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Teaching for Excellence and Equity in Mathematics

TODOS
Mathematics for ALL

An Affiliate Organization of the National Council of Teachers of Mathematics

TODOS and the editors of *TEEM*
express deep appreciation to
PEARSON EDUCATION
for its generous sponsorship of this issue.

IN THIS ISSUE

From the Editors

We have a few changes to report this year for the TEEM team. Joining and beginning their service on our distinguished Editorial Panel this year are Alejandra Sorto of Texas State University and Sylvia Celedón-Pattichis of The University of New Mexico. Also this year, Marta Civil moved from the Editorial Panel to being an Editor. Editors and Editorial Panelists depend upon quality work from external reviewers and our next issue will list those who reviewed since the publication of our fall 2011 issue. To join the ranks of reviewers, see ***www.todos-math.org*** or page six of this issue.

This has been an interesting year in the realm of equity, including a joint public statement (posted on the TODOS webpage) of TODOS, BBA (Benjamin Banneker Association), and WME (Women and Mathematics Education) on the Common Core State Standards in Mathematics (CCSSM). The statement lauds the CCSSM emphasis on excellence and calls for more attention to equity and diversity considerations.

This year yielded many recognitions for those who have contributed to the work of *TEEM* and TODOS. With apologies to those we may have inadvertently overlooked, we congratulate:

Sylvia Celedón-Pattichis and **Nora Ramirez** are editors of the 2012 book *Beyond Good Teaching: Advancing Mathematics Education for ELLs*, published by the National Council of Teachers of Mathematics.

Luciana de Oliveira received the 2012 Early Career Award in Bilingual Education Research from that SIG of the American Educational Research Association.

Carol A. Edwards received the Ross Taylor/Glenn Gilbert Leadership Award at the 2012 conference of the National Council of Supervisors of Mathematics.

Kathryn Chval received the 2011 Early Career Award of the Association of Mathematics Teacher Educators and her 2011 co-authored ELL article in *Mathematics Teaching in the Middle School* won the NCTM Research Committee's Linking Research and Practice Outstanding Publication Award for the 2011-2012 volume year of that journal.

Rochelle Gutiérrez, who received the 2011 Outstanding Mathematics Teacher Educator Award for excellence in scholarship from the Association of Mathematics Teacher Educators, is giving the keynote at the 2013 Creating Balance conference on math education and social justice.

Miriam Leiva is invited to give the inaugural Kay Gilliland Lecture on Equity at the 2013 NCSM Annual Conference.

TEEM gratefully acknowledges the support of all the leaders in our sponsoring organization, TODOS: Mathematics for ALL. We hope *TEEM* serves and increases the TODOS membership, and also serves as an inspiring pedagogical and scholarly resource for the broader mathematics education and education communities. We welcome feedback about the journal as well as results of teachers or professional developers putting the ideas (or DARE questions) of the papers into practice. We are also happy to give initial feedback to query emails about proposed ideas, including papers specifically written by and for classroom teachers. Finally, we invite readers to write "Letters to the Editor" to share their reactions to (or experiences in implementing) the ideas in TEEM papers. Send submissions (as an email attachment) to **teem@todos-math.org,** being sure to follow the Guidelines in the Call for Papers at **http://www.math.utep.edu/Faculty/lesser/TEEM.html.**

The 2012 issue of *TEEM* includes three externally peer-reviewed articles. The issue leads off with **Mathew Felton** sharing how an exploration of education test score data can educate pre-service teachers about equity as well as statistics. Next, **Anthony Fernandes** and **Laura McLeman** illustrate the importance and power of gestures in communicating mathematics. Finally, **Olga Ramirez** and **Cherie McCollough** showcase a culturally relevant mathematics activity with preservice teachers at a family math learning event through "La Lotería." With the publication of this fourth issue, the first three issues are now available to ALL at the TODOS website.

Lawrence M. Lesser   Cynthia O. Anhalt   Marta Civil

Teaching for Excellence and Equity in Mathematics

Call for Manuscripts and Reviewers

Call for Manuscripts

We encourage the submission of manuscripts that are aligned with the mission of TODOS: Mathematics for ALL (see below). Manuscripts in applied or action research, literature surveys, thematic bibliographies, commentary on critical issues in the field, professional development strategies, and classroom activities and resources are encouraged and welcome. Please see http://www.math.utep.edu/Faculty/lesser/TEEM.html for guidelines and then submit complete manuscripts to teem@todos-math.org. The *TEEM* Editors welcome query emails about the suitability of proposed topics or approaches.

Call for Reviewers

Refereeing is not only a valuable experience and service to the profession, but is also an essential means to ensure that articles of high quality and relevance are published in a timely manner. To be eligible to be a reviewer (normally, only 1-2 papers per year), we invite you to send an email to teem@todos-math.org with the following information:

(1) your full name, affiliation, and contact information (including email, phone number, fax number, and mailing address);

(2) grade levels (e.g., elementary, middle, secondary, college) where you have teaching or research experience; and

(3) thematic areas with which you have particular interest and expertise, and any other pertinent professional information.

Your information will assist the editors in assigning papers to the various reviewers.

The mission of TODOS: Mathematics for ALL is to advocate for an equitable and high quality mathematics education for all students — in particular, Hispanic/Latino students — by increasing the equity awareness of educators and their ability to foster students' proficiency in rigorous and coherent mathematics.

Test Scores in the U.S.: Introducing the Data to Pre-service Teachers

Mathew D. Felton

Abstract

I share a lesson in which I engage pre-service K-8 teachers in a mathematics content course in learning about the role of mathematics in understanding inequities in our society. Specifically, the lesson examines disparities in test scores in terms of race/ethnicity and eligibility for free/reduced lunch. I consider what messages this sends to and generates from pre-service teachers about the role of mathematics in understanding our world and I offer possible extensions of the lesson.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Why do we teach mathematics in school? In answering this question, how are you thinking about students (e.g., as future citizens, future workers, or future consumers)?
2. What do standardized test scores tell us about inequities in our educational system?
3. In what way can mathematics be used to raise awareness of and challenge inequities in our society?
4. What kinds of experiences do pre-service teachers need in their teacher education program if they are going to engage their students in these activities?

Mathew D. Felton (mdfelton@math.arizona.edu) is an Assistant Professor at The University of Arizona. His research focuses on teachers' views of mathematics as a social and political activity and how teacher education can support future teachers in understanding the social and political implications of mathematics teaching and learning.

Test Scores in the U.S.: Introducing the Data to Pre-service Teachers

Mathew D. Felton

What does equitable mathematics education look like? The equity principle in the *Principles and Standards* (National Council of Teachers of Mathematics [NCTM], 2000) states that "excellence in mathematics education requires equity—high expectations and strong support for all students" (p. 12). This is a view of equity as including *access* to high quality mathematics instruction (Gutiérrez, 2009). I wholeheartedly agree with the importance of this aspect of equity; however, this view does not question the mathematical content that students are expected to learn. Why do we teach mathematics in school, and more specifically, why do we teach the particular version of mathematics that is found in school? I view students as future citizens and as such I hope that they will leave the classroom with the ability to think deeply about the world around them, to understand the social and political issues facing our country and world, and to take action to address injustices created by our societal structures. I, and others (e.g., Gutiérrez, 2007, 2009; Gutstein, 2006), have argued that while access to high quality mathematics is absolutely necessary, equity must also include a re-envisioning of mathematical content to empower students to analyze and challenge structural forms of inequity in our society. For instance, Gutstein (2006) states that:

> Some argue that it is important that more students of color, women, and working-class students get access to mathematical courses and life trajectories. I definitely concur but argue against a presumption that more of these students in advanced mathematics classes and careers will necessarily change inequitable relations of power. I disagree with the position that urges increased access to mathematics opportunities, but that simultaneously leaves unchallenged the very structures that created the injustices. (p. 30)

In this article, I explore what it looks like to use mathematics as a tool for challenging existing injustices with pre-service K-8 teachers. I first describe the context in which I teach and a framework I use to inform my instruction and research. I then briefly describe and analyze a lesson I have used in my own teaching.

My Teaching

I currently teach mathematics content courses for pre-service K-8 teachers (PSTs) at The University of Arizona in Tucson, AZ. The courses I teach focus on mathematics knowledge for teaching (Ball, Hill, & Bass, 2005) and are part of a required sequence that all K-8 PSTs take as part of an undergraduate teacher education program. The content courses are prerequisites for the mathematics methods course, and in my experience, the vast majority of the PSTs take the content courses prior to beginning the education program.

Through research focused on my previous teaching, I have developed a framework for examining the narratives we construct about what mathematics is and what it means to do mathematics. This framework is grounded in the views of the PSTs in my courses as well as prior research literature (see Felton, 2010a, 2010b). In my teaching and research I ask three questions regarding the narratives we construct about what mathematics is and what it means to do mathematics, which I refer to as the *What*, *How*, and *Who* of mathematics:

> *What* messages do we send about mathematics?
> *How* are mathematical concepts and real world contexts related in mathematics?
> Do people (the *Who*) experience mathematics more as a *mirror* reflecting back their experiences and concerns or as a *window* into a broader perspective? (The mirror/window metaphors are from Gutiérrez, 2007.)

I ask the PSTs in my courses to explore these ideas through a series of reflection assignments in which they write about mathematics, often in response to one or more readings. I also use lessons in class that specifically highlight one or more aspects of the *What, How, Who* framework. In this article I focus primarily on what messages we send about the relationship between mathematics and the real world, and in particular the role of mathematics as a tool for social analysis (Spielman, 2009). I identify four levels of engaging in mathematics for social analysis (Felton, 2010a):

> *Real World Connections:* Using mathematics to understand everyday phenomena that are viewed or treated as neutral in nature.
> *Political Topics:* Using mathematics to understand topics that are viewed or treated as political in nature.
> *Awareness of Inequity:* Using mathematics to understand what the learner sees as systematic issues of inequity, particularly as related to race, gender, class, or other markers of difference.
> *Critique of Structural Inequity:* Using mathematics to critique the structural forces at work that produce social inequity by identifying structural causes and/or proposing alternatives.

Many of the PSTs enter my courses explicitly connecting mathematics to the real world. For instance, in a representative initial reflection one PST wrote:

> Math is needed for the success of a society and its economy. Math is used every day for this [sic] simplest of things. For instance, we count and make sure we have enough seats in the car for a group of people. We are constantly using math without even realizing it. Math is a basic need for functioning in society.

As illustrated in the above quotation, PSTs at the beginning of the course express real-life relevance of mathematics, but rarely make explicit connections between mathematics and political topics or issues of inequity. Thus, the PSTs enter my course largely echoing a traditional narrative of mathematics: that it is essentially neutral and disconnected from social and political issues facing our world. A number of authors have emphasized supporting learners of all ages in using mathematics to understand issues of inequity (see, for example, Frankenstein, 1997; Gutstein, 2006; Gutstein & Peterson, 2005). I extend this work to a mathematics content course for PSTs. One of my course goals is to challenge the PSTs' views of mathematics as neutral by highlighting ways that we can use mathematics to understand political topics, particularly issues of inequity in our society (levels 2-4 of social analysis). In the following section, I provide an example lesson I designed that highlights the role of mathematics as a tool for social analysis.

The Lesson: Test Scores in the U.S.

Description of the Lesson

I have used this lesson as an introductory lesson in data analysis in a content course for prospective K-8 teachers; the course topics are geometry, measurement, data analysis, probability, and algebra. At this point in the class we have done relatively little work on comparing and analyzing data presented in graphs or tables. I begin the lesson by introducing the National Assessment of Educational Progress (NAEP), describing it as a low stakes test, and giving the fourth-grade benchmark scores for mathematics (*Basic*: 214 or higher; *Proficient:* 249 or higher; and *Advanced:* 282 or higher). I also discuss the "achievement gap" and emphasize that many researchers prefer to call this an "opportunities gap" because it is related to the educational opportunities available to students from different groups (Flores, 2007). I then explain that we will look at race/ethnicity—specifically, NAEP scores of White, Black, and Hispanic fourth graders—and I define how NAEP uses those terms. Finally, I explain that we will also consider students' socio-economic-status (SES) as measured by the imperfect, but widely available, data point of students' eligibility for free or reduced lunch. The national data in this lesson can be found at http://nces.ed.gov/nationsreportcard/naepdata/. The PSTs are asked to analyze the two bar graphs shown in Figure 1 and to respond to prompts such as:

(a) What do you see?
(b) Make comparisons between groups in each graph.
(c) What conclusions can you reach from these data?
(d) What additional information would you like? Why?

Following this, the lesson asks: "What data would help you figure out if race/ethnicity is a factor in the U.S. educational system above and beyond issues of SES?" The PSTs are then given the information shown in Figure 2 and are asked to respond to the four prompts above for these new data.

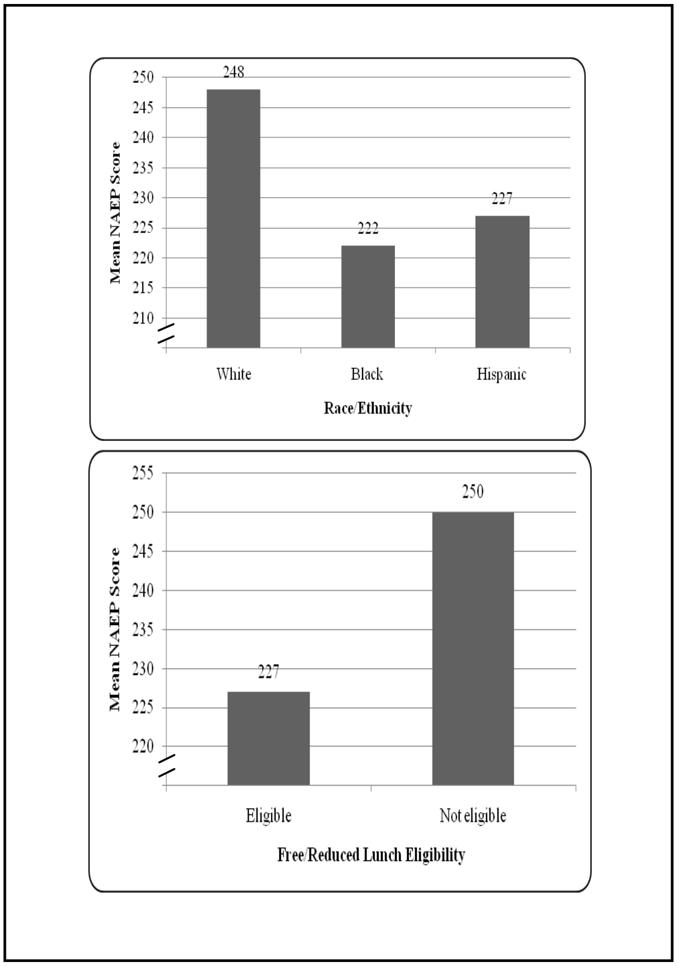

Figure 1. 2009 NAEP scores of fourth graders by race/ethnicity and lunch eligibility.

The PSTs then consider which graph is better for showing that race/ethnicity is a factor even after you account for different levels of SES and vice-versa. I use this last point to highlight that different representations of data are better for different purposes. I also mention that although we identified disparities in the bar graphs, this differs from the analysis a statistician would use to establish a statistically significant difference between groups. The PSTs are then asked to make similar graphs with the NAEP data for eighth grade students (available at http://nces.ed.gov/nationsreportcard/naepdata/). Finally, the PSTs are asked to discuss the questions shown in Figure 3, first in their groups and then as a whole class.

Analysis of the Lesson

The National Council of Teachers of Mathematics (2000) calls for students in grades 3-5 to "compare different representations of the same data and evaluate how well each representation shows important aspects of the data.… [and to] propose and justify conclusions and predictions that are based on data." (p. 176). This lesson provides the PSTs with an opportunity to engage in these aspects of data analysis while simultaneously learning about an important form of inequity in our society.

	Eligible	Not eligible
White	236	253
Black	219	232
Hispanic	225	237

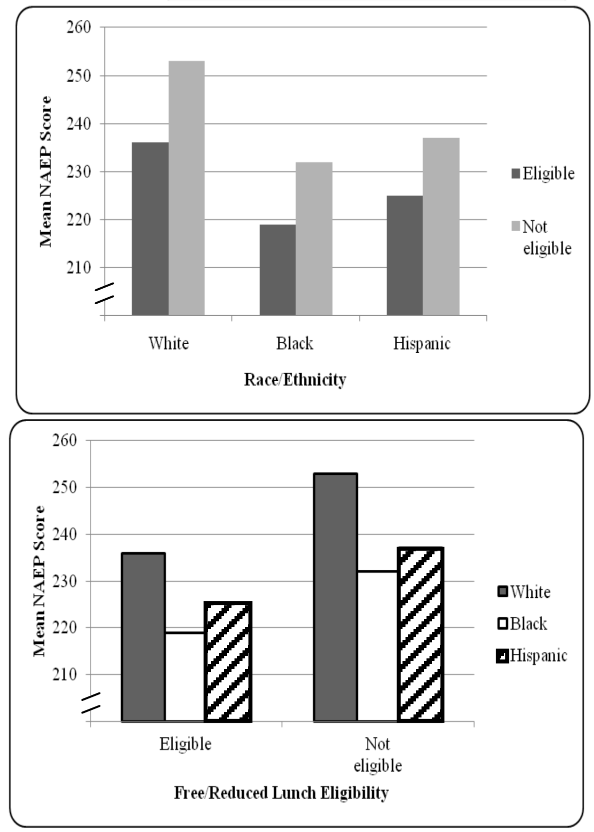

Figure 2. 2009 NAEP scores of fourth graders by race/ethnicity.

Regarding the *What* of mathematics, I intended for this lesson to send the message that mathematics is a valuable tool for understanding important real world issues. More specifically, this lesson serves as an example of how mathematics can be used to raise awareness about inequity in our society. Regarding the *How* of mathematics, the lesson highlights how real world contexts, such as test scores can be used as a meaningful context for learning mathematical concepts such as data analysis. Finally, with respect to the

Who of mathematics, this lesson may have functioned as a mirror because it connected to the PSTs' interest in and commitment to education and may have served as a window by deepening the PSTs' knowledge of this issue.

- ◊ What did you learn about educational outcomes in the U.S. today?
- ◊ Thinking about the *What*, *How*, *Who* framework from the Reflection Assignments:
- ◊ What level(s) of **What / social analysis** would you say this lesson focused on? Why?
  - ◊Level 1: Real World Connections
  - ◊Level 2: Political Topics
  - ◊Level 3: Awareness of Inequity
  - ◊Level 4: Critique of Structural Inequity
- ◊ Would you say that for **you** personally this lesson was more of a **mirror** or a **window**? Why?
- ◊ What was the relationship between the real world **context** of educational test scores, and the mathematical **concepts** (like analyzing data presented in tables and graphs)?
- ◊ What do you think about learning mathematics in this way?

Figure 3. Final reflection questions.

Examples of these three aspects of the lesson can be seen in the anonymous feedback I solicited regarding this lesson. In response to questions about this lesson and a lesson about income distributions in the U.S., the PSTs were asked whether they learned anything new about test scores or education in the U.S. and "how do you feel about *learning* (not teaching) mathematics in this way?" (emphasis in original). Out of 19 PSTs, 17 gave clearly positive responses, one indicated that "it was fine" learning mathematics in this way, and one wrote that while the visual aids in the lesson were good "the material is rather unsettling." PSTs' comments indicating further reflections or insights about testing in the U.S. included:

- ◊ "I didn't realize race had such an effect on test scores. I've always assumed it had more to do with economic status.… I like learning this way because it opens my eyes to actual facts instead of things I can't relate to."
- ◊ "I also learned that race (opportunity-wise) and SES has an affect [sic] on test scores.… I feel fine about learning mathematics in this way. I think it is very interesting and informative."
- ◊ "Yes, SES and race factors into test scores. Lower SES = lower test scores.… I enjoyed learning this way, it didn't feel like wasting time or just doing math, it was important."
- ◊ "Education: maybe because I already knew that there were differences, but I never knew how much it was.… I really enjoy it because it makes me aware about what's happening in the US."
- ◊ "It showed how much higher Whites test scores than Blacks + Hispanics, which is pretty sad.… I think it was interesting to learn this way and was easy to understand."

A consistent theme in the feedback is that using issues of inequity as a context made the mathematics more interesting and meaningful for the PSTs. An important area for further investigation would be the potential interplay of the race/ethnicity of PSTs with that of their students.

A Potential Pitfall

A number of authors have pointed to the dangers of focusing on "achievement gaps" in mathematics education. For instance, Flores (2007) has pointed to the importance of reframing the "achievement gap" as an "opportunities gap," arguing that differences in test scores indicate unequal educational opportunities rather than inherent differences in racial/ethnic or socioeconomic groups. Flores argues that "blanket statements about the low performance of certain groups of students in our schools without mentioning the underlying causes may reinforce prejudices and stereotypical images" (p. 30).

Since the lesson described in this article does not examine underlying causes for the differences in test scores, PSTs may interpret the NAEP data in a way that reinforces negative stereotypes about racial/ethnic groups and poor students. Take, for instance, the quote from above, in which one of PSTs states "I didn't realize race had such an *effect* on test scores" (emphasis added). Does this indicate a belief that some races are inherently better at mathematics than others or does this indicate recognition that opportunities to learn differ across racial/ethnic groups in the U.S.?

As discussed below, one way to address this concern is to expand this lesson to include investigation of inequities in the educational opportunities available to students. Due to time constraints this was not an option in my course. In lieu of this, I believe that two other aspects of my teaching discussed above help address this concern. First, in introducing the lesson I explicitly frame the lesson in terms of an "opportunities gap." Second, throughout the semester the PSTs reflect on the *What, How, Who* framework through a variety of readings about the teaching and learning of mathematics, some of which highlight the unequal educational opportunities available to students (e.g., Tate, 1994). Thus, although we were not in a position to examine the opportunities gap more directly, I did frame the issue in those terms.

Possible Extensions

As stated above, this lesson was used as an introduction to the data analysis portion of my course. Instructors of this course are expected to focus primarily on geometry and measurement, with some time devoted to data analysis, probability, and algebra. Therefore, while I also incorporate issues of equity into other portions of my class, there is relatively little time available in my course to expand on this lesson. However, this lesson could be extended to deal with more advanced forms of data analysis and to deepen PSTs' understanding of inequity of educational opportunities in the U.S. The NAEP website (http://nces.ed.gov/nationsreportcard/about/naeptools.asp) provides a wealth of statistical information and tools that allow the user to: (1) break down the data into other categories, such as state, English Language Learner status, gender, and student disability status; (2) run significance tests to make the analysis more rigorous (e.g., determining whether or not White students who are eligible for free/reduced lunch score statistically significantly higher than Hispanic students who are eligible for free/reduced lunch); (3) analyze changes in the scores over time; (4) create box-and-whisker plots of the distribution of student scores allowing for comparisons across student groups. Comparing box and whisker plots is found in the middle grades in both the NCTM Standards (NCTM, 2000) and the new Common Core State Standards (National Governors Association, 2010), and these comparisons can be valuable in highlighting the amount of *overlap* in NAEP scores that exists between different groups, which can counter the message that some groups are inherently better at mathematics than others. Another extension would be to explore what would happen if the benchmark scores for *basic*, *proficient*, and *advanced* were changed, thus highlighting the role of human judgment in interpreting standardized test results.

This work could also be extended by engaging the PSTs in a long term project in which they investigate the theoretical and empirical research, including the statistical data, on inequities in the educational opportunities afforded students in terms of race/ethnicity and SES in the U.S. Such analysis could include, but not be limited to, an analysis of school funding. Such an investigation would push this lesson from raising *awareness* of an existing inequity (level 3 of social analysis) to *critiquing* the inequity (level 4) by engaging the PSTs in understanding its origins and proposing possible alternatives.

Two notable examples of engaging teachers in an extended investigation of equity through a lens of data analysis can be seen in the McGraw & D'Ambrosio (2006) description of a workshop for teachers and Makar's (2004) analysis of a course for prospective secondary mathematics and science teachers. McGraw and D'Ambrosio describe a workshop designed to engage teachers in first analyzing several tables and graphs derived from the NAEP data and then using the NAEP data explorer (http://nces.ed.gov/nationsreportcard/naepdata/) to investigate state level data. Makar provides an example of a course focusing on assessment, instruction, equity, and inquiry for prospective secondary mathematics and science teachers. This course included a capstone three-week inquiry project into an issue of equity and fairness. Makar found that the prospective teachers deepened their understanding of statistics and in particular of their ability to connect statistical concepts to real-life situations. Interestingly, she also found that the depth of statistical analysis the teachers used correlated with their engagement in the topic they investigated, not with their level of statistical understanding, which points to the importance of engaging prospective teachers in connecting mathematics to contexts that they find meaningful.

Finally, data analysis is not the only content area that can be used to analyze issues of inequity in our world. For instance, in the past I have had PSTs analyze poverty by creating hypothetical budgets based on the federal poverty line, which emphasizes the NCTM's (2000) Numbers and Operations strand. A variety of resources exist for using mathematics to analyze inequity. Lesser (2007) lists a number of possible topics for investigation in his Appendix 1 and list of Selected Website Resources. The website radicalmath.org includes a number of lesson topics organized by mathematical content and social justice issue. Finally, Gutstein and Peterson (2005) and Stocker (2008) provide sample lessons and reflections on teaching mathematics for social justice. These resources can be a valuable starting place for pre- and in-service teachers interested integrating issues of inequity into their teaching while simultaneously addressing mathematics standards.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What are some common messages sent to students (and teachers) about the nature of mathematics and its role in understanding and shaping their world? How are these messages sent?
2. What alternative messages can we send about what it means to do mathematics and how can we send them?
3. What other social and political issues would be of particular importance for future teachers to analyze mathematically?
4. What social and political issues would be of particular importance for K-5, 6-8, and 9-12 grade students to analyze mathematically?
5. How might you extend the *What, How, Who* approach to topics other than data analysis?
6. Is mathematics more of a mirror or a window for you? For your students?
7. Does the discussion of underlying causes of test score differences change how you think you will handle interpretation and discussion of test scores in the future? If so, how?

Interpreting and Using Gestures of English Language Learners in Mathematics Teaching

Anthony Fernandes and Laura McLeman

Abstract

This article describes, through two vignettes, the use of gestures in the context of mathematical task-based interviews with 6th-grade students who are English language learners (ELLs). The first vignette illustrates the students' gestures associated with the concept of area and perimeter and the second vignette displays the effective use of gestures in supporting the students' thinking. Implications for teaching are also discussed.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions:

1. How are gestures connected to speech?

2. List some of the gestures you have seen ELLs make as they communicate their mathematical thinking. How have you used these gestures to interpret and build upon their mathematical thinking?

Anthony Fernandes (anthony.fernandes@uncc.edu) is an Assistant Professor at University of North Carolina Charlotte. His research examines effective ways of preparing mathematics preservice teachers to work with ELLs and measuring their beliefs about the mathematics education of ELLs. He is also interested in the use of various resources ELLs draw upon to communicate their mathematical thinking, especially gestures.

Laura McLeman (lauramcl@umflint.edu) is an Assistant Professor at the University of Michigan-Flint. Her research interests include preparing mathematics preservice teachers to work with underserved and underrepresented populations, especially ELLs. She is also interested in preparing mathematics teachers and mathematics teacher educators to integrate issues of equity and social justice in their instruction.

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Interpreting and Using Gestures of English Language Learners in Mathematics Teaching

Anthony Fernandes and Laura McLeman

Spontaneous hand movements that usually occur when people speak are a very common type of gesture (McNeill, 1992). The goal of this article is to examine how such gestures can prove useful when teaching mathematics to English Language Learners (ELLs). In particular, we outline how gestures can play a role in (1) interpreting ELLs' thinking and (2) guiding ELLs' mathematical work. Each point will be illustrated through a vignette that occurred in the context of task-based interviews with ELLs at two locations in the United States.

One theory about gestures considers them an add-on to speech that provides no information beyond that conveyed in the associated speech (Hadar & Butterworth, 1997). Another theory posits that gestures and speech form an integrated system that provides more insight into a person's thinking (Goldin-Meadow, 2005; McNeill, 1992). This latter theory is bolstered by theories of embodied cognition which assume that knowledge is constructed through bodily experiences in the environment (Lakoff & Núñez, 2000; Radford, 2003, 2009; Wilson, 2002). As such thinking, learning and teaching are multimodal processes that involve speech, words, signs, gestures, artifacts, and visual representations (Arazello, Paola, Robutti, & Sabena, 2009). This paper highlights the gesture mode in interactions with ELLs.

Gestures rely on visual imagery and convey meaning globally, unlike speech which relies on linear segmentation and conveys meaning discretely. For example, moving the forefinger in a circle can describe at once the spatial arrangement of people sitting in a circle, as opposed to the sentence "The people were sitting in a circle". Further, if the gesture and speech co-occur, the information encoded in the gesture is not necessarily redundant as it can also indicate to the listener the location of the seating arrangement in the room. The different information conveyed in the speech and gesture modes can prove to be valuable in educational contexts. For example, Perry, Church, and Goldin-Meadow (1988) found that when some 10-year-old children were asked to solve missing value problems like 3 + 4 + 5 = _ + 5, their strategies in speech and gesture differed. In their speech, the students conveyed a strategy that involved adding the numbers on the left to get 12, which they put in the blank (an incorrect solution). However, the students simultaneously pointed to the numbers 3 and 4, which -- if added -- would give the correct solution of 7. The gestures and speech of these students differed from another group of students who pointed to 3, 4, and 5 in succession as they put 12 in the blank. The researchers observed that, after receiving instruction, the former group performed better on similar items on the post-test.

Gestures are especially useful in communication when the speech may not be understood by the listener (Gullberg, 1998; Church, Ayman-Nolley, & Mahootian, 2004). Using gestures with students who are beginning to learn English can be very fruitful. For example, Church et al. used a pre-test to divide 51 first-graders consisting of 26 native English speakers and 25 native Spanish speakers into two groups, with each group including half of the students from each language category. One group was then provided instruction on conservation tasks involving volume and quantity (e.g., pouring water from one container to another with a different shape preserving the amount of water even though the water levels varied) using speech (first in English and then in Spanish) and gestures, while the other group was provided the same instruction in speech only. Both groups were later tested on conservation tasks. Questions were posed verbally in testing videos, both in English and Spanish, and the students were required to circle on an answer sheet if the quantity was conserved or not (S- Same, D-Different or M-Mismo, D-Diferente). Findings indicated that both language populations in the group that was exposed to gestures and speech scored higher on the post-test. Shein (2012) describes the use of a 5th-grade mathematics teacher's gestures to support her discourse practices of questioning and re-voicing as she engaged her students, all of whom were ELLs, in mathematical discussions to correct errors they made in finding the areas of geometric shapes. For example, the teacher built on the students' gestures to clarify the meaning of height by gesturing vertically and diagonally. Thus, through the gestures she intuitively conveyed the idea of the height being perpendicular to the base – something that may have been more challenging just to convey in words for her students. Neu (1990) also found that ELLs could "stretch" their linguistic competence through the use of gestures. In the case of ELLs who are still in the process of learning academic language, gestures can assume a larger significance in interpreting their thinking and helping them learn. Moschkovich (1999, 2002) asserted that bilingual students' mathematics work is best interpreted within a situated-sociocultural perspective where resources like gestures, along with objects and the use of native language, are integral to making meaning in mathematical communication.

Context

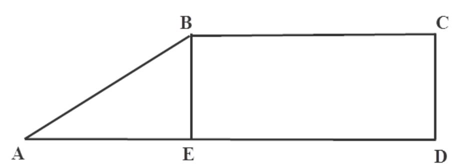
The two vignettes were chosen from a corpus of task-based interviews that were conducted in two phases with ELLs in grades 4-8 in the Southwest or Southeast regions of the United States. Phase one aimed to understand the linguistic challenges ELLs faced as they communicated their thinking and the resources that they used in the process. In phase two, the first author sought to extend this method to prepare PSTs to work with ELLs by understanding the involved challenges and resources.

Three researchers (including the first author) conducted the first phase, which yielded a vignette involving a 6th-grade Spanish-speaking student from a school that was located in a working class neighborhood in the southwestern United States. The second vignette, selected from the second phase of the project, involved 32 preservice teachers (PSTs) from the first author's geometry class interviewing ELLs at a local school. This vignette involved a 6th-grade Spanish-speaking student who attended a school in an urban city in the southeast United States with a rapidly growing Latina/o population.

Four National Assessment of Educational Progress (NAEP) measurement tasks that varied in difficulty (as judged by NAEP) and grade level were selected from a larger collection of NAEP measurement problems. Note that the NAEP is conducted only with students at the 4th, 8th and 12th grade levels, thus the tasks for 6th-graders were a blend of tasks from the 4th and 8th grade assessments. The tasks were tested in initial rounds of phase one and were selected because they involved linguistic facets that challenged ELLs, but at the same time garnered creative responses that included informal thinking, drawings, and concrete materials to solve the task. In both phases, the student was first asked to solve the problems independently and then explain his/her thinking process. Further probing questions were always asked to uncover the student's thinking process. The interviews were usually conducted by two persons, one interviewing and the other videotaping.

Vignette 1: Area and perimeter gestures

This vignette[1] focuses on Rita, a 6th-grade Latina student who was interviewed in English by a university researcher named Carol. Rita was working on the Trapezoid Problem in Figure 1. In particular, we discuss Rita's use of gestures as she discussed the concepts of area and perimeter.



The area of rectangle *BCDE* shown above is 60 square inches. If the length of *AE* is 10 inches and the length of *ED* is 15 inches, what is the area of trapezoid *ABCD*, in square inches? (NAEP 1992, 8th Grade, question #6 from Block M12; http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx?subject=mathematics)

Figure 1. The trapezoid problem.

After reading the problem, Rita mentions that she is unable to understand the problem and Carol proceeds to uncover what she knows, with the intention of assisting her to solve the problem. The interaction below involves Carol trying to get Rita to interpret the information given in the problem and write the measurements in the figure at appropriate places. Rita's gestures are particularly interesting to note when she is asked to put the "60" in the figure.

C: Okay, so… what they are talking about?

R: [*After a slight pause*] That all of this [*points to the base with the pencil* (see Figure 2)] this [*changes the pointing to BCDE* (see Figure 3)] rectangle is 60.

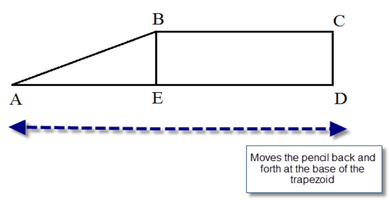


Figure 2. Rita's movement along the base of the trapezoid.

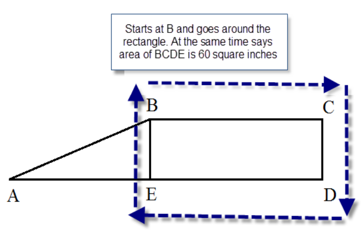


Figure 3. Rita traces along the outside of the rectangle.

C: Okay, so can you put the 60 somewhere in the figure and to keep in mind this information?
R: Right here [*points to BC with the pencil* (*see Fig. 4*)]
R: [*Writes 60 inside the rectangle just below BC*]

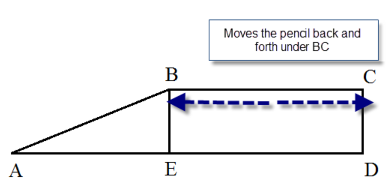


Figure 4. Rita traces along the length of the rectangle.

The gestures that Rita uses are linear in all instances. She initially points to the base of the trapezoid while verbalizing area of rectangle *BCDE* and then changes her gesture to outline the rectangle *BCDE* in the figure. What is noteworthy is that both gestures indicate a linear measurement, not a display of space, as given in the problem [see Figure 1]. Again, when asked to write the 60, Rita points to *BC* and writes 60 below it, as one would indicate a line segment's length.

Further into the interaction, Carol tries to scaffold the Trapezoid problem by asking Rita to draw a separate rectangle with lengths of her choice and work out the area. Rita mentions that the lengths of the sides of the rectangle should be added to get the area. Carol draws Rita's attention to a previous problem where she had added the sides to get the perimeter, implying that perimeter and area could not be calculated in the same way. Referring to the rectangle that was drawn, Carol probes Rita about perimeter and area.

C: If you were to explain to a 4th-grader what perimeter is, how would you explain it?

R: The inside of any shape [*simultaneously outlines a square or rectangle* (Figure 5) *on the table with the eraser that was on the pencil she had in her hand*]

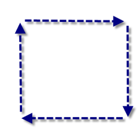


Figure 5. Rita's movement while explaining the concept of perimeter.

…
[*8-second pause*]
C: What is area?
R: Oh the inside [*simultaneously outlines a square or rectangle* (Figure 5) *on the table with the pencil eraser*] and the perimeter is the outside [*shifts hand from the table to the paper and moves the pencil eraser around the outside of the square provided in a previous problem that she solved correctly (Fig. 6)*]

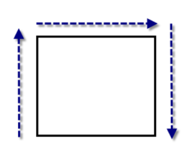


Figure 6. Rita's movement while explaining the concept of area.

Rita uses the same gestures to refer to the area and the perimeter of the rectangle. When referring to area, she said 'inside' but outlines line segments to indicate a square or rectangle. When discussing perimeter, Rita goes back to the figure of the square that was provided in the previous problem and outlines line segments, though these are now literally on the outside of the square. Rita probably interpreted the 'inside of the shape' (when referring to area) to mean just inside the boundary of the object, but still measuring length instead of space and the 'outside of the shape' (when referring to perimeter) to mean measuring lengths outside the boundary of the shape. At no point did her gestures for area seem to indicate the enclosed space in an object. When probed by Carol, Rita's use of similar gestures could account for the same calculations (adding up the sides) she was doing to work out the perimeter and area of the rectangle.

Vignette 2: Gestures supporting instruction

In this vignette, Tess and Matt, both preservice teachers, interviewed[2] Carla, a 6th-grade ELL on the Tile Problem (Fig. 7). Carla was provided a 0.5-inch graph sheet, and she worked on the problem independently for 12 minutes before she indicated that she was ready to discuss her thinking.

The Tile Problem

How many square tiles, 5 inches on a side, does it take to cover a rectangular area that is 50 inches wide and 100 inches long?

(NAEP 2009, 8th Grade, question #17 from block M5; http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx?subject=mathematics)

Figure 7. The interview task given to Carla.

Tess, who was the interviewer, asked Carla to explain her solution. Carla used a lot of pointing gestures to explain that she counted the squares along the length (39) and width (30) of the graph sheet, oriented in landscape view in front of her, and multiplied these numbers to get a total of 1170 tiles (Note that Fig. 8 shows a cropped graph sheet with the entire width, but only part of the length). Tess then asked Carla what information was provided in the "story problem".

> T: And how many [tiles] did it tell you in the story problem?
>
> C: [*Reading from the problem*] If 5 inches of side (sic) [*starts to move the pencil to point to the bottom of the graph sheet where she had counted 10 tiles and had written "5 inches on a sid[e]" (Fig. 8)*]. It says if 5 inches of side (sic) [*moves pencil in the air from the left end to the right end of the graph paper as she utters "side" and then retracts the pencil to the initial point (Fig. 8)*]

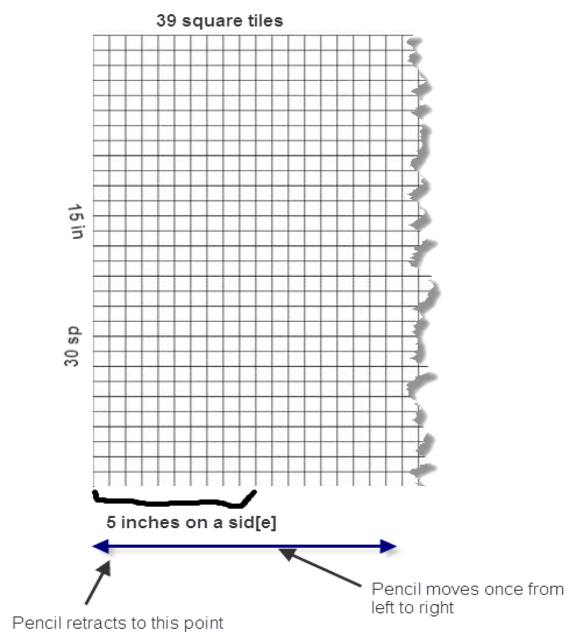


Figure 8. Reproduction of Carla's work (in boldface) on the graph paper.

Note that since this was a 0.5-inch graph sheet, the 10 squares that she marked would represent an actual measurement of five inches. Key to Carla's explanation of the tile measurement is her gesture as she utters "side" and moves her pencil, in the air, along the side of the graph sheet. She did not refer to the square on the graph sheet as a tile, even though she made this assumption for her initial solution. Her initial work shows that Carla has some idea how to solve the problem but she is unsure how to interpret the phrase "5 inches on a side," which she assumes is a linear measurement related to the side of the rectangle. The artifacts themselves do not create meaning; instead, they act as a background on which the student draws certain features to the attention of the listener. Thus, the gesture employed by Carla clarifies to Tess her meaning of "side" in the phrase "5 inches on a side".

In further interactions, Tess attempts to support Carla through mostly verbal instructions with a few pointing gestures. First, she instructed Carla to draw a rectangle of any size in order to represent the 100" by 50" rectangle given in the problem. Using this representation, Tess then asked Carla to "pretend that there are 5-inch tiles inside [of the rectangle]." When this strategy of imagination failed to help Carla progress in the problem, Tess instead asked Carla to think about how division might help her. Through these questions and different strategies, it seems that Tess was challenged to pinpoint exactly why Carla was struggling as well as how to help her proceed.

After observing Carla's interaction with Tess, Matt interjects and assumes the role of the interviewer. Matt uses concrete materials in conjunction with gestures to mediate the interactions with Carla. He uses a graph sheet to represent the 100" by 50" rectangle and a square cutout to represent the tile.

> M: Ok so can we just take this, so here is the graph paper [*takes the paper that is lying on the table*] and let's just pretend that this [*moves finger up and down along the longer side of the sheet oriented in portrait view*] is a 100 inches long and this is going to be 50 inches this way [*moves finger side to side along the shorter side of the sheet oriented in portrait view*]. So this is one of the shapes that you used in the other problem [*holding up the tile*]. So if that [*holds up the cutout*] was the tile that is 5 inches on each side, what would the measurement be on this side? [*points to one of the sides*]
> C: [*points with pencil in space*] five
> M: Five. How about his side? [*points to the next side*]
> C: Five
> M: This side? [*points to the next side as he rotates the cutout*]
> C: Five
> M: And this side? [*points to the last side as he rotates the cutout*]
> C: Five
> M: It's five on all sides, right? So could we go ahead and if this [*points to the length of the graph sheet*] was a hundred inches long, can we put this on here [*places tile in the bottom left corner of the sheet* (see Fig. 9)]. How many of these would it take [*starts moving the tile up the sheet and emphasizes this motion by tapping down on the table after each addition*] if it was a hundred inches. [*C starts doing some calculations on her paper*]
> C: Twenty!

Carla successfully works out the tiles that would fit along the width and the total number of tiles it would take to cover the entire rectangle.

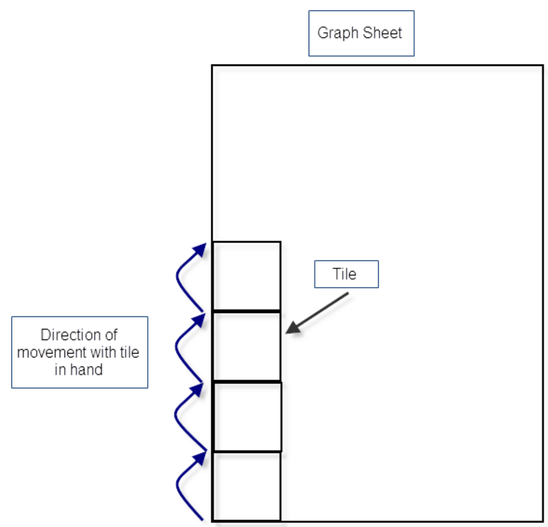


Figure 9. A representation of Matt's support for Carla.

In a written report[3], Matt explains the reasoning behind the guidance he provided. In particular, he shares how he focused his attention on a possible confusion that Carla may have had with the wording of the problem. Based on how Carla drew a line to represent the "5 inches on a side" (Fig. 8), he conjectured that the 5 inches referred to a measurement on "a side" of the larger rectangle rather than all four sides of the square tile. Matt also conjectured that this was because she may have encountered the dimensions written in the more familiar format of 'each side was 5 inches' or '5 by 5'. Matt first takes the graph sheet and makes salient the dimensions of 100 inches and 50 inches by pointing to the length and width respectively. By using the word "pretend", Matt attempts to distinguish this approach from Carla's first attempt where she worked out the area by multiplying the actual number of squares along the length and the width. He takes the cutout from a prior task and once again emphasizes the dimension of each side by pointing to it and asking Carla to confirm the measure. This move was key for Carla to understand the phrase "5 inches on a side" and both the cutout and the gestures play an important role in confirming, for Carla, the word "side" in the phrase. Finally, Matt's gesture of moving the cutout up the side of the sheet and simultaneously using a rhythmic tapping gesture adapts Carla's first method (where she counted squares along the length and width) and she is able to find the solution. Matt's speech — "How many of these would it take if it was 100 inches" — is not enough to convey the method he had in mind. It is the gesture (in Fig. 9), in conjunction with the speech and concrete materials, which conveys the complete meaning.

It is interesting to note that Tess' approach, at some point in her probing, resembled the approach by Matt. She asked Carla to draw a 100" by 50" rectangle and imagine that there were 5-inch square tiles on the inside. Tess conveyed most of the information verbally to Carla and was unable to move her forward. Matt, however, used gestures, grounded in the materials, as a bridge to connect the information in the problem successfully.

Conclusion

Gestures can be used to interpret the thinking of all students, but may be especially useful and revealing in the case of students who are still learning academic language in English. Teachers should pay attention to the gestures where conflicting ideas are conveyed in the speech and gesture channels (e.g., making gestures for perimeter when talking about area). These differences could indicate possible misconceptions that permeate the students' thinking.

Campbell, Adams, and Davis (2007) point out that the cognitive load on ELLs is greater than non-ELLs when solving mathematics problems due to the added linguistic demands they face. In our observations, the ELLs in the task-based interviews used materials, gestures, and speech to distribute part of this cognitive load in the environment. For example, by using gestures to make reference to features in the drawings or materials, these students could create a meaningful argument without using precise academic language. By accepting and legitimizing these arguments, teachers will go a long way in encouraging the participation of ELLs in the classroom. Note that we are not advocating that ELLs should not be expected to create precise verbal arguments; instead, we see the use of gestures as a transition towards these types of precise arguments. Roth (2002) demonstrated this progression through his observations of the use of gestures by high-school students in a Grade 10 Physics course. When asked to explain the phenomena in experiments they were conducting, the students initially relied heavily on the equipment from the experiment to construct explanations. Gradually, gestures dominated their explanations as the students could talk about the phenomena without the presence of the equipment. Towards the end of the course, the researcher observed that the students were providing most of their explanations through speech with use of academic vocabulary they were learning in the course.

In addition to understanding and paying attention to ELLs' gestures, teachers can also employ gestures to make abstract content of mathematics comprehensible (Short & Echevarria, 2004/2005). Gestures can play a vital role in this process as they can act as the "glue" (Alibali & Nathan, 2007) that binds the speech and the concrete materials that the teacher uses. Given the embodied nature of mathematical ideas (Lakoff & Núñez, 2000), engaging students in concrete activities such as shading areas of different shapes to understand the meaning of 'inside of the shape' when referring to area, can provide a platform for a robust understanding of the concept. By expanding students' inputs to include gestures, teachers' understanding of student thinking increases and this knowledge can inform future instructional design. By focusing on the gestures in addition to the speech, it is possible that teachers will see ELLs' thinking at a higher level, and in turn, develop higher expectations of them.

Notes

[1]See Fernandes (2012a) for more elaboration of Vignette 1; all names used are pseudonyms.

[2]This interview was part of phase two of the project, designed to foster awareness among preservice teachers of linguistic challenges faced by ELLs and the resources they draw upon as they communicate their mathematical thinking in English (Fernandes, 2012b). Pairs of preservice teachers from a geometry content course interviewed an ELL on four NAEP measurement problems and wrote a report based on questions designed by the instructor. Later in the semester, each pair conducted another interview with a different ELL and turned in a second report.

[3]Both Tess and Matt were required to provide written reports in which they reflected on specific elements within the task-based interview, such as the language used by the student and why they made certain instructional decisions.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions:

1. How do the findings from our two vignettes build upon the framing of gestures in Moschkovich (1999)? Specifically, how do the students in Moschkovich's article differ in their use of gestures to communicate their mathematical thinking with that of the students in the two vignettes?

2. Lesser and Winsor (2009) provide an example of how an individual can use gestures to illustrate understanding of a statistical concept. With this consideration and the difficulties some ELLs face in lecture-style presentations (see pp. 17, 19 of their paper), how might you use gestures to convey the meaning of statistical concepts such as standard deviation?

3. Warkentin (2000) provides examples of how gestures may facilitate communication in a geometry class. What examples do you think would be most effective in the classes you teach and why?

4. In the conclusion, we argue that teachers should pay attention to the gestures where different ideas are conveyed in the speech and gesture channels. What specific instructional practices could you employ in order to notice the gestures that ELLs are using to communicate their mathematical thinking? Further, videotape yourself interacting with an ELL and analyze the gestures you and your student used to communicate mathematical thinking.

5. What gestures do you observe when ELLs use technology and how do they differ from the gestures used in face-to-face interactions? How might the use of technology affect the use of gestures to communicate mathematical thinking?

"La Lotería" - Using a Culturally Relevant Mathematics Activity with Pre-service Teachers at a Family Math Learning Event

Olga M. Ramirez and Cherie A. McCollough

Abstract

This paper aims to raise awareness of how university content faculty prepare pre-service teachers to implement culturally relevant math activities involving Hispanic families in an after-school Family Math Learning Event (FMLE). By exploring the game of chance "La Lotería" from Hispanic culture with math objectives appropriate for elementary and middle school grades, this paper illustrates the potential impact on future teachers and Hispanic families participating in the FMLE.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What do you know about typical games played in Hispanic households?
2. How can the "bingo-like" board game "La Lotería" played by many Hispanic families be used to teach mathematical concepts?
3. What do you know about Family Math Learning Events (FMLEs)?
4. Why are the objectives listed in Table 1 (on the following page) important for pre-service teachers involved in a FMLE? To what extent are these objectives aligned with the mission and goals of TODOS: Mathematics for All?

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"La Lotería" - Using a Culturally Relevant Mathematics Activity with Pre-service Teachers at a Family Math Learning Event

Olga M. Ramirez and Cherie A. McCollough

"La Lotería," a colorful and historical game of chance, holds a special place in Hispanic tradition (see Figure 1). Many Hispanic families, especially those with parents and grandparents in South Texas and México, enjoy playing this game together. While "La Lotería" has structural similarities to U.S. bingo, Lotería boards are very colorful, featuring vivid pictures accompanied by a number and a word describing the scene. We incorporated this rich cultural resource into a Family Math Learning Event (FMLE) sponsored by the first author's university and a local school and results are included in this paper.

Presenting such culturally responsive math activities at a FMLE helped inform pre-service teachers' future classroom experiences by providing awareness early in their teacher preparation program of the importance of relating to Hispanic families in a manner that values their culture and everyday circumstances, consistent with the objectives in Table 1.

Culturally Responsive Mathematics Teaching

This work regarding culturally responsive mathematics and pre-service teachers (PSTs) is informed by the work of Ladson-Billings (1995, 2001), who notes the challenge that teacher educators (88% of whom are White) have leading PSTs to experience teaching diverse cultures. Ladson-Billings (1995, 2001) remarks that while our students are becoming more diverse, the majority of the teacher work force is white as are PSTs and their educators. Yet, making radical changes to reorient teaching faculty to other types of cultures or values is labor intensive and leaves less time for traditional activities of research and publication (Young, 2011). Educators need to be aware of their own culture and of their perceptions of other cultures in order to gain understanding of the role culture plays in connecting to families as part of the learning process (Morrison, Robbins, & Rose, 2008).

Furthermore, PSTs are generally not provided with culturally responsive mathematics activities. They lack knowledge of cultures different than their own, and they experience a level of discomfort when challenged with the notion that they may have misconceptions of culture. They also lack awareness of how mathematics and culture improve teaching and learning (Gutstein, Lipman, Hernandez, & de los Reyes, 1997; McCollough & Ramirez, 2012b). Ladson-Billings (1995) suggests that in order to achieve equity and excellence in diverse classrooms, teachers should include student culture in the classroom as official knowledge. Additionally, families should also be included as research suggests that community-based experiences are more powerful than stand-alone multicultural education programs (Sleeter, 2001). Consequently, parent and community involvement in the public schools has become a venue that helps improve the quality of children's cultural and educational experiences (Morrison et al., 2008). This involvement is said to help increase student achievement, is important for children of low socioeconomic status, and has

Table 1
Objectives for Pre-service Teachers Involved in a FMLE

OBJECTIVE	DESCRIPTOR
Content Knowledge	The pre-service teacher must prepare and deliver a culturally relevant math lesson that incorporates concept learning, procedures with understanding, and processes that support problem-solving to help develop their comfort and confidence with the math content.
Teaching Skill and Pedagogy	The pre-service teacher must practice planning and conducting a culturally relevant math lesson with an instructional activity that helps enhance their teaching and communication skills and that will bring them close to the learner early in their teacher preparation program.
Professional Disposition	The pre-service teacher must practice developing professional teaching behaviors that support the belief that all students can learn and that reflect a caring and supportive culturally respectful learning environment with families.

positive effects on the development of linguistic minority students (Tomás Rivera Policy Institute, 2007).

The main purpose for infusing culturally relevant mathematics in pre-service teacher preparation programs is to help PSTs examine their own cultural perceptions and, in turn, learn to apply what they know about their students' culture to mathematics. In addition, through FMLEs, the PSTs develop critical approaches to knowledge and skills they will need to be culturally responsive teachers. This work on culturally responsive mathematics has grown from projects with PSTs participating in FMLEs where culturally relevant examples, materials and activities are structured to encourage parent and child teams to work together to solve problems or investigate natural phenomena (McCollough & Ramirez, 2010; McDonald, 1997). In these events, parents and children self-select learning activities that have been designed by the PSTs and implemented after school in a venue such as the cafeteria or gymnasium, moving at their own pace with the pre-service teacher-facilitators providing materials and encouragement. Exploring how pre-service mathematics teachers incorporate mathematics in a culturally relevant and engaging way in a FMLE is a way for practitioners to merge formal mathematics classroom teaching with informal settings that bridge schools, mathematics, culture, and families.

Exploring "La Lotería" for a FMLE

In the first author's content mathematics course for teachers at her university, one of the PSTs knew that "La Lotería" was a popular game among Hispanics, especially in her family, and she wanted to prepare a mathematical lesson with historical connections related to this game. It did not take her long to notice something that spurred her curiosity. We had been studying the problem-solving strategy 'look for a pattern' and arithmetic sequences (Billstein, Libeskind, & Lott, 2007) and this helped her to notice that "La Lotería" boards had rectangular groupings of varying sizes. Could a pattern help her find all possible rectangles on the board? As she pondered these ideas, she concluded that this would be her FMLE activity and she began her research regarding the cultural relevancy of the activity and how to solve the problem.

This activity would be one among others implemented as part of a math content course FMLE requirement for which the pre-service elementary and middle school teachers, under faculty supervision, thoughtfully conceptualize, construct and implement culturally relevant mathematics activities as part of their undergraduate mathematics class that prepares them to see mathematics from a teacher's perspective. This assignment is required in a math content course taken prior to their final year of their student teaching internship. The 48 mathematics activities implemented for this assignment included 20 with a culturally relevant focus. This paper showcases the activities based on the popular game of "Lotería," because of the ease with which Hispanic families gravitated to this activity while at the same time expressing curiosity about how mathematics could be learned with this familiar family game.

Aside from sharing the historical and traditional aspects of the Lotería board game at the FMLE in a Hispanic community, the pre-service teacher working on this project would connect how mathematical principles can help families see mathematics even in basic things as board games. This is important since board games may motivate students to learn and use mathematics (Ramani & Siegler, 2008; Siegler & Ramani, 2008) to determine good winning strategies. Furthermore, board games have been used in pre-service teacher programs to help local school students develop an understanding of various mathematical concepts (Jiménez-Silva, White-Taylor, & Gómez, 2010) and to motivate them to practice skills previously learned (Jiménez-Silva, Gómez, & White-Taylor, 2010).

"La Lotería" Cards and Cultural Connections

Another PST found on Wikipedia (2009a) a picture of all 54 colorful "La Lotería" cards such as: "El Diablo" (The Devil), "La Dama" (The Lady), "El Catrin" (The Gentleman), "El Paraguas" (The Umbrella), and "La Muerte" (Death). Unknown to the PSTs prior to their research, "La Lotería" boards historically depict a form of folk art as each of the colorful cards have images of popular Mexican figures. There are many variations of these colorful cards, including "La Lotería" boards with images of fruits and other unusual figures from the Day of the Dead celebrated in Mexico on the second day of November which parallels the Western celebration of All Souls' Day celebrated on the first day of November (Chisholm, 1911, as cited in Wikipedia, 2009b). Also, a PST was fascinated by how the Rodriguez and Herrera (1999) book accompanied each Lotería card with an artist's linoleum print and a poet's linguistic riffs. In that book's forward, Rupert Garcia explains that "La Lotería" came from a secular, colonial Spanish card game that arrived in México during the latter 18th-century, first played as a parlor amusement game by the colonial social elite, but eventually played by all social classes.

Immersed in the historical origin of "La Lotería," the PSTs purchased "La Lotería" cards at local stores (or from Amazon.com) and played the game. Much like U.S. bingo, when a Lotería card is drawn and the player has a picture of the card on his/her "La Lotería" board, the picture is covered with some type of counter such as a coin or (as often done in Hispanic homes) an uncooked pinto bean. The winner of "La Lotería" is the first to make any of the winning moves depicted in Figure 1. Students can verify that there are 12 ways a person can win: four ways by rows, four ways by columns, one way by the corners, two ways by diagonals, and one way by center rectangle).

Interviewing Families

To gain understanding and sensitivity of culturally relevant math activities, each of the 52 PSTs was asked to do four in-person interviews prior to the FMLE – interviewing two people about the Lotería activity and two people about a Quinceañera activity. Thus, 104 people in the community served by the first author's university were interviewed about the Lotería board game. The interviewees' ages spanned 13-85 and 86% were female. Responses to the interview questions in Table 2 indicated that 96% were very familiar with "La Lotería" and 92% agreed that it was a good way for Hispanics to preserve their culture. They felt this way because "La Lotería" is a game that has been played for many generations and is part of the social life of many Mexican families.

In addition, they felt that the game provides children an opportunity to practice their ancestral language of Spanish and to learn the spelling of Spanish words correctly, as this may be one of the few times their children are exposed to the written language of Spanish.

When asked if they could relate the game to mathematics, 77% stated that probabilities were involved in playing the game. While they did not specifically mention what probability concepts are learned by playing this game, it was ap-

Table 2
Interview Questions

- ◊ Are you familiar with the game "La Lotería"?
- ◊ Do you know how to play this board game?
- ◊ Do you or members of your family play this game? How often? If so, do they pay to play?
- ◊ When the dealer calls out each card, what are some of the quips (or rhymes) they state?
- ◊ Where do you think this game originated?
- ◊ Do you think that playing "La Lotería" in family gatherings is a good way for Hispanics to preserve their culture or to have children learn some Spanish words? Why?
- ◊ Can you relate this game to mathematics in any way? If so how?
- ◊ Can you find all the rectangles in one card? How many? Can you find a pattern?
- ◊ Is there anything else you would like to share about the game "La Lotería"?

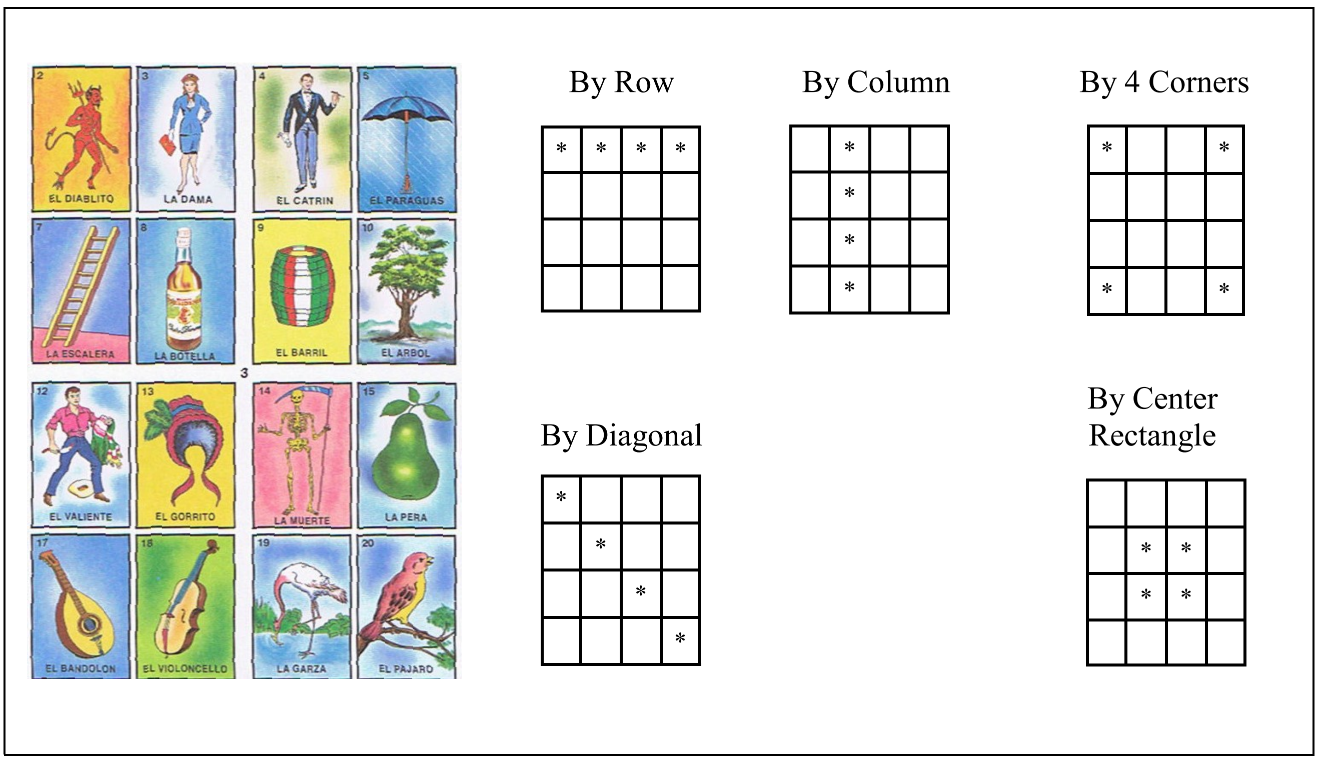

Figure 1. Sample "La Lotería" board and types of winning moves.

parent that the respondents felt that this game is a game of chance and they knew that the more game boards they play, the more chances they have of winning. Further connections that could be made include the relative likelihoods of each of the winning patterns in Figure 1 (e.g., the probability of winning with a horizontal row is four times greater than winning with "four corners"). Also, playing with money offered the opportunity to do counting as well as operations like addition or subtraction.

In classroom discussions, PSTs shared that they were impressed with the interview responses and felt that the interviewees showed genuine interest in "La Lotería." Also, the PSTs reported that an elderly lady mentioned that when a card was drawn, the announcer (also called the dealer) would often call it out with whimsical humor and a poetic flair that resembles what Garcia states in his introduction of Rodriguez and Herrera (1999) that when a "La Lotería announcer chooses a card from the deck, he doesn't simply call out its name, but rather, he either improvises a short poem or uses a stock phrase that makes a poetic allusion to the character on the card" (pg. xiv). For this reason, families like to appoint as the dealer a lively individual who will offer some funny or interesting quips, poems, stock phrases, or quotes. Some sample quips and quotes collected

Table 3
Sample Quips and Quotes Used when Playing "La Lotería"

Card	Spanish Quips and Quotes	English Translation
Rooster	¡El Gallo que alardea y alardea, despierta la mañana ha venido!	The Rooster that crows and crows: wake up! the morning has come!
Death	¡La Muerte, no tengas miedo; un día vendrá, pero estarás listo!	Death, do not be afraid; one day it will come, but you will be ready!
Sun	¡El Sol, la manta para el pobre!	The Sun, the blanket for the poor!
Shrimp	¡Camarón que se duerme se lo lleva la corriente!	The Shrimp that falls asleep is swept away by the tide!

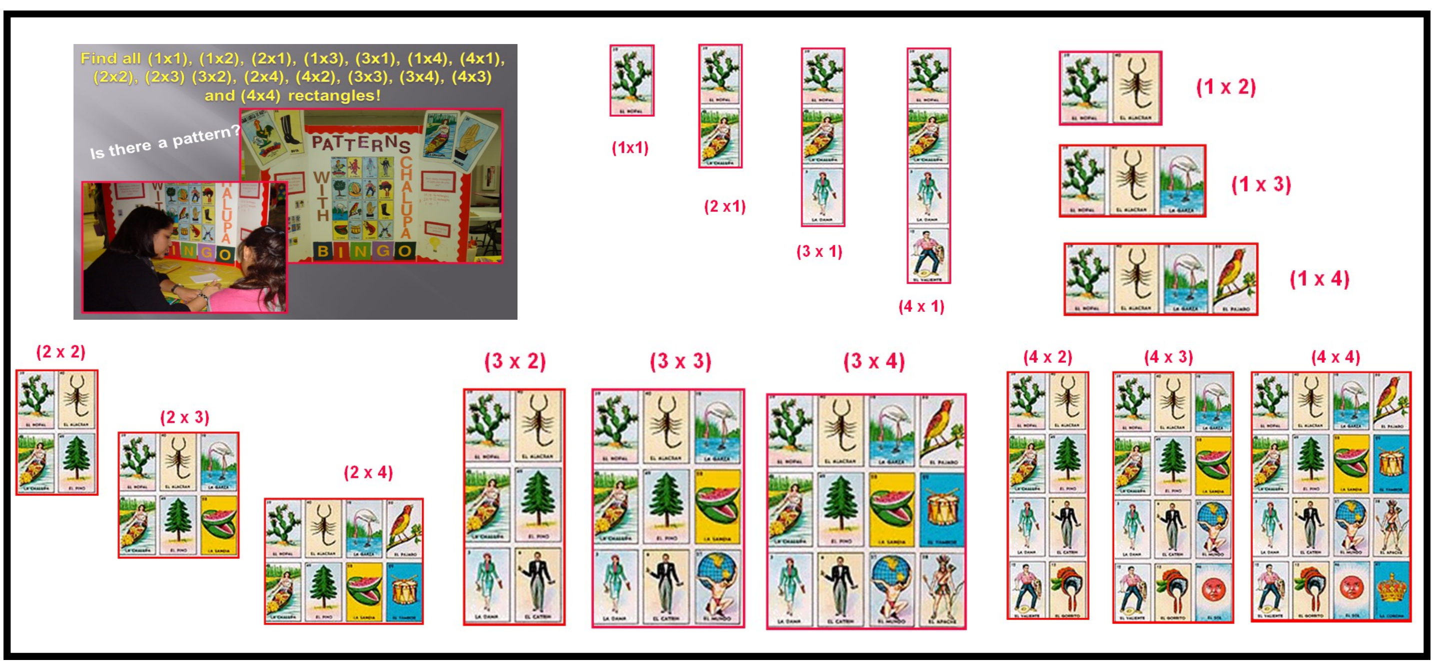

Figure 2. FMLE participants finding the rectangles on a "La Lotería" game board.

by students during the interviews are listed in Table 3, with English translations supplied by the first author.

PSTs found that the Lotería game is a favorite pastime for elderly Hispanics, but can often be a game played by entire families for hours at a time. Interviews indicated that games are often played with an entry "pot" fee per game (ranging from a nickel to a dollar), but some families play without the entry fees (as would any use in schools). However, an essential characteristic that PSTs discovered about Hispanic families playing "La Lotería" is that this group event brings families together to have fun and to socialize while helping children learn Spanish as previously stated. In addition, playing this game also gives them the opportunity to discuss with their elders cultural connotations portrayed in the colorful Lotería cards.

One PST who was interested in "La Lotería" for the FMLE asked the Hispanic family audiences to find all the rectangles on the Lotería game board. She knew this would be a challenging problem and she began by seeing what the families would do without much guidance or minimal assistance. Figure 2 illustrates the tri-fold poster she used for her presentation. When some families focused only on individual (1×1) oblong rectangles, the PST was ready to help them identify different arrays of oblong rectangles using strips with rows and columns cut from the Lotería cards as noted on Figure 2.

The PST showed them how rectangles overlapped with one another on the game board. After families' initial attempts, she furnished them with a problem-solving strategy known as the systematic list strategy (Johnson, Herr, & Kysh, 2004) as shown on Table 4. The table was provided (without the answers already filled in) so the families could organize their thinking to record their findings while searching for the various types of rectangles on the Lotería board. Once the families used the systematic list strategy, they were more successful at finding more of the known 100 rectangles on the 4 × 4 "La Lotería" board as so noted on Table 4. After this and subsequent FMLEs where the systematic list was used as part of a "Loteria" activity, the PSTs often shared in the content course with their classmates and instructor the effective use by families of this problem-solving strategy. This authentically supports the instructor's communication to the PSTs of the importance

Table 4

Systematic List Strategy for Finding Rectangular Arrays

Rectangular array dimensions	Number of such arrays	Rectangular array dimensions	Number of such arrays	Row Total
1 × 1	16	1 × 1	Already counted	16
2 × 1	12	1 × 2	12	24
3 × 1	8	1 × 3	8	16
4 × 1	4	1 × 4	4	8
2 × 2	9	2 × 2	Already counted	9
3 × 2	6	2 × 3	6	12
4 × 2	3	2 × 4	3	6
3 × 3	4	3 × 3	Already counted	4
3 × 4	2	4 × 3	2	4
4 × 4	1	4 × 4	Already counted	1
GRAND TOTAL				100

of math problem-solving strategies. PSTs would also mention that the families who used the systematic list to guide their thinking and to organize the number of rectangles often found more of the rectangles than the families who randomly looked for the rectangles. Other representative comments some family members made regarding this activity included “We didn’t know how to get started” and “The table helped us a lot!” The value of 100 conveniently offers an opportunity to ask questions involving percent (e.g., what percent of the possible rectangular arrays is a certain size).

For younger children in attendance, another PST took the opportunity to introduce the rectangular array model of multiplication with the picture models of the Lotería rectangular arrays as shown on Figure 1. She also illustrated how to model the Distributive Property of Multiplication over Addition [$a(b + c) = ab + ac$, where a, b, and c are counting numbers] by using La Lotería cards that depict examples such as: 2 (2 + 3) = 2 (2) + 2 (3) as shown in Figure 3.

Another mathematical question posed by another PST to the families included asking them to guess how many “Lotería” game boards (each with 16 different pictures) could be created with the set of 54 Lotería picture cards. She helped them begin to generate an answer pattern by using the problem-solving strategy of simple cases shown on Table 5. This was a bit difficult to teach, but the families realized with some help that the number of possible different 4×4 “La Lotería” boards (i.e., arrangements of 16 images chosen without replacement from a set of 54 distinct images) was enormous (a 27-digit number!) but that there was a way to determine the answer. Because the location of an image on the card can affect whether one of the winning moves noted in Figure 1 is attained, this could provide a motivation in a formal instructional setting to discuss why the mathematical term “permutation” is more appropriate than “combination.”

The mathematical problems posed in using the Lotería board game, are not necessarily unique to the Lotería game. However, by using the Lotería card instead of a traditional bingo card or other rectangular array, this culturally familiar game draws Hispanic adults and children to look at the Lotería card in a new and interesting way. Whether the Lotería board game is used to pose the oblong rectangle problem, used as a means to show rectangular arrays with multiplication connections, or used as the launching pad for asking how many Lotería board games can be created with the 54 Lotería cards, the PSTs (as well as their course instructor) who did or witnessed the Lotería activities at this FMLE and at subsequent FMLEs suggest that the cultural familiarity with the game excites and motivates the Hispanic families to solve the mathematical problems because of the Lotería game’s cultural connection. The Hispanic families’ cultural identification with the Lotería game is so strong that mathematical problems, despite their difficulty, do not seem to pose a threat to them. This is important in creating a welcoming and non-threatening Family Math environment for Hispanic families to learn and enjoy mathematical activities.

Discussion

The FMLE experience helped the PSTs realize the value of culturally relevant mathematical topics. They noted that the Hispanic families felt comfortable when they saw activities

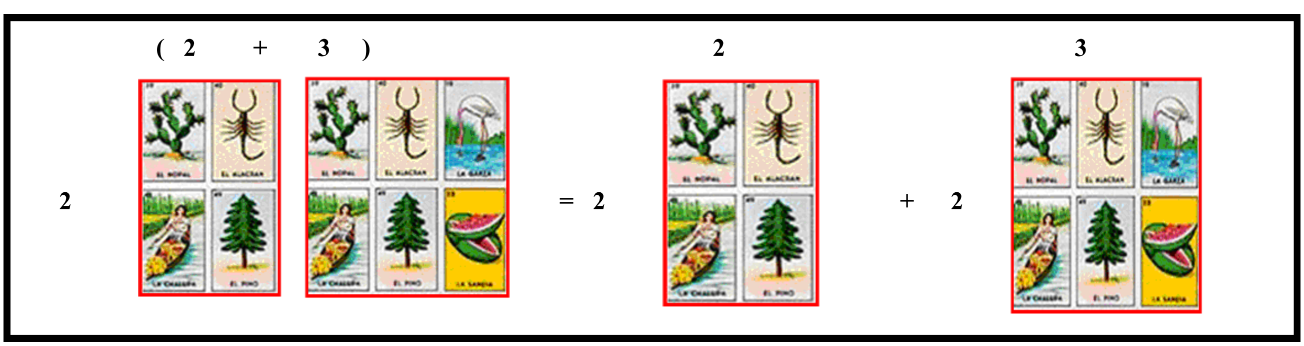

Figure 3. The distributive property of multiplication over addition using “la lotería” cards.

Table 5
"Simple Cases" Strategy for Finding Number of Boards

Given a Lotería board with *n* positions	Number of Possible "La Lotería" Boards Using 54 Cards
1	54
2	54×53
3	$54 \times 53 \times 52$
4	$54 \times 53 \times 52 \times 51$
…	…
16	$_{54}P_{16} = \frac{54!}{(54-16)!} = \frac{54!}{38!} = \frac{54 \times 53 \times 52 \times 51 \times \cdots \times 3 \times 2 \times 1}{38 \times 37 \times 36 \times \cdots \times 3 \times 2 \times 1}$ $_{54}P_{16} = 54 \times 53 \times 52 \times \cdots \times 39$

that involved something they recognized and that was part of their heritage. In fact, they noted that parents made comments such as "Wow! I didn't know I could help my children with math using these games we have at home." These PSTs also learned that to engage family members of various age groups, they needed to adapt their activities for multiple mathematical levels. Furthermore, they were better able to comprehend the value of using appropriate mathematical vocabulary to make the topics personally meaningful and to avoid teaching mathematical concepts with non-contextualized approaches that use rote memorization with irrelevant content.

The implementation of culturally relevant mathematics activities suggest that participation by pre-service mathematics teachers in FMLEs can be a powerful facilitator of learning for all involved (McCollough & Ramirez, 2010; McCollough, Ramirez, & Canales, 2009). Those who participated in FMLEs had the rich opportunity to observe mathematics learning in progress, and perhaps experience their only opportunity as a PST to work with Hispanic families. Often, a PST expressed surprise at the students' and parents' high ability to learn and solve problems, beyond what the PST had expected for their number of years of formal education. This suggests that informal educational settings such as FMLEs have the potential of informing educators that ability is not limited to those with formal educations. Since we have observed that PSTs in our local context have little or no knowledge of student and parent background knowledge, FMLEs may serve an important purpose in providing experience in working with students and parents of different cultures, changing prior misconceptions and creating positive perceptions of those cultures (McCollough & Ramirez, 2012b).

Future mathematics teachers report that they learn to see mathematics in relevant and engaging situations, to integrate mathematics with authentic and fun activities, to make mathematics culturally relevant, and to enjoy teaching mathematics. Because these PSTs are provided an authentic environment within their community to foster connections between theory and practice, they increase their confidence in teaching science and mathematics (McCollough & Ramirez, 2010, 2012a, 2012b). These pre-service teachers become familiar with and change their thinking about diverse student groups before starting their semester of student teaching. Most importantly, the teaching experiences using culturally relevant math activities such as the one highlighted in this paper include opportunities for self-reflection in examining perceptions regarding teaching (in family settings) students from minority ethnic groups and different socio-economic groups. By implementing the Table 1 objectives, PSTs realized that their knowledge of math was paramount, that pedagogical skills improved with practice, and that "all students can learn" was not just a slogan. Indeed, these types of rich culturally responsive FMLE opportunities may help shape future classroom experiences and lead to the success of all students (especially Hispanics) in mathematics.

Notably, we encountered numerous challenges as we incorporated culturally responsive mathematics lessons in our work, but none were insurmountable. Teaching PSTs how to incorporate culturally relevant mathematics in FMLEs reminded us of Cochran-Smith (2004) who states that teachers must "teach against the grain" both within and around the culture of teaching at their particular schools. This applies to accepting the challenge of teaching culturally responsive lessons even if the school or district is charged with using a prescribed lockstep curriculum with little room or precedent for tailoring lessons to students' culture. Further, they must depend on the strength and convictions of their beliefs that their work ultimately makes a difference in the arena of social responsibility. Paris (2012) writes that multiethnic and multilingual students must be supported with culturally sustaining pedagogies, fostering and sustaining "linguistic, literate and cultural pluralism as part of the democratic process of schooling" (p. 95). Without these efforts, students will continue to lose their heritage and community practices in order to achieve in U.S. schools (Paris). For this reason, the authors continue to implement culturally responsive mathematics/science in their PST curriculum. Each author also conducts Family Math/Science Learning Events within her respective preservice teacher preparation program that serves a large Hispanic community.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Why are activities such as FMLEs important teacher preparation activities? Read, summarize, then discuss at least two articles from Edge (2000).

2. Find a culturally relevant topic in Hispanic culture and discuss the mathematics that can be associated with the activity. An example is Lesser (2010).

3. Do some research and create a culturally relevant math activity lesson for a target audience of students in elementary or middle school whose culture is different than your own. Use the objectives and descriptors in Table 1 as a guide. To what extent are the objective descriptors found in Table 1 applicable to the activity? If weak, modify the activity so that it can utilize the principles inherent in each of the objective descriptors.

   Note: For interesting activities, you can use the *Family Math* book (Stenmark, Thompson & Cossey, 1986) or you can refer to http://lawrencehallofscience.stores.yahoo.net/familymath.html.

4. Study a math game that can be useful in planning a math lesson for a FMLE target audience. Create a free-standing tri-fold poster (47.5" wide by 36" high) for the math game and present the activity to a target audience. You can refer to the website http://www.funmathsgames123.com/.

5. From your experiences with items 1-4 above, do you have suggestions that can help teachers enhance mathematics learning for children of cultures other than your own? How do your suggestions align with the goals and mission of TODOS?

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