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# Teaching for Excellence and Equity In Mathematics

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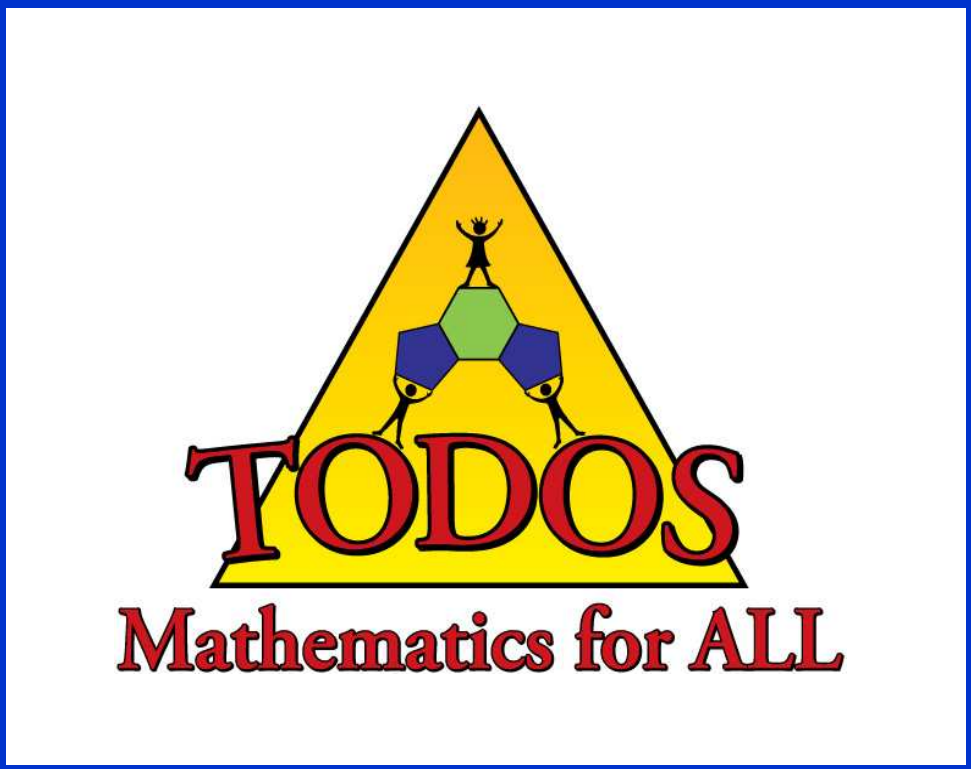



an Affiliate Organization of the

National Council of Teachers of Mathematics

# From the Editors

*¡Bienvenidos!* It is an exciting privilege to welcome everyone to the debut issue of this refereed TODOS journal: *Teaching for Excellence and Equity in Mathematics*.

Let us introduce our *TEEM* team of editors. **Cynthia Anhalt** has been an editor and contributor for *Noticias de TODOS* and is currently on the Instructional Faculty in Mathematics Education at The University of Arizona in Tucson, AZ. **Larry Lesser** has also been an editor and contributor for *Noticias de TODOS* and is an Associate Professor of Mathematics Education at The University of Texas at El Paso. **Miriam Leiva** is the founding president of TODOS, is Distinguished Professor of Mathematics Emerita at the University of North Carolina Charlotte, and is an author of Houghton Mifflin Harcourt Mathematics. Collectively, the editors have served on several national editorial or research boards, have published on equity/ELL issues in mathematics and statistics education, and have precollege teaching experience.

Since its launch in Spring 2005, the semiannual periodical *Noticias de TODOS* has served as much more than TODOS' newsletter by also including quality peer-reviewed and invited articles on pedagogical activities, curriculum, and issues on topics of interest. This past winter, the Board of TODOS voted to create *TEEM* as a separate (for now, annual) publication to focus on teacher-oriented articles to incorporate ideas for excellence and equity into teaching practices.

As stated in the flyer released at the 2009 NCTM annual meeting, *TEEM*'s intended audience includes math educators, practitioners, leaders, and administrators at all levels. The journal aims especially to engage mathematics education topics involving excellence and equity simultaneously (rather than either in isolation) in a way that connects research to classroom practice and can inform the practice of teachers or professional developers. One feature that helps ensure this accessibility and practicality is the "DARE" (Discussion And Reflection Enhancement) questions that bookend each paper. Information for prospective authors (and referees) may be found at **http://www.math.utep.edu/Faculty/lesser/TEEM.html** or by contacting us at **teem@todos-math.org.**

This inaugural issue is a bit unusual in that it consists of articles that were competitively selected by us (with input from key TODOS officers and former *Noticias* editors) as the most "*TEEM*-like" peer-reviewed papers from past issues of *Noticias de TODOS*, and then enhanced by the addition of a concise abstract, the thoughtful DARE questions, and some editorial processing. Future *TEEM* issues will generally consist of new papers submitted during the annual submission months (November and April) using guidelines at the website mentioned above.

We hope *TEEM* not only serves the current members of TODOS, but also brings in new members. We also hope *TEEM* serves as an inspiring pedagogical and scholarly resource for the broader mathematics education and education communities.

Enjoy the debut issue!

*The Editors*

*The mission of TODOS: Mathematics for ALL is to advocate for an equitable and high quality mathematics education for all students—in particular, Hispanic/Latino students— by increasing the equity awareness of educators and their ability to foster students' proficiency in rigorous and coherent mathematics.*

## ARTICLES IN THIS ISSUE

# Framing Equity: Helping Students "Play the Game" and "Change the Game"

Rochelle Gutiérrez

**Abstract**

This article introduces a framework for equity that entails the dimensions of Access, Achievement, Identity, and Power. Beyond knowledge and skills, teachers need an "equity stance" that embraces and works to balance the tensions between these four dimensions.

**Discussion And Reflection Enhancement (DARE) Pre-Reading questions:**

1. How do people in your working context define equity and what words do they use to discuss it?

2. How do *you* define equity and how do you know you are addressing it in your everyday practice?

"DARE" Post-Reading questions appear at the end of the article. This article (without DARE questions) originally appeared in Spring 2008 *Noticias de TODOS.*

**Rochelle Gutiérrez** (rg1@illinois.edu) is an Associate Professor at the University of Illinois at Urbana-Champaign, and her research focuses on equity in mathematics education, race/class/language issues in teaching and learning, effective teacher communities, and social justice.

# Framing Equity: Helping Students "Play the Game" and "Change the Game"

Rochelle Gutiérrez

"Equity" is a hot topic in mathematics education these days. However, for many people, addressing equity issues rarely moves beyond the goal of closing the achievement gap (Gutiérrez, 2008). For me, equity is ultimately about the distribution of power--power in the classroom, power in future schooling, power in one's everyday life, and power in a global society (Gutiérrez, 2002). I draw on the idea that equity must be framed with both dominant and critical definitions. In working with teachers, I have found it useful to explicate four key dimensions (Access, Achievement, Identity, and Power) and to highlight the relationships and tensions between them. Let me explain.

*Access* relates to the resources that students have available to them to participate in mathematics, including such things as: quality mathematics teachers, adequate technology and supplies in the classroom, a rigorous curriculum, a classroom environment that invites participation, and infrastructure for learning outside of class hours. The Access dimension reflects the idea that students are affected by their "opportunity to learn." However, a focus on access is a necessary but insufficient approach to equity, in part because it fails to redress past injustices. Besides giving students necessary resources, we also care about student outcomes, or what I categorize as *Achievement*. This dimension is measured by tangible results for students at all levels of mathematics, including such things as participation in a given class, course taking patterns, standardized test scores, and participation in the math pipeline (e.g., majoring in mathematics in college, having a math-based career). Moving from mere access to achievement is important when considering that there are serious economic and social consequences for not having enough math credits to graduate from high school, not scoring high enough on a standardized achievement test to gain acceptance to college, or not being able to major in a math-based field that can confer a higher salary and prestige in society.

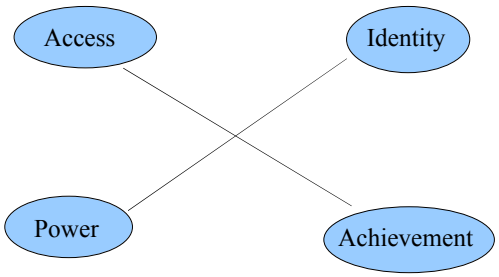
However, because many students find themselves down playing some of their personal, cultural, or linguistic capacities in order to participate in the classroom or the math pipeline and because some groups of students historically have experienced greater discrimination in schools, issues of *Identity* are also important to consider. For many mathematics educators, attending to students' identities means focusing on students' pasts (e.g., including the contributions of their ancestors). But, the identity dimension also concerns itself with a balance between self and others in a global society and acknowledges ways students are racialized (Martin, 2007), gendered and classed (Walkerdine, 1988). It includes whether students have opportunities to draw upon their cultural and linguistic resources (e.g., other languages and dialects, algorithms from other countries, different frames of reference) when doing mathematics, paying attention to the contexts of schooling and to whose perspectives and practices are "socially valorized" (Abreu & Cline, 2007; Civil, 2006). The goal is not to replace traditional mathematics with a predefined "culturally relevant mathematics" in an essentialistic way, but rather to strike a balance between opportunities to reflect on oneself and others as part of the mathematics learning experience.

The Power dimension takes up issues of social transformation at many levels. This dimension could be measured in voice in the classroom (e.g., who gets to talk, who decides the curriculum) (Morales, 2007; Zevenbergen, 2000; Adler, 1998), opportunities for students to use math as an analytic tool to critique society (e.g., exploring "risk" in society) (Mukhophadyay & Greer, 2001; Skovsmose & Valero, 2001; Gutstein, 2006), alternative notions of knowledge (D'Ambrosio, 2006), and rethinking the field of mathematics as a more humanistic enterprise (Gutiérrez, 2002).

Access and Achievement can be thought of as comprising the dominant axis, preparing students to participate economically in society and privileging a status quo. The dominant axis, where access is a precursor to achievement, measures how well students can play the game called mathematics. Identity and Power make up the critical axis. The critical axis, where identity can be seen as a precursor to power, ensures that students' frames of reference and resources are acknowledged in ways that help build critical citizens so that they may change the game. All four dimensions are necessary if we are to have true equity. Learning dominant mathematics may be necessary for students to be able to critically analyze the world, while being able to critically analyze the world may provide entrance into dominant mathematics. It is not enough to learn how to play the game; students must also be able to change the game. As educators, we need to be clear on our stance--that we are advocates for our students to do both. Doing so requires situating ourselves in the tensions that exist in this work (Gutiérrez, 2009).

This equity diagram seeks not to simplify the complexity but rather to offer a useful "mapping space" for ideas when trying to reflect on one's practice. As a researcher, it is useful for me to see the kinds of approaches that teachers and families take to address equity. Take, for example, the issue of "power." While teachers in interviews may say they "want to empower students," they almost always mean it only as it relates to achievement, not with respect to helping students reach personal goals of excellence that may intersect with the doing of mathematics (e.g., helping their communities solve a local problem).

Dimensions of Equity




I am not implying that at the heart of all teachers' equity agendas is Access and/or Achievement while it is Identity and/or Power for most marginalized students and their families. Many educators already embrace the idea that students need to see themselves reflected in the curriculum and be offered opportunities to develop further agency in the world. My experience in working in urban communities is that some marginalized families do not want their students to develop "agency" in the ways that critical researchers seem to think is important, as they worry that it will take away from schools giving their students the tools to excel in school, or they feel they are already doing this "critical" work with their children at home.

As a researcher dedicated to equity, I attempt to situate myself in "Nepantla," the crossroads of these tensions, to highlight the phenomena at hand. Being able to name the dimensions helps us move toward highlighting tensions between the dimensions so that we might be more reflective about how we can successfully balance attending to them all.

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# Discussion And Reflection Enhancement (DARE)
# Post-Reading Questions

1. What do you think it means to help students "play the game"? What do you think it means to help them "change the game"? In what way(s) do you believe you work to help students "change the game"? Why do we need to change the game? Is "the game" referring to mathematics or mathematics education or both?

2. Which of the four dimensions are addressed by an achievement gap focus? What is missing? Why is that important?

3. What are some examples of "past injustices" that an opportunity-to-learn view misses?

4. In what way(s) does Gutiérrez' concept of equity overlap with or depart from the way in which equity is articulated in the National Council of Teachers of Mathematics' position statement (accessed at http://www.nctm.org/about/content.aspx?id=13490)?

5. The four equity dimensions are written from the point of view of students and learning. Do these dimensions also apply to teachers and teaching? In other words, do teachers, administrators, and teacher educators need to be thinking about access, achievement, identity, and power with respect to teaching? If so, what might this involve?

6. Gutiérrez claims, "As educators, we need to be clear on our stance--that we are advocates for our students to do both. Doing so requires situating ourselves in the tensions that exist in this work." What might it mean to situate oneself in the tensions that exist in this work?

7. Try this: The next time you teach, *make note of the ways in which you are attempting positively to address dimensions of identity and/or power in your classroom*. Would students agree with your list? What would it take to address more strategically these dimensions? Are there individuals in your working context that you feel are already doing this better with whom you can become an ally?

8. Read the Gutiérrez (2009) paper listed in the References. Come up with a tension you have experienced that is not highlighted in the article. Why is this tension important in an equity stance? In what ways do you embrace this tension and in what ways do you reject it?

"DARE to Reach ALL Students!"

# A Reflection on my Work with Latino Parents and Mathematics

Marta Civil

**Abstract**

This article describes research from different parental engagement projects in mathematics. Through Latino/a parents' voices, we learn about their beliefs and values about mathematics education and these findings can inform those who work with students or parents.

## Discussion And Reflection Enhancement (DARE) Pre-Reading questions:

1. What experiences have you had or do you know of that involve families and mathematics teaching/learning?

2. Based on your prior experiences and/or readings, what are some resources (specific to mathematics learning) that Latino/a parents (and their children) bring to school?

3. What are some challenges that you think Latino/a parents (and their children) face in terms of the teaching and learning of school mathematics?

4. What is your definition of "parental involvement" for this context?

"DARE" Post-Reading questions appear at the end of the article. This article (without DARE questions) originally appeared in Spring 2007 *Noticias de TODOS*.

**Marta Civil** (civil@math.arizona.edu) is a professor in Mathematics at the University of Arizona. Her research in mathematics education spreads over two areas: teacher education and equity in mathematics education— in particular, on a sociocultural approach to the mathematics education of ethnic and language minority school age students and adults.

# A Reflection on my Work with Latino Parents and Mathematics

Marta Civil

I have been working on issues related to Latino parents' engagement in mathematics education for almost fifteen years. Research with parents is indeed one of the key areas of research in our NSF-funded center called CEMELA (http://cemela.math.arizona.edu; Center for the mathematics education of Latinos/as). Currently our main research study in this area is looking at Latino parents' perceptions of the teaching and learning of mathematics. In this reflection, however, I focus on some of the highlights of the work I did prior to CEMELA as a way to explain the background for our current work.

One of the original motivations for my current work was and continues to be the idea of mathematics teaching innovations that could help us bridge the gap between in-school and out-of-school learning. Our work takes places in Mexican / Mexican-American, working class communities in the Southwestern United States. Our efforts have been geared towards the development of learning environments that build on the students' and their families' knowledge and skills. But, how do we uncover that knowledge and those skills that all families have? Through the Funds of Knowledge for Teaching project and then later during the Bridge project, the teachers (sometimes accompanied by university researchers) visited the homes of some of their students. Using in-depth questionnaires (on family history, labor history, perspectives on education, uses of mathematics at home and at the work place), these teachers were able to uncover some of this knowledge and those skills/experiences that reside within the households. Our challenge then was to use this knowledge in learning modules that would make mathematics more meaningful (both from a cognitive and from an affective point of view) for the school children. These household visits certainly had an impact on the teachers in that they saw the families and "the home as a real learning place, real learning environment, you know I didn't think it was so much a learning environment as it is" (teacher's interview).

But to me the most rewarding and eye-opening experience was another activity we developed towards this effort to bridge the gap between in-school and out-of-school, namely, our mathematics workshops for parents. Through these workshops, we do not only learn about the parents' ideas and perceptions about the uses of mathematics and about their children's mathematical education, but we engage in joint explorations that allow us to establish a two-way conversation grounded on the learning and teaching of mathematics. It is important to note that the success of this approach is in its continuity: these are not isolated workshops, but rather series of sessions with the same group of parents: sometimes we call them "math for parents courses", but our preferred term lately is "tertulias matemáticas" (mathematical circles). We (parents, university personnel, and sometimes teachers) come together to do and talk about mathematics. Our workshops are modeled after the work we have been doing for years in professional development for teachers. They are highly participatory, hands-on, and centered on what we view as meaningful school mathematics tasks.

What have we found out from these "tertulias matemáticas"?

*Parents engaged in mathematics*

**Parents like being learners of mathematics**

Although most of the parents originally come because they want to help their children, they soon become interested in the content as learners themselves. The parents in our projects have made it very clear that academic mathematics is important to them. They want to learn this type of mathematics to help their children, but also for themselves, as this excerpt from a mother's reflection captures quite well:

*I am so happy with all these mathematics workshops because I realize how to help my children understand mathematics in a different way, from a fun approach, all together as a family. … And also for us, because one never knows when we may use it, and this way we move forward, and no one is going to mandate that it has to be the way they say, because we also think and solve problems.*

**Parents value teaching for understanding**

Parents enjoy finding out the "why" behind the many things they had memorized as children in school.

*[This project] has been very different from my previous experience (with math). I went through my whole life being told how things were not and not given any freedom to figure it out on my own. Being able to experiment with blocks or whatever is much more interesting.*

*I'm amazed because [I see] something that I didn't see before, and it clicks in my mind and I understand why things are the way they are. I get excited because now I know, I'm not accepting it, now I know why that is the way it is.*

We argue that if parents learn mathematics with an emphasis on understanding rather than rote memorization, they are more likely to become quite vocal about the importance of understanding for their children's mathematics education. As one mother very eloquently said, "I don't want them [teachers] to teach to the test. You have to be versatile in many things. If you don't understand, what's the point?"

**Parents bring their own beliefs and values**

Like everybody else, parents often have deep–rooted beliefs about the teaching and learning of mathematics. A clear example of this is with the algorithms for division. In all the workshops where we have parents who learned how to divide in México, this topic comes up. As they compare the methods traditionally taught in México and in the U.S., comments along the lines of "their" method being more efficient (because they write less as they do the subtraction in their heads) always come up. These differences in approaches are fantastic opportunities to engage in not only the mathematics behind the different methods but also in a conversation about issues related to the teaching and learning of mathematics. Another salient topic among immigrant parents is the

differences between the educational systems:

*No, I'm not happy [with the system at her son's school]. I feel that there is repetition of a lot of things; I don't understand why the teaching is so slow, I don't like it, I don't like the system, I don't like it at all. When we go to México, my nieces and nephews or my husband's nieces and nephews, they are children that are more or less the same age as Jaime and I see that Jaime is behind. Here they tell me that Jaime is really excellent.*

To me, this is not about discussing which system is "better." But we have to be aware that parents are going to bring up these comparisons and that these perceived differences may lead to conflict between parents and teachers. We all bring our valorization of knowledge. But these differences may also lead to conflict between parents and children:

*Last night my son said to me that school from México was not valued the same as school here, that is, it doesn't count. What I studied there doesn't count here. He knows that what is taught here is different from what is taught there and so he says, 'why would I ask my Mom for help if she's not going to know.' So, there is a barrier.*

Parents have mentioned to us that when they try to help their children with the mathematics homework, they often run into two obstacles—the language (English / Spanish) and the mathematical approach. Several of these parents have mentioned how they know the content but they do not know it the way their children are learning it and then they (or sometimes their children) feel that they cannot help them:

*He [her son] doesn't feel very sure that I am understanding him because the problem is written in English. I don't know how to read it and he doesn't…know how to translate well for me because he speaks Spanish and reads Spanish, but we say different things for the same words and questions, I think he thinks I studied differently.*

### Parents value "confianza" (trust)

As I mentioned earlier, this kind of work requires time. It is not about isolated workshops but about establishing rapport and connections with the families. I want to end stressing the importance of the concept of "confianza" when working with parents.

*When I joined this group, for me the most important foundation was the confianza that each one offered me…. I can say that all that I now know and have learned has been accomplished by means of the confianza (a mother reflecting on her experience with the math workshops).*

*At last, I also have someone that more than a teacher is a friend and most importantly inspires me: Confianza, the confianza that I in particular never had with any other teacher of mathematics. …Thanks to the confianza that exists in the group we can work without problems and pose any sort of question without fear.*

This concept of "confianza" is not only important for the parents in the workshops. The final excerpt below is from a fifteen-year old reflecting on the impact that these workshops had on his mother:

*Now that she [his mother] is attending these workshops she is learning in a different way, understanding the why of the formulas and where they come from and how they can be applied in her life; she shares it with the entire family and we all get involved in a mathematical gathering that is*

*fun. We are all teachers and students at the same time, there is no difference and that there be much respect and confianza is most important.*

Acknowledgement: The work reported here was supported by the Educational Research and Development Centers Program, PR/Award Number R306A60001, as administered by the OERI (U.S. Department of Education) and by the National Science Foundation (NSF) under grants ESI-99-01275 and ESI-0424983. The views expressed here are those of the author and do not necessarily reflect the views of the funding agencies.

## Discussion And Reflection Enhancement (DARE)
## Post-Reading Questions

1. What are some experiences you could develop to engage with parents in a two-way dialogue?

2. The article's author believes in encouraging the sharing of different approaches to doing mathematics such as multiple algorithms for the basic arithmetic operations. What experiences have you had (e.g., with your students) in sharing different approaches? What are some benefits and challenges involved with this?

3. What strategies do you (or could you) use to help overcome "the language barrier" that parents who do not feel comfortable with English may encounter when it comes to helping their children with homework or other school-related issues?

4. What are some implications of the concept of "valorization of knowledge" for the teaching and learning of mathematics?

5. What strategies could you use to learn more about the knowledge and experiences that your students and their families have? What implications would this additional information have for your teaching of mathematics?

"DARE to Reach ALL Students!"

# Advancing Equity and High Quality Mathematics Education with Actions Drawn from Ethnomathematics

**Fredrick L. "Rick" Silverman, Gary H. Fertig, Jennifer Harding-DeKam, & Susan Conklin Thompson**

**Abstract**

This article advocates for the (not yet common) goal of mathematics education to advance peace, harmony, and respect among people and, consequently, to reduce discord. This outcome can be accomplished by utilizing Ethnomathematics, and we offer suggestions for cultivating an Ethnomathematics orientation and employing associated classroom practices.

**Discussion And Reflection Enhancement (DARE) Pre-Reading questions:**

1. Where have you heard or read the term "ethnomathematics" before reading this article? If so, what does ethnomathematics mean to you and what thoughts do you have about how it connects to the diversity of the students you teach?

2. *Principles and Standards for School Mathematics* (NCTM, 2000) elaborate an Equity Principle: http://standards.nctm.org/document/chapter2/equity.htm. What are some inferences you can make for what equity means in the context of mathematics education?

3. Several of the words in the title of this article might have stirred your thinking about social studies, most likely, "equity" and "ethnomathematics." In that vein, how might mathematics and social studies education be integrated in ways that children might come to value both disciplines as complementary means for uncovering social and cultural inequity and proposing possible solutions to achieve social justice?

"DARE" Post-Reading questions appear at the end of the article. This article (without DARE questions) originally appeared in Fall 2007 *Noticias de TODOS*.

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# Advancing Equity and High Quality Mathematics Education with Actions Drawn from Ethnomathematics

Fredrick L. "Rick" Silverman, Gary H. Fertig,
Jennifer Harding-DeKam, & Susan Conklin Thompson

Let's implement experiences in mathematics education that advance the aims of creating more peace, harmony, and respect in our world! Striving to achieve that goal has a place in the classroom. That's the perspective that Ubi D'Ambrosio (2006a, 2006b) has expressed as a critical element of ethnomathematics, an element that can be a foundation for a mathematics education framework that promotes high quality mathematics education for all students, thus advancing the increasingly urgent need for equity in teaching and learning mathematics. Let there be no more students whom our mathematics education teaching, programs, curricula, or leaders, wittingly or unwittingly, marginalize! D'Ambrosio, one of the founders and international leaders of ethnomathematics, urges that mathematics educators organize and lead students for constructing mathematical knowledge based on harmony among individuals, society, and nature.

Harmony in this vein means enacting mathematics education that is more inclusive of all students than is common practice. Hence, this essay advocates for access to equitable and high quality mathematics education for all children, and especially for those whose home language is not the language of the school, for children of poverty, for minority children, for children whose families are mobile, for children who have special needs, and for all other children whom for one reason or another the school experience often marginalizes. And let us not forget that school mathematics experiences for girls and young women have frequently been seriously lacking. Too often, mathematics education has been an elitist discipline and has had the effect of advantaging some students while disadvantaging others by the exclusivity of the processes, unintentionally or intentionally, by which they encounter mathematics during formal school lessons in the discipline, lessons that have been short on developmental, cultural, or social appropriateness.

Mathematics education can be a contributing experience to the cultivation and nurturing of democratic citizenship and democratic dispositions that value diversity that is a hallmark of a pluralistic society that seeks high quality education for all. Such an outcome of mathematics education would certainly lead to a more harmonious society, perhaps even to a more harmonious world. If a teacher promotes such positive societal outcomes, then using open ended problems, seeking and sharing explanations for solutions, appreciating explanations of others, valuing other's points of view, and raising conversations in connection with learning and applying mathematics are means to such a socially beneficial outcome (Simmt, 2001).

On the other hand, such approaches as the following are likely to hinder the cultivation and nurturing of good citizenship, democratic dispositions, and inclusiveness: teaching mathematics as a set of facts, skills, and

procedures; teaching mathematics as a fact set that has no utility in the real world - little purpose except for practicing routines for textbook exercises, readying for tests, and preparing for the next year's mathematics course; and teaching mathematics as a discipline in which outcomes are either right or wrong, the adjudication of outcomes resting solely with the teacher as authority (Simmt, 2001). Six categories that A. J. Bishop (1988) formulated are an ethnomathematical context, a set of behaviors, in this case, that are inherently encounters with mathematics that children and others have in their lives. Ethnomathematics impels teachers to be cognizant of the styles and techniques that people, including children, use to make sense of the cultural, social, linguistic, and natural environment in which they live. It also evokes learners' interests in the styles and techniques by which others than themselves come to know the world and enact their lives, as when children discover that a game they play and enjoy, perhaps the top-spinning game Toma-Todo from México, is very similar to a game played by children of another culture, as is true with Dreidel, a top-spinning game in Jewish tradition (Zaslavsky, 1998).

Bishop's (1988) six categories in which mathematics arises naturally in people's lives across cultures, societies, and the linguistic landscape are these: Counting, Measuring, Locating, Designing and Building, Playing, and Explaining. Let's look through the lenses of Bishop's categories at a few examples of encounters with mathematics that arise subtly and unmistakably in the lives of many children:

- Biking to and from school and describing the route, which aligns with *locating* and *explaining*
- Playing Rayuela, a hopscotch game from Colombia, aligns with *playing and locating*
- Making miniature furniture with toothpicks and spice drops for *Grandmother's Adobe Dollhouse* (Smith, 1984) aligns with *designing and building*
- Buying or trading one sports card for another aligns with *explaining*
- Determining the rate at which water is flowing in the street gutter after a rain aligns with *measuring* and *explaining*

The above examples from children's lives are both mathematical and social behaviors. Teachers who seek, represent, and share examples of mathematics in their own lives are practiced sufficiently to be able skillfully to scaffold their students to do the same. In that way, children learn that mathematics is a discipline of this world, and it is an active, or perhaps more quiet, presence in nearly everything they do. Mathematics lives in social contexts of children and adults, no matter their linguistic, cultural, religious, ethnic, racial, or other personal characteristic, heritage, or way of life.

Children sharing their mathematical encounters with one another engenders respect and appreciation of diversity. Teachers who make use of their knowledge of ways children encounter mathematics are in better position to scaffold the link between more formal mathematics learning experiences of the school and the more informal ways in which children experience mathematics outside the classroom. The answer to the question "When are we ever going to use this stuff?" is "Everybody uses it every day already." And then one day after the rain has stopped, the teacher can say, "Let's hurry outside and see how fast the water in the gutter is flowing." Now that's an authentic, substantive, equitable mathematics, and ethnomathematics, experience that might interest just about every child in the class!

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## Discussion And Reflection Enhancement (DARE)
## Post-Reading Questions

1. What role does mathematics play in socializing students in schools and in the world outside school? The article indicates that traditional teaching of mathematics is often not responsive to selected groups of students such as those whose home language is not the language of the school. Do you agree or disagree with that claim? Why?

2. What are examples of how the teaching of mathematics might contribute to the development and nurturing of democratic dispositions or to might inhibit those dispositions? (By the way, examples of democratic dispositions can be found in the URL for Simmt (2001) in the References above.)

3. How can you use ethnomathematics to help your students perceive, form, maintain, and use connections between everyday informal mathematics students learn outside of school and the more formal mathematics in school? What ethnomathematics do you find in children's play?

"DARE to Reach ALL Students!"

# Communicating Mathematically: English Language Learners in the Mathematics Classroom

Debra Coggins, Drew Kravin, Grace Dávila Coates,
& Maria Dreux Carroll

**Abstract**

This article explores the essential role of communication and language in learning mathematics. Implications for English language learners taught primarily by English-speaking teachers are highlighted. In this paper (and their related book), the authors advocate regular use of pedagogical strategies such as "help English learners talk-to-learn during mathematics lessons" and "provide mathematical and organizational representations."

**Discussion And Reflection Enhancement (DARE) Pre-Reading questions:**

1. Think of when you were communicating in a foreign language to get directions to a place or to learn something unfamiliar. What was confusing and what helped you understand?

2. Think of a time when you learned specialized vocabulary associated with a new sport or hobby. What did you do to be able to develop meaning for, and to recall the new words? Did you learn the concept or the word first?

3. What do you believe are necessary components/attributes of effective mathematics lessons that are taught in English to students who are still in the process of learning English?

4. Have you purposefully added a new instructional technique to your practice to the point where it became routine? What process worked for you, from the time that you decided to expand your practice to the time when you could almost automatically use the technique? What types of support were helpful?

"DARE" Post-Reading questions appear at the end of the article. This article (without DARE questions) originally appeared in Fall 2007 *Noticias de TODOS.*

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# Communicating Mathematically: English Language Learners in the Mathematics Classroom

**Debra Coggins, Drew Kravin, Grace Dávila Coates, & Maria Dreux Carroll**

"The ability to read, write, listen, think, and communicate about problems will develop and deepen students' understanding of mathematics."

— Communication Standard, NCTM (2000)

One essential aspect of *participation* in a high-level mathematics lesson is written and oral communication, including requirements for students to explain their mathematical thinking. The Communication Standard from NCTM (2000) states that instructional programs from PreK-12 should enable students to organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze the mathematical thinking and strategies of others; and use the language of mathematics to analyze and express mathematical ideas precisely.

These recommendations have profound implications for the pedagogical strategies used by teachers of English learners (ELs) as well as for the achievement of all their students. These multiple abilities and higher expectations of mathematics learning present a particular difficulty for second language learners who are taught by traditional methods that depend on considerable teacher talk and relatively passive student involvement. *It is evident that use of language is essential for mathematics learning, and mathematical activities provide opportunities to extend language skills.*

The authors [of this paper and of Coggins et al., 2007] aimed to address the critical issue of the level of mathematics learning by the growing numbers of ELs in American classrooms. Based on experience as professional development designers and providers, as college instructors, as researchers and writers, and as classroom teachers, we knew that the responsibility to seek and implement strategies for ensuring each student's development of communication skills and mathematical ideas ultimately falls upon the classroom teacher.

A major purpose of the book was to provide teachers a context for focused conversations about expectations and possibilities for all learners. The accessible writing style and examples used, along with specific information and guidance related to several practical teaching strategies and a lesson map, are intended to spark reflections on current teaching practices. The hope is to inspire teachers purposefully to adopt effective strategies for teaching mathematics. *We believe that the recommended strategies are more than just good teaching practices — they are essential to the education of ELs.* We envision the use of our book in preservice courses, in collaborative inquiry teams, and as a vehicle for school-wide or grade level implementation of effective strategies for teaching ELs.

Each chapter includes several components: mathematics teaching examples, each focused on a specific mathematics problem; a discussion of the use of the chapter's focus strategy in the teaching example; an overview of the research or theoretical basis for each chapter topic; specific teaching tips; further discussion of the focus strategy, and practice/discussion questions.

An extensive lesson-planning map is provided not as a template, but tool for teachers to think about the myriad of planning aspects that are part of an excellent lesson.

Samples of the chapter components are included in relation to the seven chapters listed below. For the sake of coherence, examples related to communication and language have been selected.

**Developing Conversational Language: Help ELs Talk-to-Learn During Mathematics Lessons** – Teaching Tips –Tips include suggestions to include brief Think-Pair-Share sessions on focused discussion topics and suggestions of sentence starters that can expand ELs' participation in discussions.

**Developing Academic Language: Develop Mathematics Concepts and Vocabulary for English Learners** – Discussion of the lesson vignette–The discussion of this lesson on comparative relationships emphasizes the need to provide a positive environment replete with opportunities to use academic language: The questions, verbal and physical models, diagrams, "talk to your neighbor" directions, and partner games all lead to an increased likelihood that students will have frequent meaningful encounters with the mathematics vocabulary.

**Scaffolding: Give Support for Both Mathematics and Language Learning** – Theoretical basis for the strategy – "Another [scaffolding] technique, is to provide challenging tasks, with collaborative support, including considerable social interaction" (National Research Council, 2001).

**The Role of Concrete Materials – Utilize Objects to Develop Mathematical Understanding for English Learners** – Research. While tactile and visual learning are significant meaning-centered components of instruction, benefits include not only increased access to ideas, but also multiple ways of thinking and communicating.

**Visual Learning: Provide Mathematical and Organizational Representations as a Regular Component of Instruction Focused** – Discussion of the Topic– Focus questions, such as "What are graphic organizers, advance organizers, and diagrams, and why are they important?" are included. Each visual tool is defined, discussed, and examples are given. For example, "Graphic organizers are visual structures that make it possible to organize words, ideas, information and so on to further learning goals such as understanding, communicating, and remembering."

**Questioning Strategies: Ask Questions to Foster Students' Learning of Mathematics and English** – Mathematics teaching example – The lesson vignette about Snail Races, a probability game, includes many questions from the teacher, such as "Help me label our bar graph. How can we use this chart to find out which snail won the most in our class?…How did I choose the numbers [to use to label this tally chart]? How should we use this chart?" The questions are also identified as to type of question.

**Comprehensible Input: Combine Many Strategies to Develop Mathematics Concepts Through Clear and Effective Instruction** – Practice and Discussion Questions - Question 1. Think about teaching a small group of intermediate-level English learners the concept of perimeter. What would you say and how would you create access to your explanation? What would the students see? How would you assess students' learning?

The book aims to serve as a resource to those who work to further the goals of mathematics for ALL. It is intended to show the high level of mathematical learning and increased use of language that result when specific and purposeful planning occurs in designing mathematics and ELD (English language development) lessons. These strategies and components are designed to promote communication, mathematical understanding, increased skill development, and confidence for ELs.

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## Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Do you believe that communicating about mathematical problems is essential to the development of a deep understanding of mathematics? Why?

2. What is necessary in order to communicate about a mathematical idea with others?

3. In the preface of the Coggins et al. (2007) book (http://www.corwinpress.com/upm-data/19571 Preface.pdf), the authors contend "if we teach mathematics by following commonly accepted 'best practices,' we may actually overlook English learners, because they have very specific needs." Identify and discuss the evidence for some of those specific needs.

4. Name a strategy or topic listed in this *TEEM* article that you think warrants further consideration for teachers. Examples of strategies are: Help ELLs Talk-to-Learn during mathematics lessons; utilize objects; provide mathematical and organizational representations; and ask questions. Outline what you will do to use your identified strategy purposefully in an instructional context in the next few weeks. Arrange to discuss your progress with the strategy with a colleague and then identify modifications to make when you next use the strategy.

5. Reflective lesson preparation, either by a single teacher or as part of a collaborative team, is needed to address the learning needs of English learners better. For an upcoming math lesson or unit, outline how you could identify and introduce key vocabulary and develop in advance plans for exposure, sense-making, practice, repetition, reinforcement, and application.

6. In a 2007 position paper titled "Improving Student Achievement by Leading *Effective and Collaborative Teams* of Mathematics Teachers" (http://mathedleadership.org/docs/PositionPapers/NCSMPositionPaper1.pdf), the NCSM states that effective collaborative teams of mathematics teachers meet to "[s]hare teaching strategies and analysis of the effectiveness of those strategies" and to engage in reflective discussions about observations of teacher practice. How could you become part of a collaborative team that focuses on effective instructional strategies for English language learners?

"DARE to Reach ALL Students!"

# Equity, Social Justice, and the Mission of TODOS: Connections and Motivations

Larry Lesser

**Abstract**

Equity and social justice are shown to be intertwined with each other and with the TODOS mission. Also, Shaughnessy (2007) and the author's pilot survey of in-service secondary teachers suggest interaction (or even interference) between students' prior concepts of fairness and certain mathematics/statistics topics. Recommendations for exploration are provided.

## Discussion And Reflection Enhancement (DARE) Pre-Reading questions:

1. How are equity and social justice different?

2. How are equity and social justice connected?

3. How are equity and social justice related to the mission of TODOS (or to the theme of this journal)?

"DARE" Post-Reading questions appear at the end of the article. This article (without DARE questions) originally appeared in Fall 2008 *Noticias de TODOS*.

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# Equity, Social Justice, and the Mission of TODOS

Larry Lesser

## Equity Involves Social Justice

Equity has recently drawn much attention in mainstream mathematics education, as reflected by the first Mathematics Education Equity Summit (hosted by NCTM in February 2008 and also attended by representatives of Association of Mathematics Teacher Educators, Benjamin Banneker Association, North American Study Group on Ethnomathematics, National Council of Supervisors of Mathematics, TODOS, and Women and Mathematics Education), the first leadership principle of NCSM (2008b), the position statement by NCTM (2008), the inauguration of the Iris M. Carl Equity Address (at NCTM's 2008 Annual Meeting), the professional development focus for 2008-09 by NCTM, and classroom explorations (e.g., McCoy, 2008).

TODOS' mission advocates for an *equitable* mathematics education. Having to ask for something equitable, however, suggests that there have been inequities and that addressing them is a matter of social justice. Indeed, the first *Standards* document of NCTM (1989, p. 4) asserts: "The social injustices of past schooling practices can no longer be tolerated." Unfortunately, some inequities persist in varying degrees and forms. For example, public schools in heavily-minority neighborhoods are less likely to be well-funded or have teachers who are highly qualified or certified in the subject they are assigned to teach (Flores, 2008). And the report by The Education Trust (Arroyo, 2008) shows that high-ELL districts generally receive less financial support than do districts with few or no ELLs.

Complete understanding and attainment of equity, therefore, is not possible without the perspective of social justice. An explicit connection between equity and social justice also appears in the position paper of NCSM (2008a, p. 1):

> "We believe inequities caused by lack of student access to mathematical knowledge and the opportunity to learn this mathematics knowledge must be addressed using a systematic process. This can best be accomplished when all mathematics educators respond to equity as a meaningful process to address social justice issues of race, language, gender, and class bias."

The equity model of Gutiérrez (2008) [see first article of this issue] entails not just equality on conventional measures of access and achievement, but also empowerment of students to use their distinctive strengths and mathematics tools to address social problems or injustices where they live.

## Exploring Social Justice

Nevertheless, some individuals express discomfort about connections between equity

and social justice, labelling the latter as unduly radical, communist, etc. So the aim of this article is to reflect on how social justice connects to the TODOS mission in more than just a narrow "radical" way. Last year, at the first annual Conference on Math Education and Social Justice (attended by over 500 participants from a broad spectrum nationwide), TODOS was an official partner/exhibitor and had members who gave talks, including plenary panel speaker Rico Gutstein. Gutstein is a prominent leader in social justice math and his work (e.g., Gutstein and Peterson 2005, Gutstein 2006) inspired the first comprehensive application to the context of statistics education (Lesser, 2007).

Those new to the topic could start with Murrey and Sapp (2008) and then move on to the aforementioned Gutstein or Lesser references. Two distinctive features of Lesser (2007) are its extensive, readily browsable resources (because the article is electronic) and also how it situates the topic among mainstream precedents -- as potentially neutral or apolitical as mainstream curriculum. Lesser (2007, p. 10) illustrates this claim with this question from Long Island University Professor Kathleen Kesson:

> "Why is there an assumption that people who wish to bring real world social justice issues into the math curriculum are any more 'ideological' than teachers who teach from a math textbook in which the word problems feature product placement for Nike shoes, Barbie dolls, or Cocoa Frosted flakes? Both approaches claim that their goal is 'relevance'. While the former might actually get students to think about housing patterns or the incidence of asthma in their neighborhoods, the other seems geared to encourage mindless consumption. Now isn't that just a tad ideological?"

In a related spirit, Eglash (2008, p. 11), while making a case for ethnomathematics in the classroom, states:

> "While the political right has a tradition of covering up the human rights violations of capitalism, the political left has been guilty of avoiding critique of human rights violations of socialism…..We need not worry about imparting some particular political line in order to convey social justice; it is enough to provide students with the tools of thought and the information about the world that will allow them to make their own decisions."

And regardless of an educator's own beliefs about any particular context, she can always play devil's advocate to encourage students to develop their own reasoning skills by asking questions like "What other interpretations are consistent with this data?" or "What further data would you need to collect to investigate that conjecture you just made?"

Educators can integrate social justice into curricula in a variety of ways and there is a continuum of levels of involvement, ranging from having students apply predetermined statistical methods to predetermined datasets to offering students opportunity to discuss the context, choose the social justice topic(s), and find (or even collect) the data.

## Additional Motivation

While offering equal opportunities to learn is itself a matter of social justice, social justice can also play a role in helping engage less-motivated students by offering curricular contexts authentically and perhaps connected intimately with their own reality. For example, there was recently a story in the *El Paso Times* with the headline "Study: Hispanics, blacks pay more for mortgages." Students seeing such articles should feel empowered to apply their mathematics knowledge to explore what this story says and doesn't say. Perhaps this is one way to move students toward "rigorous and coherent mathematics" as the TODOS mission demands. When Kitchen (1999, p. 321) asked his pre-service high school teachers to look at each day's local newspaper over a two-month period to examine what types of articles incorporate statistics, they found the most common theme in the articles was race or ethnicity and that "the more relevant that the data were to the students, the more willing that they were to analyze the data."

Finally, all teachers – regardless of their level of commitment to social justice – must be prepared to address how students' concepts of "fairness" may impact how they encounter standard mathematics concepts. For example, consider the "fairness" of random assignment of treatments in the context of doing experiments. Vogt (2007) presents a counterargument some students may believe that treatment resources should be assigned to be neediest students or patients, not the luckiest. Also, Shaughnessy (2007, p. 985) reports on research by himself and others that shows that precollege students do not value randomization in surveys because they find it "unfair" that each student doesn't get to choose whether to be in the survey or if a random sample does not happen to pick students from all possible subgroups.

Even when $N = 8$ inservice secondary school teachers in the author's graduate mathematics education research class were asked to evaluate the results of a random sampling process from a given hypothetical school population, five of them gave answers that focused not on the purpose of the survey or on the randomization in the process, but on the bottom line of whether all demographic groups were represented or represented in proportion to the population. So, the values of social justice people already have when they enter the classroom can impact the learning process.

In summary, social justice is not only a goal consistent with TODOS' call for equity, but can also be a vehicle, through supplying meaningful real-world contexts that motivate commitment to mathematics/statistics as a set of tools (e.g., proportional reasoning, expected value, probability, regression, etc.) that allow us to identify and quantify inequities that help us understand (and maybe even improve) some of our society's most profound or pressing problems. Engaged by these topics, students may want to be more meaningfully engaged with our subject and with our world and thus help make our subject matter (Lesser, 2007).

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**"The vision of equity in mathematics education challenges a pervasive societal belief in North America that only some students are capable of learning mathematics."**

***—Principles and Standards of School Mathematics* (NCTM, 2000, p. 12)**

# Discussion And Reflection Enhancement (DARE)
# Post-Reading Questions

1. What are some specific ways in which social justice can be a vehicle for introducing and engaging specific mainstream mathematics topics? [hint: see Lesser(2007) from the References]

2. Watch http://www.amstat.org/education/webinars/UsingSocialJusticeExamples.wmv (a half-hour ASA webinar recorded live November 25, 2008 on teaching K-12 mathematics/statistics using social justice examples) or browse the collection of social justice examples at http://www.radicalmath.org. Discuss how you might adapt one of the examples for your classroom.

3. Is Timothy Chambers' letter about expected value on p. 172 of the October 2009 *Mathematics Teacher* another example of Lesser's claim that "the values of social justice people already have when they enter the classroom can impact the learning process"? Explain.

4. What might be some specific ways in which, as the article says, your students might "use their distinctive strengths and mathematics tools to address social problems or injustices where they live"?

5. How can social justice examples readily involve all six levels of Bloom's Taxonomy, the "use real data" recommendation of ASA's *Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education* (http://www.amstat.org/education/gaise/), and, in general, offer an opportunity for learning mathematics at a high level of quality, rigor, coherence, and excellence?

6. Is incorporating awareness of equity and social justice into one's teaching a neutral, "value-free" act? Why? Is ignoring equity and social justice a neutral, "value-free" act? Why?

7. What actual and potential roles for equity and social justice awareness do you see at your school or in the mathematics education organizations to which you belong?

8. Why do you think equity and social justice have recently received such increased attention?

"DARE to Reach ALL Students!"

# Mayan Mathematics:
# Connecting History and Culture in the Classroom

Joseph M. Furner

**Abstract**

This paper discusses incorporating historical and cultural connections into one's teaching to bridge cultural gaps, foster appreciation for diversity, and promote sound understanding of mathematics and other cultures' contributions to mathematics. Studying civilizations such as the Maya helps many young learners appreciate their heritage and the evolution and logic of today's mathematics.

## Discussion And Reflection Enhancement (DARE) Pre-Reading questions:

1. What part of the population at the school(s) where you live/work is ELL?

2. How can using history and cultural components of our students help us better reach them in the teaching of mathematics?

3. Does integrating cultural connections better help to interest students when teaching mathematics?

4. What are some challenges or barriers to incorporating history in the teaching of mathematics?

"DARE" Post-Reading questions appear at the end of the article. This article (without DARE questions) originally appeared in Fall 2008 *Noticias de TODOS.*

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# Mayan Mathematics:
# Connecting History and Culture in the Classroom

Joseph M. Furner

Just south of West Palm Beach, FL, in the City of Lake Worth, is one of the largest populations of Mayans outside of México and Guatemala. Schools like Highland Elementary School in Lake Worth on average have 93% English Language Learners (ELL), many whose parents are from México, Guatemala, and Central America originally, but with their children born here in South Florida. It is important that schools recognize the heritages of their populations now in the USA. Math can be made more meaningful for these ELL students and all students when educators make meaningful historical and cultural connections to the math they are learning while valuing the heritage of a large population of the students.

The study of the cultural and historical contexts of ancient civilizations can be an intriguing way to introduce students to the evolution and logic of today's mathematics (Bidwell, 1993; Furner, Doan-Holbein, & Scullion-Jackson, 2000; Furner, 2008; Zaslavsky, 2002; Farmer & Powers, 2005). While people in Europe were struggling with the Roman numeral system whose symbols lacked both representation for zero and a calculated correlation with the numbers they represented, the Mayans in Mesoamerica were developing a system which modern day scholars find to be sophisticated, beautiful, and logical (Gilbert, 2006; Hand Clow, 2007). The Mayans invented a counting system which could represent very large numbers by using only 3 symbols: a dot, a bar, and a shell symbol for zero (see Figure 1).

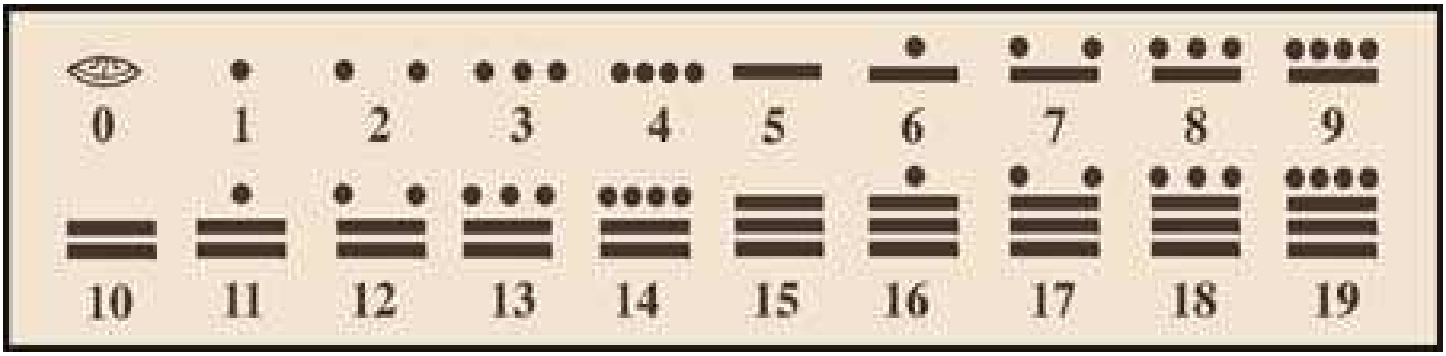



Figure 1.

Mayan Number base-20 System

The following lesson can serve to bridge historical and cultural connections while our students learn mathematics:

## Mayan Mathematics Lesson Plan

The following are suggestions for incorporating the historical context and symbolic notation system of the Mayans into a teaching unit (see Figure 2). The lessons might be taught from an interdisciplinary, integrated curriculum perspective, and modified to meet age appropriate needs of the students. This lesson too can serve in reinforcing concepts related to place value and number systems with different bases in mathematics. The strategies specifically include effective learning techniques for ELL students or English for Speakers of Other Languages (ESOL) but also brings to the learning meaning for all students involved.

The students create base-20 Mayan counters with corn kernels and popsicle sticks as the teacher guides and questions them. The students make base-20 Mayan corn stick counters to learn about place value and base-20. The students glue corn kernels on popsicle sticks to make base-20 counters to learn about Mayan mathematics and place value.

**Figure 2.**

**Students making base-20 corn stick counters**

### Objectives

Students will:

(a) explore similarities and differences of number systems from other cultures, particularly the Mayan system.

(b) calculate place value with base-10 and base-20 systems.

(c) Reinforce student understanding of place value.

(d) apply student technology skills.

(e) develop an appreciation for the culture and math of the Mayans.

### Menu of Motivation (Initiating) Activities

1. Pairs of students (one of which is an ESOL student) will take a web quest and visit websites to read about Mayan numerical systems.
2. Students view the video *Mystery of the Maya* (see References).
3. Students meet in discussion groups. Possible topics for discussion might be:

   a. defining terms such as decimal system, non decimal system, place value.

   b. describe similarities and differences between (among) systems.

### Menu of Core Activities

1. Create Mayan manipulatives to use in Mayan calculations.
2. Calculate and solve problems using their Mayan manipulatives.
3. Create and solve coded puzzles (see Example 1).
4. Read literature which relates to cultural differences in mathematics (particularly Mayan Mathematics). Suggested books are *Arithmetic in Maya* and *Skywatchers of Ancient México* (see References).

5. Develop and refine discussion groups and paired activities. Possible questions:

   a. Can you translate Mayan numbers into our numeral system?

   b. How about translating our numbers into Mayan numbers?

   c. Why do you think the Mayans chose a base-20 numeral system?

   d. How does the Mayan system compare to that of the Egyptians or Romans?

   e. Why do you think the Mayans chose a seashell to symbolize zero?

   f. What symbol for zero would you choose? Why?

6. Practice writing Mayan numerals 0 to 100.
7. Write reflective essays on website visits.
8. Create a fictional number system with a unique base and symbols.
9. Keep a journal of math activities and ideas.
10. Illustrate Mayan mathematics with a selected artistic medium such a magazine photo collage, penciled sketch, etc.
11. Locate additional books and websites about Mayan math and other math systems.
12. Invite parents and selected community guest speakers who are knowledgeable about the Mayan or other number systems (e.g., Lake Worth, FL has the Guatemalan-Mayan Center).

The lesson plan suggestions include provisions which are appropriate for all students/grade levels and especially ELL students. Realia and demonstrations develop vocabulary through WebQuests, literature, and study of artifacts of the culture. Prior knowledge and background are enriched through the study of the historical context of the evolution of number systems while developmentally appropriate activities using manipulatives provide concrete examples which reinforce concept development.

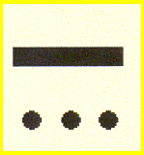
**Example 1:** Can you translate this into an equation using our base-10 number system?

**Example 2:** Can you write the number 103 using the base-20 Mayan Number system? (Remember to write from top to bottom, group in 20's, and use Mayan symbols.)

**ANSWERS**:

Example 1: 6 + 7 = 13

Example 2: (5 twenties and 3 ones; remember to leave a little space between the twenties place and the ones place)



Exploring various media such as drawing, painting, sketching, and creating collages addresses learning styles and promote creativity. Discussion about readings, web quests/field trips (actual or Internet), activities, and guest speakers prompt analytical and critical thinking as well as metacognition by encouraging students to verbalize their perceptions of learning. Interdisciplinary and cultural connections are established through historical and literary readings, discussions of economic and marketplace functions, and explorations of artistic and scientific contributions such as the Mayan calendar.

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***Map of Ancient Maya***


***The above map is reprinted (with permission) from www.mesoweb.com/resources/maps/crystal_map.html***

**"There is no conflict between equity and excellence."**

***—Principles and Standards of School Mathematics* (NCTM, 2000, p. 5)**

# Discussion And Reflection Enhancement (DARE)
# Post-Reading Questions

1. What might be some specific ways in which teachers can reach a particular ELL population based on their cultural background in mathematics? i.e. teaching Mayan math to children of Mayan descent (México, Guatemala, and Central America). Choose a cultural population and create some math activities around that particular culture.

2. How might you adapt one of the examples from the movie/DVD *Mystery of the Maya* for use in your classroom?

3. In an effort to reach "all students" in our classrooms today, how can using history and culture help to create stronger mathematics students in our ever-growing globally competitive world and provide equity for all learners? What are other methods?

4. Victorian-era math professor James W. L. Glaisher stated: "I am sure that no subject loses more than mathematics by any attempt to dissociate it from its history." How can mathematics teachers incorporate more history into the teaching of mathematics?

5. Visit the website http://www.lost-civilizations.net/mayan-history.html and create a lesson plan/activity to use in the classroom to integrate Mayan culture and mathematics.

6. Interdisciplinary and cultural connections are established through historical and literary readings, discussions of economic and marketplace functions, and explorations of artistic and scientific contributions such as the Mayan calendar. Explore some of the special occurrences the Mayans believed would happen in 2012.

7. The Mayans are well known for their expertise in astronomy. In fact, their calendar system by some scientists is believed to be more precise than what we currently use today. With the emphasis on strengthening math, science, and technology education, can we infuse more cultural connections like Mayan math into the curriculum to better reach minority students in the STEM (science, technology, engineering, and mathematics) fields?

8. Why do you think our society has recently given increased attention and emphasis on reaching the larger ESOL populations in our schools?

"DARE to Reach ALL Students!"

# Improving the Teaching and Learning Culture of Mathematics for Immigrant Children

Guillermo Mendieta

**Abstract**

Immigrant children are bombarded with negative messages that impact their beliefs and dispositions about schooling, authority, and themselves. Schools can counteract this by providing instruction that includes strategies such as: faculty discussing challenges immigrant students face, focusing on the big mathematics ideas, using multiple representations, and using generative language.

## Discussion And Reflection Enhancement (DARE) Pre-Reading questions:

1. How far are you personally willing to extend yourself to advocate (at the school or district level) for curriculum that serves all students? What are some specific actions you could do?

2. Have you had the experience of being able to teach and reach children more effectively after first establishing a culture of mutual respect and understanding? What specific things did you do to establish and maintain this culture?

3. What can we do to make immigrant children feel valued just as much as all other children?

4. How are roles of teacher and advocate similar? How are they different?

"DARE" Post-Reading questions appear at the end of the article. This article (without DARE questions) originally appeared in Fall 2006 *Noticias de TODOS.*

**Guillermo Mendieta** (pictorialmath@yahoo.com) is the author of *Pictorial Mathematics: An Engaging Visual Approach to the Teaching and Learning of Mathematics.* For more information and strategies to improve the teaching and learning of mathematics for minority students, visit www.PictorialMath.com.

# Improving the Teaching and Learning Culture of Mathematics for Immigrant Children

Guillermo Mendieta

A few years ago, I made the decision to go on a hunger strike to try to reverse the Los Angeles Unified School District's decision to eliminate all NCTM aligned curricula. The district's own data indicated that such curricula had significantly increased the percentage of Latino and African American students who successfully completed the college preparatory course sequence. Scores of activists, researchers, teachers and students spoke and wrote to the board about how their decision was going to deny tens of thousands of minorities access to college. They told the board that since the difference between the income of college and non-college graduates is around $15,000 per year, this decision was going to rob minority communities in Los Angeles of billions of dollars in future income. However, decisions about teaching and learning, like for most things, are often not made based on knowledge, on research or even on the best interest of students. Decisions are made based on politics: who has the power, who knows who, and who is well-connected.

Today we find ourselves in a similar, and perhaps worse situation. This time, the issue is not just about deciding between hands-on learning and the back-to-basics curricula. We are facing a nationwide anti-immigrant movement that has added fuel to the fire of most educational policy decisions. Unfortunately, the most vulnerable, the ones without power or the vote, the children of the poor, the children of minority parents, are the ones that suffer the long-term consequences of the bad educational policies that spring from elitist and xenophobic political rhetoric.

Imagine yourself as an immigrant child in this country. Every day, in the media you see and hear authority figures refer to immigrants as undesirable, criminals, and worse. You hear them say immigrants should be denied health care and education. As a child, you don't hear or understand the nuances about immigration law. All you hear is that you are not wanted, that you are a criminal, that you are undesirable. Now picture yourself going to school and trying to learn in the middle of this hostile environment. What should the school do to help you learn, to help you feel safe and to help you trust enough to take the types of risks that learning requires of you? As I thought about these issues, Herbert Kohl's book *I Won't Learn From You* came to mind (Kohl, 1994). Like Mr. Kohl, I have come across hundreds of Latino students who will actively resist learning, who will not give a teacher the pleasure of learning from them if they do not feel respected.

There are so many different issues and challenges intrinsically woven and spinning seemingly out of control for teachers working with immigrant students, there is not enough room to address them in this article. However, given my background as a math educator and as an advocate for minority students, I would like to offer four suggestions that would help schools in general,

and math teachers in particular, create a more supportive and effective learning environment for their immigrant students.

**Learn and Discuss The Specific Challenges Facing Immigrant Children With The Entire School Staff**.

Regardless of the judgments anyone might make about the decisions made by the parents of immigrant children, most people would agree that immigrant children are facing a multitude of challenges and pressures that impact the schools' teaching and learning culture. A staff that knows and understands the challenges immigrant children face, from the perspective of their students, will be better able to connect, support, and create the type of relationships and learning environment that are needed to improve teaching and learning.

**Focus In-Depth On Three To Four Key Standards Or Main Concepts Throughout The Year While Making Connections With Other Strands.**

Researchers have pointed out that the curriculum in countries where students perform well on international comparison tests, such as Japan, is an inch wide but a mile deep, while the curriculum in the U.S. is a mile long and an inch deep. Disadvantaged students in general and ELL students in particular are particularly vulnerable to the prevailing "coverage-driven curriculum" that emphasizes superficial coverage of material at the expense of in-depth teaching. Kindergarten teachers might focus on number representations and patterns, first-grade teachers might focus on addition and subtraction, and fifth grade teachers might focus on fractions, ratios and percents, etc.

**Help Teachers Learn To Use Effective Multiple Representations In Their Teaching.** Focusing on fewer concepts should not mean giving more symbolic representations to students. The most important and effective teaching strategies in mathematics are the use of effective multiple representations, the translations among them, and the transformations within them.

**Use Model-Generative Academic Language To Teach Mathematical Concepts.**

The language we use when teaching a mathematical concept with ELL students can either open up or obscure the ideas we are trying to represent. Teachers should learn to use language that carries with it a mental model of the ideas or processes involved. For example, when teaching how to multiply 1 ½ x 2 ½, a teacher might read and talk about this exercise in these ways:

One and half groups of two
and half each

Two and half repeated one
and a half times

One group of 2 ½ combined
with half of 2 ½

Associating such language with a pictorial example will help students generate a mental model of how to work such problems.

**Conclusion**

Challenges faced by immigrant children and their teachers are being magnified by anti-immigrant rhetoric sweeping the country. Schools that want to be more effective in helping immigrant children learn should help teachers learn more about specific challenges faced by immigrant children.

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## Discussion And Reflection Enhancement (DARE)
## Post-Reading Questions

1. Which of Mendieta's four suggestions have you tried (or would be most likely to try)? Discuss any possible pitfalls and benefits of this approach.

2. Are Mendieta's suggestions aligned with the NCTM Standards? Discuss.

3. Do his suggestions seem like they would be effective for non-immigrant children as well? Explain.

4. Compare the fourth suggestion with the way traditional books in the past represented fraction arithmetic. What are advantages of Mendieta's approach?

5. Read Mendieta's book or visit his website (mentioned in the author bio at the beginning of this article) and identify an additional useful idea.

**"The Equity Principle demands that high expectations for mathematics learning be communicated in words and deeds to all students."**

**—*Principles and Standards of School Mathematics* (NCTM, 2000, p. 13).**

"DARE to Reach ALL Students!"

# Lesson Study: Collaboration among Middle School Mathematics Teachers of Latino Students

Cynthia O. Anhalt, Laura Farias, Salvador Farias,
Josie Olivas, & Melanie Ulliman

**Abstract**

This lesson study experience occurred within a partnership between mathematics educators and four middle school mathematics teachers of ELLs. The lesson focus was addition of fractions of unlike denominators. The students were given opportunities to think individually and then work with a partner using fraction bars to explain and justify their solutions.

## Discussion And Reflection Enhancement (DARE) Pre-Reading questions:

1. Reflect and brainstorm effective teaching practices that you are familiar with for teaching mathematics. Share with the group and find commonalities and differences in the approaches shared.

2. Share specific strategies and practices that you are familiar with for effectively teaching Latino students in your school.

3. Share what you know about Lesson Study. What are its components? Discuss the potential professional growth in participating in a lesson study cycle.

"DARE" Post-Reading questions appear at the end of the article. This article (without DARE questions) originally appeared in Spring 2007 *Noticias de TODOS.*

**Cynthia O. Anhalt** (canhalt@math.arizona.edu) is Instructional Faculty at the University of Arizona, in the Department of Mathematics, and the Director of the Secondary Mathematics Education Program. Lesson study was a research project with the Center for the Mathematics Education of Latinos/as (CEMELA, NSF Award No. ESI-042983) and with Chaparral Middle School mathematics teachers **Laura Farias**, **Salvador Farias**, **Josie Olivas**, and **Melanie Ulliman**.

# Lesson Study: Collaboration among Middle School Mathematics Teachers of Latino Students

Cynthia O. Anhalt, Laura Farias, Salvador Farias,
Josie Olivas, & Melanie Ulliman

Lesson study is a collegial form of professional development, based on the Japanese model of "*kenkyuu jugyou*" -- or research lessons. In the US, lesson study focuses primarily on mathematics, while in Japan it is also used for other subject areas. Lesson study allows teachers to examine each other's approach to teaching subject matter, in order to collaborate and improve the overall teaching experience in the school. Common characteristics include:

- shared goal for improvement
- focus on learning of a specific aspect of academic content
- oriented toward gathering evidence on students' learning, engagement, and treatment of one another as a result of the teacher's lesson, in order to examine student thinking, or "the eyes to see children (*kodomo wo miru me*)."
- based on shared observation of actual lessons in progress. Videotape, written cases, lesson plans, photographs, and student work are used to aid teachers in analysis and improvement of instruction.

In the fall of 2006 in Arizona, middle school mathematics teachers of predominantly Latino students teamed up with a researcher at the Center for the Mathematics Education of Latinos/as [CEMELA] to examine effective strategies in teaching mathematics. The project centered around a lesson study cycle.

We followed the traditional lesson study structure that allowed all of us to design a lesson. Then, each of the participating teachers taught the lesson while the rest of us on the team observed during the teaching. Each rendition of the lesson contained modifications based on the team's feedback and debriefings. Our discussions and negotiations that took place on what constitutes effective teaching were based on and influenced by our individual teaching experiences, as well as on the book *The Teaching Gap* (Stigler & Hiebert, 1999). Our overall goal for the evolving lesson was for students to initially think independently and then to collaborate with a partner to arrive at a solution.

### Adding Fractions

The lesson involved addition of fractions with unlike denominators, which the middle school students had not worked on yet this year. First, students were given the following word problem:

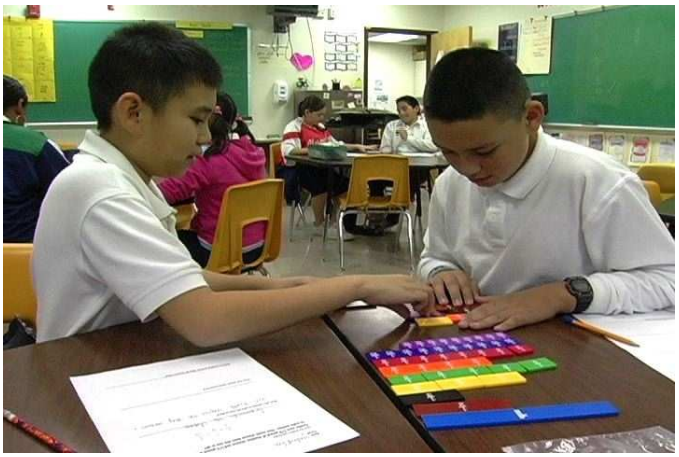
> *Cecilia uses 2/6 pound of cheddar cheese and 1/4 pound of mozzarella cheese to make nachos. How much cheese does she use in all?*

Students were initially given time to think independently. Most of them readily added the fractions by adding the numerators and by adding the denominators, and wrote down a solution of 3/10. After discussions took place among partners, only a few pairs of the students changed their solutions to

7/12, recognizing that it was necessary to find a common denominator. Not until additional hands-on tools were provided to students, namely fraction bars, did they begin to think critically about their approach, since they had to "justify" their solutions using the fraction bars. It was at this point that most of the students were able to make connections between the fractions, the fraction bars, and why they needed to find a common denominator to solve the problem.

## Making Sense of Mathematics

Our reflections on this lesson involve issues surrounding teaching mathematics to Latino English Learning students. By allowing students to make mistakes initially (adding the numerators and the denominators) we were able to accomplish a salient goal of observing how students would later correct their own thinking after identifying their own errors. The impact of students realizing their own mistakes and understanding why a common denominator is necessary for adding fractions will have a lasting effect on their continued learning of fractions. We realize that all students need to make these connections; however this approach proved to be especially crucial for English Language Learners (ELLs), as they may not always "fully" understand everything the teacher is saying during English instruction.



*Students justifying their solution with fraction bars*

Teaching-by-telling often has a negative impact, because some students whose academic English may not be developed appropriately for grade level expectations will never have an opportunity to learn particular concepts. In our research lesson, we deliberately planned for students to fully engage during a problem-solving and concrete experience, with the concept of the need for a common denominator, and did not focus on telling them initially how to add fractions with unlike denominators by a procedure.

While it was difficult to observe exactly when they were making a mistake in the process of adding fractions, it was powerful to observe the "discovery" students later made on their own. Teachers did intervene, but only to ask questions and to continue engaging students in the discourse of the mathematics that was involved in the lesson. We found that, by designing the lesson as we did, the ELLs were able to actively engage in academic discourse regarding fractions in a meaningful way.

## Collaboration is Key

During the teaching and critiquing of the lesson, lesson study allowed us to work collaboratively in planning, teaching, and observing each other. Ultimately, when we modified the lesson to improve it, lesson study was a way for us to systematically examine our practice, while at the same time focusing on the mathematics in the lesson. Collaboration was a key factor, because we found that our combined strengths for designing effective instruction for our students are greater than our individual efforts. Engaging in the collaborative lesson study process allowed us to improve the lesson with students' best interests in mind.

Each time we observed the evolving lesson, our attention was directed to the students' learning, interaction, and engagement in class. Our experiences in doing lesson study allowed reflection time in the midst of debriefing and gave us an opportunity to grow in our individual teaching practices, with a focus on our ELL student population.

Acknowledgement: The authors thank Marianne Smith for editorial assistance on this article.

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## Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Consider the initial problem that was posed to the students about the amount of cheese needed to make nachos. Would you consider this problem reasonable for 6th grade students to know how to solve? Why or why not?

2. What do you think of these teachers' approach to pose the problem and not show the students a procedure for adding fractions with unlike denominators? What are the benefits and the drawbacks of posing a problem for students to figure out how to solve the problem and then to justify their solution?

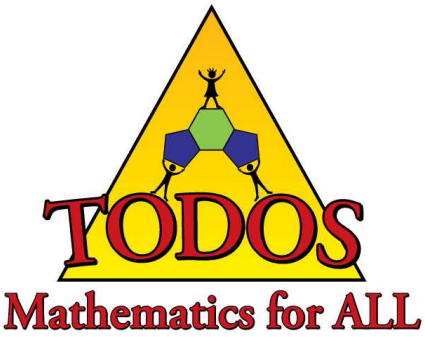
3. What do you suppose happens in a lesson when students realize that adding the numerators and denominators does not produce a reasonable solution when they are asked to justify and prove their solutions using an alternate representation of the solution?

4. What was the teacher's role during the students' work of adding fractions with unlike denominators?

5. What were the benefits for this group of teachers in using a lesson study format for discussing what occurred during the lesson?

6. What are the benefits of including ELLs in the academic discourse of mathematics? What has been your experience in having your students participate in math class discussions?

"DARE to Reach ALL Students!"

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**TODOS Research Monograph #2**

The second monograph is expected to be published this school year and it focuses on the assessment of Hispanic/Latino students in mathematics. The monograph will inform the education community and policy makers about large-scale assessment formats and classroom-level assessment strategies that foster or have the potential to foster greater achievement and learning of Hispanic/Latino students in mathematics.

**"Equity without quality is useless, quality without equity is unjust."**

***—Guillermo Mendieta,***

*Recipient of the first TODOS Iris Carl Leadership and Equity Award, 2006*

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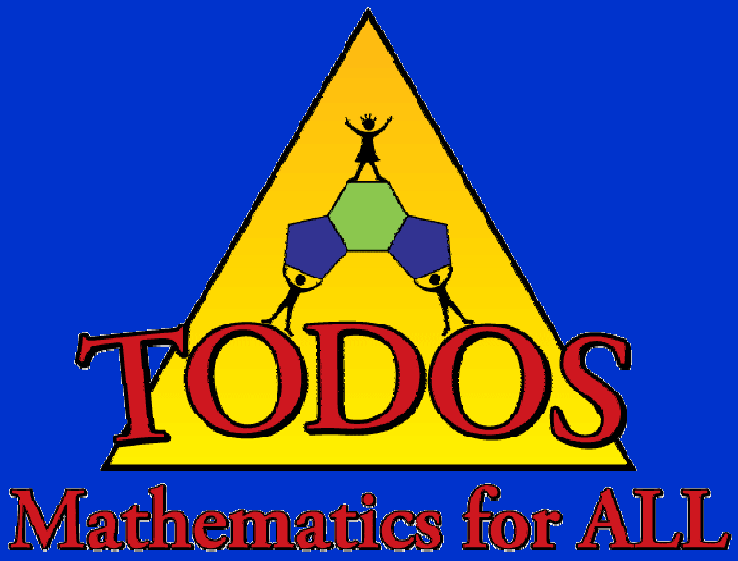


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