledge and experiences as resources for mathematical reasoning. There are many different types of cognitive resources. There should be many opportunities for students to participate in mathematical talk in multiple ways. But talk should not be the only resource: students should also have opportunities to draw flexibly on multiple resources, such as drawings, written text, mathematical representations, gestures (Fernandes & McLeman, 2012; Moschkovich, 2002), and manipulative objects, etc. As described above, instruction should support students in using multiple languages and dialects, as well as express their mathematical thinking in everyday ways. Other cognitive resources include stories (for example in story problems) and physical activity (using a motion detector, or walking on a number line).

Equitable Practices for English Learners

Although it is difficult to make generalizations about the instructional needs of all students who are learning English, research suggests that high-quality instruction for English Learners (ELs) that supports student achievement has two general characteristics: a view of language as a resource rather than a deficiency, and an emphasis on academic achievement, not only on learning English (Gándara & Contreras, 2009). Mathematics teachers who work with ELs need to know some things that are specific to their students. They also need to be aware of mathematics notation in other countries. Lastly, they need to know some things about language in general and about bilingualism in particular.

First, mathematics instruction should be informed by knowledge of students' experiences with mathematics instruction, language history, and educational background (Moschkovich, 2010). Teachers need to know the details of a student's history with formal schooling, for example which grades they attended, where, and in what language (or languages). They should have some information about their language history, for example are they literate in their home language, what is their reading and writing competence in the home language. Some students may not have had any formal instruction in the language spoken at home. Another important information is the students' history with school mathematics instruction: when they had mathematics classes, in what language, and for which topics.

Mathematics teachers who work with ELs also need to know a few things about language and bilingualism. A few assumptions about language that come from research in linguistics (for more details, see Wong-Fillmore & Snow, 2000) include the following: a) language involves meaning, action, purpose, and discourse practices (not just vocabulary or single words); b) we learn language by using it to communicate (rather than by memorizing definitions and lists of words), and c) learning a second language is long-term process (at least several years).

Teachers also need to be familiar with the findings from current research on bilingual mathematics learners (for a short summary of this research, see Moschkovich, 2009; for a longer version, see Moschkovich, 2007b). Native-like control of two or more languages is an unrealistic definition of bilingualism that does not reflect evidence that the majority of bilinguals are rarely equally fluent in both languages. Teachers need to know and build on the fluencies their students bring rather than comparing bilinguals to monolinguals or focus on how bilingual students miss the mark in comparison to monolinguals. Because bilinguals have a wide range of proficiencies in two languages, teachers should not expect mathematics students to know mathematical terms in a first or second language unless they have had mathematics instruction in that language. Bilinguals have a wide range of proficiencies in modes (listening, writing, speaking, and reading) in their two languages. Teachers should not assume that proficiency in one mode implies proficiency in another mode and should provide mathematics assessment and instruction across all modes. Switching languages is not a sign of a deficiency. In fact, this skill is a complex cognitive and linguistic resource (Moschkovich, 2007a, 2007b, 2009; Valdés-Fallis, 1978; Zentella, 1981). Teachers should not imagine that switching languages is related to mathematical thinking or understanding in any simple way.

In Closing

There are many ways to define equitable practices in mathematics classrooms. I am certain that the definition and framing I have provided here leave out important aspects and work that is relevant. However, my intention was not to provide the perfect definition, but instead to establish some common ground. It is my sincere hope that the resources I provided here prove useful for designing equitable mathematics instruction.

References

American Educational Research Association (2006). Do the math: Cognitive demand makes a difference. *Research Points, 4*(2), 1-4.

Au, K. (1980). Participation structures in reading lessons: Analysis of a culturally appropriate instructional event. *Anthropology and Education Quarterly*, *11*(2), 91-115.

Brenner, M. E. (1998). Adding cognition to the formula for culturally relevant instruction in mathematics, *Anthropology and Education Quarterly*, *29*(2), 213-244.

Civil, M. (2002). Culture and mathematics: A community approach. *Journal of Intercultural Studies*, *23*(2), 133-148.

Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. In N. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 105-117). New York: Teachers College Press.

Civil, M., & Planas, N. (2010). Latino/a immigrant parents' voices in mathematics education. In E. Grigorenko & R. Takanishi (Eds.), *Immigration, diversity, and education* (pp. 130-150). New York: Routledge.

Fernandes, A., & McLeman, L. (2012). Interpreting and using gestures of English language learners in mathematics teaching. *Teaching for Excellence in Equity and Mathematics, 4*(1), 15-23.

Finegan, E., & Besnier, N. (1989). *Language: Its structure and its use*. New York: Harcourt Brace Jovanovich.

Fuson, K. C., Smith, S. T., & Lo Cicero, A. M. (1997). Supporting Latino first graders' ten-structured thinking in urban classrooms. *Journal for Research in Mathematics Education*, *28*(6), 738-766.

Gándara, P., & Contreras, F. (2009). *The Latino education crisis: The consequences of failed social policies*. Cambridge, MA: Harvard, University Press.

García, E., & Gonzalez, R. (1995). Issues in systemic reform for culturally and linguistically diverse students. *Teachers College Record*, *96*(3), 418-431.

González, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk*, *6*(1-2), 115-132.

Gutiérrez, K. D., & Orellana, M. F. (2006). At last: The 'problem' of English learners: Constructing genres of difference. *Research in the Teaching of English*, *40*(4), 502-507.

Gutiérrez, K. D., & Rogoff, B. (2003). Cultural ways of learning: Individual traits or repertoires of practice? *Educational Researcher, 32*(5), 19-25.

Gutiérrez, R. (2009). Framing equity: Helping students "play the game" and "change the game". *Teaching for Excellence and Equity in Mathematics*, *1*(1), 4-8.

Gutiérrez, R. (2012). Context matters: How should we conceptualize equity in mathematics education? In B. Herbel-Eisenmann, J. Choppin, D. Wagner & D. Pimm (Eds.), *Equity in discourse for mathematics education: Theories, practices, and policies*. New York: Springer.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education*, *34*(1), 37-73.

Gutstein, E., & Peterson, B. (Eds.). (2005)[1]. *Rethinking mathematics: Teaching social justice by the numbers*. Milwaukee, WI: Rethinking Schools Limited.

Lee, C. (2003). Why we need to re-think race and ethnicity in educational research. *Educational Researcher*, *32* (5), 3–5.

Lee, C. (1993). *Signifying as a scaffold for literary interpretation: The pedagogical implications of an African American discourse genre*. (Research Rep. No. 26). Urbana, IL: National Council of Teachers of English.

Lipka, J. (Ed.). (1998). *Transforming the culture of schools: Yup'ik Eskimo examples*. Mahwah, NJ: Lawrence Erlbaum.

Lipka, J., & Adams, E. (2004). Culturally based math education as a way to improve Alaska Native students' math performance. *The Appalachian Collaborative Center for Learning, Assessment and Instruction in Mathematics Series.* Available: http://acclaim.coe.ohiou.edu/rc/rc_sub/pub/3_wp/list.asp

Lipka, J., Webster, J., & Yanez, E. (2005). Factors that affect Alaska Native students' mathematical performance. *Journal of American Indian Education*, *44* (3), 1-8.

Moll, L. C., & González, N. (2004). Engaging life: A funds-of-knowledge approach to multicultural education. In J. Banks & C. A. McGee Banks (Eds.), *Handbook of research on multicultural education* (pp. 699-715). San Francisco: Jossey-Bass.

Moschkovich, J. N. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning*, *4*(2&3), 189-212.

Moschkovich, J. N. (2007a). Bilingual mathematics learners: How views of language, bilingual learners, and mathematical communication impact instruction. In N. Nasir & P. Cobb (Eds.), *Diversity, equity, and access to mathematical ideas* (pp. 89-104). New York: Teachers College Press.

Moschkovich, J. N. (2007b). Using two languages while learning mathematics. *Educational Studies in Mathematics*, *64*(2), 121-144.

Moschkovich, J. N. (2009). Using two languages when learning mathematics: How can research help us understand mathematics learners who use two languages? Research Brief and Clip, National Council

[1] Editors' Note: A second edition of Gutstein and Peterson's *Rethinking mathematics: Teaching social justice by the numbers* has been released in 2013 that includes the first edition material plus roughly the same amount of new material.

of Teachers of Mathematics, available at http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/Research_brief_12_Using_2.pdf

Moschkovich, J. N. (2010). Language(s) and learning mathematics: Resources, challenges, and issues for research. In J. N. Moschkovich (Ed.), *Language and mathematics education: Multiple perspectives and directions for research* (pp. 1-27). Charlotte, NC: Information Age Publishing.

Moschkovich, J. N. (2012). How equity concerns lead to attention to mathematical discourse. In B. Herbel-Eisenmann, J. Choppin, D. Wagner & D. Pimm (Eds.), *Equity in discourse for mathematics education: Theories, practices, and policies* (pp. 89-105). New York: Springer.

Moschkovich, J. N., & Nelson-Barber, S. (2009). What mathematics teachers need to know about culture and language. In B. Greer, S. Mukhopadhyay, S. Nelson-Barber, & A. Powell (Eds.), *Culturally responsive mathematics education* (pp. 111-136). New York: Routledge, Taylor & Francis Group.

Nelson-Barber, S., & Lipka, J. (2008). Rethinking the case for culture-based curriculum: Conditions that support improved mathematics performance in diverse classrooms. In M. Brisk (Ed.), *Language, culture and community in teacher education* (pp. 99-123). New York: Lawrence Erlbaum Associates.

Orey, D. C. (2004). The algorithm collection project. A preliminary report on the preliminary findings using the ethnomathematics of basic number sense acquisition. Sacramento, CA: California State University.

Perkins, I., & Flores, A. (2002). Mathematical notations and procedures of recent immigrant students. *Mathematics Teaching in the Middle School*, *7*(6), 346-351.

Powell, A. B., & Frankenstein, M. (Eds.). (1997). *Ethnomathematics: Challenging Eurocentrism in mathematics education*. Albany, NY: State University of New York Press.

Rogoff, B., Paradise, R., Arauz, R. M., Correa-Chavez, M., & Angelillo, C. (2003). Firsthand learning through intent participation. *Annual Review of Psychology*, *54*(1), 175-203.

Secada, W. (1983). The educational background of Limited English Proficient students: Implications for the arithmetic classroom. Arlington Heights, IL: Bilingual Education Service Center. (ERIC Document Reproduction No. ED 237 318)

Trumbull, E., Nelson-Barber, S., & Mitchell, J. (2002). Enhancing mathematics instruction for indigenous American students. In J. Hankes & G. Fast (Eds.). *Changing the faces of mathematics: Perspectives of indigenous people of North America* (pp. 1-18). Reston, VA: National Council of Teachers of Mathematics.

Valdés-Fallis, G. (1978). Code-switching and the classroom teacher. *Language in education: Theory and practice* (Vol. 4). Wellington, VA: Center for Applied Linguistics. (ERIC Document Reproduction No. ED 153 506)

Wong-Fillmore, L., & Snow, C. (2000). *What teachers need to know about language*. Washington, D.C.: U.S. Department of Education, Office of Educational Research and Improvement.

Zentella, A. (1981). Tá bien, you could answer me en cualquier idioma: Puerto Rican code-switching in bilingual classrooms. In R. Durán (Ed.), *Latino language and communicative behavior* (pp. 109-130). Norwood, NJ: Ablex Publishing Corporation.

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Consider the four dimensions of equity (access, achievement, identity, and power) and discuss how each plays out in your classroom or in your school.
2. Consider the three dimensions for equitable practices: cultural content, social organization, and cognitive resources. Discuss how these play out in your classroom and what you could do to improve one of these dimensions in your teaching.
3. What teaching characteristics are successful with students from non-dominant communities? Which characteristic do you think is the most essential and why? Which characteristic do you think is the most challenging and why?
4. What is an example of an alternative algorithm or notation that is important for teachers to recognize as equally valid if they see their students using it?
5. Did any of the claims about how bilingual mathematics learners use language surprise you? What could you do to learn more about bilingual learners?
6. Describe three things you could do to learn more about the cultural, linguistic, and mathematical practices of the students in your classrooms.

"DARE to Reach ALL Students!" TODOS Mathematics for ALL

Acknowledgement of TEEM Reviewers

We thank these reviewers for their service during the period January 2012-July 2013 (since the previous listing of referees published in the third issue of *TEEM*), and we are always interested in having more people available to review (normally one manuscript per year). If willing, please email teem@todos-math.org with your (1) contact information, (2) grade levels of teaching or research experience, and (3) thematic areas of greatest interest and expertise. And if there is someone you would like to nominate, please relay the necessary information for the editors to be able to send an invitation.

Harold Asturias

Susan Beal

Kathryn Chval

Kelly Costner

Peggy Cuevas

Anthony Fernandes

Thomas Gilsdorf

Lena Licón Khisty

Rick Kitchen

Brian Lawler

Laura McLeman

José María Menéndez

Sandra Musanti

Janet Pittock

Nora Ramirez

Ksenija Simic Muller

Angela Thompson

Michelle Vaughn

Craig Willey

William Zahner

“DARE to Share!” - Tell us how you’ve used *TEEM* articles! - Invitation to Readers

TEEM invites readers to write in with experiences they have had applying or extending the articles they see in the journal, and we may publish them in the journal or its website.

For example, while preparing a breakout workshop on culturally relevant mathematics for the 2013 NCTM High School Institute, *TEEM* co-editor Larry Lesser reread Olga Ramirez and Cherie McCollough’s paper “La Lotería: Using a Culturally Relevant Mathematics Activity with Pre-service Teachers at a Family Math Learning Event” from the fall 2012 TEEM issue *4*(1), 24-33 and was inspired to write these additional mathematics questions (and solutions) for the context of the board game “La Lotería.”

1. What’s the smallest number of cards (out of 54) that the dealer could call before your 4x4 board MUST win? (*Hint:* first consider what is the largest number of uncovered spaces your board could have where you don’t have a win yet, but the very next card called MUST give you a win)
2. What’s the probability that neither of the first two cards called are on your 4x4 board?
3. What’s the probability that you have a win after the dealer calls *exactly* 4 cards?

Solutions:

1. Suppose 12 cards have been called that are on your game board, as shown by X’s:

X	X	X	
	X	X	X
X	X		X
X		X	X

   As you can see, it is possible to have this many spaces covered and yet not have one of the winning combinations (of Figure 1 in Ramirez and McCollough, 2012). Assume the 4 uncovered spaces on your game board are the only 4 cards that have not yet been called in the deck of 54 cards. Then, this means that 50 cards have been called so far, and the very next card must produce a win, and so the answer to the question is 51.

2. The probability that the first card IS on your board is 16/54, so its complement is 1 – (16/54) = 38/54. Since cards are drawn independently without replacement, the probability that both the first and second cards are not on your board is (38/54)*(37/53), which is approximately .49.
3. The probability of this (very unlikely) event can be obtained in more than one way. Ramirez and McCollough (2012, pp. 26-27) enumerate the 12 ways a person can win and so we can divide 12 by the total number of ways the dealer can choose 4 cards from 54. In other words, 12/ C(54,4), and this is less than 1 in 26,000. Another way to look at it is to find the probability that the first four cards called by the dealer happen to all be on the player’s game board, and then multiply that answer by the probability that set of 4 cards happens to be one of the 12 ways of winning. And so, we obtain (16/54)(15/53)(14/52)(13/51)(12/ C(16,4)), which yields the same (tiny) answer!

TODOS 2013-14 ELECTED LEADERSHIP

Don S. Balka President	Professor Emeritus, Saint Mary's College Notre Dame, IN
Susie W. Håkansson President-Elect	Executive Director, University of California, Los Angeles Mathematics Project Los Angeles, CA
Gil Cuevas Vice-President	Professor, Texas State University, San Marcos Austin, TX
Florence Glanfield Director	Associate Professor, University of Alberta Edmonton, AB, Canada
Bill Jasper Director	Professor, Sam Houston State University College Station, TX
Janie Zimmer Director	Consultant, Research-Based Education Reading, PA
Sylvia Celedón-Pattichis Director	Professor, University of New Mexico Albuquerque, NM
Diane Kinch Director	Secondary Mathematics Specialist, Pomona, California Claremont, CA
Carol A. Edwards Executive Secretary	Professor of Mathematics Emerita, St. Louis Community College at Florissant Valley, St. Louis, MO; Facilitator, Chandler-Gilbert Community College Chandler, AZ

TODOS: MATHEMATICS FOR ALL
2014 CONFERENCE ANNOUNCEMENT

TODOS PUBLICATIONS

Library Subscription Information:
http://www.todos-math.org/assets/documents/TODOS%20LIBRARY%20SUBSCRIPTIONS.02.01.11.pdf

Teaching for Excellence and Equity in Mathematics Journal
Refereed Journal
2009, 2010, 2011, 2012, 2013

TODOS Research Monographs
(2008, 2010)
Publication and dissemination of both monographs were supported by the National Education Association.

Bibliography of Diversity and Equity in Mathematics Education
1st Edition (2004), 2nd Edition (2007)

NOTICIAS de TODOS—Semiannual newsletters
(Spring 2005 - present)

See ***www.todos-math.org*** to join and have access to all publications.

TODOS
Mathematics for ALL
www.todos-math.org