A Framework for Modifying Mathematics Tasks for Accessibility

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Abstract

For our work in *Language in Mathematics*, we developed a framework for analyzing mathematics tasks along lines of mathematics concepts, mathematics practices, contexts, and language demands. By referencing these features, we worked across our distinct academic specializations of mathematics education and language/literacy education more easily. They also helped us to draw important distinctions between task characteristics (concepts and practices) that cannot be modified without changing what is being assessed mathematically; and those that can be changed (context and language demands) as long as the changes are done with care. We share our framework, which can be used for curricular and instructional purposes, in hopes it can help other educators to work cross disciplinary areas for improving the accessibility of mathematics tasks more generally.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What is an example when the real-world context of a mathematics problem seemed to affect that task's accessibility for English language learners in your classroom?

2. What is an example when the language of a mathematics problem seemed to affect that task's accessibility for English language learners in your classroom?

3. What does the CCSSM say about #1?

4. What does the CCSSM say about #2?

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This article provides an illustrative summary of a four-dimensional framework that we used in creating an assessment of academic language in mathematics for *Language in Math* (*LiM*), a research and development project funded by the Institute for Educational Sciences (IES). *LiM* aimed to combine what we know about how upper-elementary and middle-school students learn mathematics with what we know about how students who speak Spanish as their first language acquire English as a later language. In *LiM*, we worked with certified grade 4-8 teachers who taught mathematics to self-contained classes that included students who had been identified by the school as "limited English proficient." At the time that *LiM* was implemented, "LEP" was Florida's terminology, though we prefer the term English-language learner (ELL). Almost all the students in our study were at the intermediate or advanced proficiency stage of learning English. The assessment of academic language in mathematics was meant to give us a sense of how changes in mathematics-relevant language would affect ELL students' performance on and reasoning about tasks that are typically administered in mathematics tests.

The Framework

Mathematical content, *mathematical practices*, *context*, and *language demands* are terms that gloss over some important distinctions found in the research literature. Yet these terms provided a good starting point for us to communicate ideas among ourselves, and now to teachers and other colleagues, without getting too bogged down in details.

Mathematical Content

A task's mathematical content is the mathematical idea(s) or concept(s) that an individual must call upon in order to solve that task. *On the Shoulders of Giants* (Steen, 1990) and the domains found in the Common Core State Standards in Mathematics (CCSSM; National Governors Association and Council of Chief State School Officers, 2010) provide ways of describing the "big ideas" of mathematics.

Mathematical content can also entail somewhat smaller-sized ideas such as place value, fraction equivalence, or linear expressions. Mathematics content may become even more narrowly focused as in "knowing that fractions, percents, and decimals are all different ways of expressing the same number" and/or "adding two fractions with the same denominator."

Mathematical Practices

A task's mathematical practices are the social and conceptual processes that an individual must often call upon to solve tasks; these may differ depending upon the task's content and context. Heuristics described in *How to Solve It* (Pólya, 1957) reflect practices one may use when solving problems, and the eight cross-cutting practices found in the Common Core State Standards in Mathematics (NGA & CCSSO, 2010) provide examples of practices being promoted for school-mathematics. The CCSSM practices have social and psychological aspects: (1) make sense of problems and persevere in solving them; (2) reason abstractly and quantitatively; (3) construct viable arguments and critique the reasoning of others; (4) model with mathematics; (5) use appropriate tools strategically; (6) attend to precision; (7) look for and make use of structure and (8) look for and express regularity in repeated reasoning.

Context

Context refers to the setting within which a mathematics task is found and which gives rise to that mathematical problem. Tasks vary in how much support their contexts provide: familiar or even personally-interesting contexts might motivate and help someone to access mathematics concepts and to engage in the mathematical practices that are needed to solve the task because they know and understand the context in which the task is embedded. Unfortunately, too many tasks incorporate contexts that create barriers and/or have no meaning for students who become confused and unmotivated (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). Computational tasks are often said to have "no contexts", though computations can also be thought of as purely "mathematical contexts."

Language Demands

Language demands refer to the language-processing demands that are placed on the reader of a task. Because we were focused on text-based assessment tasks (that include printed words, graphic representations, symbols, and numbers) that had to be seen, this dimension excluded the demands of certain language modalities (speaking, listening, writing). When extending this framework to instruction, all five language modalities (reading, writing, listening, speaking, and viewing and representing) need to be considered. For example, in Avalos, Medina and Secada (2015), we included attention to oral and graphical forms of communication in our presentation on how teachers might use visual graphics to help multilingual students access algebraic word problems.

An Example

Our goal was to create or revise assessment tasks to be more accessible to English language learners and so that we could better understand how modifications in a task's non-core-mathematical features (i.e., context and language demands) affect student performance. By comparing performance on tasks with relatively higher to relatively lower language demands, we hoped to better understand how students' academic language proficiency in mathematics affects their performance.

The following example is drawn from the Florida Comprehensive Assessment Test 2.0 (FCAT; Florida Department of Education, 2011). According to the Test Book and Answer Key, this task corresponds to Benchmark Code MA.8.A.1.1 (FLDOE, 2011), which in turn refers to Mathematics, 8th grade, Big Idea 1 (Analyze and represent linear functions, and solve linear equations and systems of linear equations), Sub-idea 1 (Create and interpret tables, graphs, and models to represent, analyze, and solve problems related to linear equations, including analysis of domain, range, and the difference between discrete and continuous data) (CPALMS, 2008).

Sami installed a 6-foot-tall cylindrical storage tank to collect rainwater from the roof of her house. She used the rainwater to water the lawn and garden during dry spells. Sami recorded the rise in the water level in her storage tank after each of 3 rainstorms. Her results are shown in the table below.

Rainfall (in inches)	Rise of Water Level in Storage Tank (in inches)
1.5	24
0.5	8
2.5	40

Which is the best prediction of the rise of the water level, in inches, in her tank after a storm produced 2.25 inches of rain?

A. 16 inches B. 28 inches
C. 32 inches D. 36 inches

Figure 1. Sample Task 1 (FCAT; Florida Department of Education, 2011)

Mathematical Content and Mathematical Practices

From sub-idea 1, above, this task requires students to "… interpret tables, … and models to … solve problems related to linear equations …." This task does not include specific mathematical practices in the sense that students are not required to demonstrate or to use them as a condition of being scored right. Any changes in either content or practice would undermine this task's validity when it is used for assessment. Hence, we kept these features of the task, while changing other task features in an effort to make it more accessible.

Context

The context for Sample Task 1 entails a large container filling with rain water for use in gardening. In Southeast Florida, rainy weather is common; water pooling in pots or containers to use for other purposes could link to the problem's context of water collection. Hence ELL students may be familiar with a task entailing water collection. On the other hand, tasks like this one are criticized for failing to motivate a need for storing rainwater in the first place and for predicting the amount of rainwater rise in a tank based on predicted rainfall.

We tried to increase the likelihood that students would be familiar with the problem context by referencing a school's garden, something that many middle schools are planting and that need to be watered regularly. We hypothesized that students would also find this setting more motivating than a context involving an unknown individual. Another alternative is simply to strip away all contexts, thereby converting this task into something that is purely symbolic.

Language Demands

Among the features that make text more difficult to read and understand are the unnecessary inclusion and/or use of:

- extraneous information, such as the location of water storage cylinder on the roof;
- overly long sentences, such as the problem question;
- technical vocabulary, such as stating that the storage container is cylindrical.[1]

In revising this task, we addressed the above language demand concerns. Also, we did not use a picture to try to reduce language load because we were not sure that ELL students would understand how the picture referred to what had been written, which is necessary for the picture to be helpful.

Result: Two Revised Tasks

Informed by our analysis, we created two revised tasks (see Figures 2 and 3).

A 6-foot-tall storage tank is used to collect rainwater which is then used to water the school's garden during dry spells. Sami recorded how much the water level rises in the storage tank after each rainstorm. Her results for 3 rainstorms are in the table below.

Rainfall (in inches)	Rise of Water Level in Storage Tank (in inches)
1.5	24
0.5	8
2.5	40

A recent storm produced 2.25 inches of rain. How much did the water in the tank rise?

A. 16 inches C. 32 inches
B. 28 inches D. 36 inches

Figure 2. Alternative Wording

Given the following relationship between x and y:

x	y
1.5	24
0.5	8
2.5	40

If $x = 2.25$, then $y = ?$

A. 16 C. 32
B. 28 D. 36

Figure 3. Purely symbolic

[1] We understand that the cistern's placement on the roof allows gravity to empty it; but so does its being placed anywhere above the ground. That the cistern is a cylinder may explain why there is a continuously linear relationship between the amount of rainfall and the rising water levels; yet the same would be true for a cube or rectangular polyhedron. Hence while correct, this information is not central to the problem's statement.

Figures 2 and 3 maintain the original task's mathematical content and non-specification of mathematical practices. Figure 2 modifies the context to be more motivating; and it modifies the language demands so that the resulting task would be more accessible to English language learners. Figure 3 removes all context and strips language demands to a minimum in case purely symbolic problems are, in fact, more accessible to ELL students.

Extension to Curriculum and Teaching

Concerns for construct validity in assessment limit our ability to modify a task's mathematical content and mathematical practice. However, no such constraints limit our ability to modify the tasks that comprise students' mathematics curriculum and its teaching.

This task could be modified by changing some combination of its mathematics content, mathematics practices, context, and language demands for purposes of curriculum and teaching. Some modifications might work alone or in tandem to make the task more accessible to ELL students; others, to make it more difficult. The changes would depend on teachers' instructional goals.

Mathematical Content

Changes in the shape of the water container could motivate exploration of non-linear functions. Instead of being cylindrical, the container could be spherical (as in the case of some containers that sit atop water towers) or even a series of pyramids and polyhedra (as in the case of swimming pools). The resulting tables would represent non-linear functions.

Switching over to real-world water containment systems, such as lakes or ponds, would require the use of some combination of shapes to approximate their volume. The resulting tables relating rainfall to the rise in the containers' water levels would be quite complex as is the case for functions that are piece-wise linear or non-linear.

For these examples, changes in context would lead to changes in mathematical content. Furthermore, the shapes of the water containers would actually matter; and hence, the task's language demands would also be affected.

Mathematical Practices

Two mathematics practices found in the Common Core are implicit.

- If students drew a picture to represent the container and sketched it filled at various levels, they could be making sense of the problem (practice #1);
- If students reorganized the table so that rainfall amounts and corresponding rises in water level were ordered from lowest to highest, they are looking for structure (practice #7);
- If students halved the amount of rainwater rise corresponding to 0.5 inches and then, either (a) built up from 1.5 inches of rain to 2.25 inches by adding the amounts of rainwater rise corresponding to 0.5 and 0.25 inches or (b) reduced the rainwater rise corresponding to 2.5 inches of rain by the amount corresponding to 0.25 inches, they are making use of structure (practice #7).

Alternate strategies for solving this task could include (a) plotting the graph (using a graphing calculator, if appropriate) corresponding to the table presented above and interpolating between 1.5 and 2.50 inches to see how much the rainwater rises in the containment structure when 2.25 inches of rain falls; (b) computing the amount of rainwater that the container rises per inch and then multiplying that by 2.25; and/or (c) deriving an equation from the table and "plugging in" 2.25 for x.

Very often, these sorts of tasks are used to teach eighth graders about the rise-over-run method of computing slope. However, an open-ended in-class discussion of how students made sense of and solved this task would allow students to engage the Common Core practices numbers 1 and 7. In addition, if classroom norms permitted, in-class discussion would encourage students to construct viable arguments and critique the reasoning of others (practice #3). If the original task were extended to the use of different shaped containments, students would have to model with mathematics (practice #4).

Context

As noted in our discussion about assessment, one could argue that the original task lacks relevance for the students and may not seem to provide a compelling reason for being solved. On the other hand, flooding devastation is one of many reasons why we might want to predict when a natural or human-made water containment structure might overflow. Yet even in the case of more localized water collection system as in the case of watering a garden, the subsequent cleanup to the container's flooding can be time-consuming and messy.

The threat of hurricanes in Florida often leads to the draining of water from Lake Okeechobee in an effort to stave off flooding (Reid et al., 2016). Similarly, floods caused by snow melt or thunderstorms take place throughout much of United States and students see the resulting damage in the news. Even though drought has plagued much of the Western United States (Pacific Institute, 2017), including Lake Mead (Worland 2016), increased rainfall has ameliorated many of those concerns and seems to be motivating questions about the impact of too much rainfall on surrounding areas and the possibility of draining some water to avoid flooding. These real-world contexts of how too much rainfall can lead to rising waters and flooding are too complex to be incorporated into students' mathematics curriculum without modification. But if modified tasks were presented in conjunction with science and social studies lessons on the environment, it may be possible to use such settings to motivate sets of tasks that, individually, are accessible to ELL students and that, in the aggregate, lead to a more sophisticated set of mathematical understandings.

Language Demands

Assessment tasks should be as easy to read and understand as possible because students must read the texts by themselves and solve the resulting problems without the social processes that provide support during instruction. Also, busy teachers cannot revise every task found in their students' mathematics books. However, it is possible for teachers to scaffold a task's language demands in anticipation of when students first read them and to be sensitive to those demands during the rapid give-and-take of a mathematics lesson. For example, teachers can discuss a text's technical vocabulary, its cultural references, and other features as part of instruction and students can create and maintain their own glossaries of unfamiliar terminology. The glossary may be further refined by similar or different uses of a particular term in other disciplines and contexts.

In planning language-focused class discussions, teachers should remember that, as general rules of thumb, for ELLs:

- passive voice is more difficult to understand than active voice;
- past and complex tenses are more difficult to understand than present tense;
- longer sentences are more difficult to understand than shorter sentences when a student has limited knowledge of the mathematical concept(s) within the text;
- when precision is required and students have some prior understanding of the concepts that are involved, technical vocabulary may actually help them to understand what is being asked because of its precision;
- when technical vocabulary provides false precision or when a student has not encountered the basic conceptual underpinnings of that terminology, technical vocabulary may render a task more difficult;
- a picture may be "worth a thousand words", but students have to understand what the various components of mathematical illustrations refer to in order to make use of them;
- mathematical symbols place their own unique demands on someone's ability to read and to understand the information that a task provides and what is being asked.

Concluding Comments

Figure 4 provides a visual summary of our Framework and some of the salient issues that arise when thinking about its utility for classroom instruction.

Dimension → Application ↓	Mathematical Concepts	Mathematical Practices	(Mathematical) Context	Language Demands (Math, Academic, and Everyday)
Classroom Assessment	Cannot be modified without raising validity issues	Cannot be modified without raising validity issues	Can be modified, with care, to motivate and to increase text comprehensibility	Can be modified with care to increase text comprehensibility; mathematics terminology can provide precision
Curriculum and Teaching	Can be modified with care to related concepts, depending on specificity	Can be modified with care to related practices, depending on specificity	Can be modified, with care, to motivate, to increase text comprehensibility, and/or to extend to new concepts or new practices	Can be modified with care to increase text comprehensibility; mathematics terminology can provide precision

Figure 4. Annotated Framework

From the *Language in Math* project we learned a lot about the challenges of meaningfully teaching mathematics to ELL students in ways that allow students to understand mathematics and that are consistent with the standards set out in the CCSSM. Being from different disciplines -- mathematics education and language and literacy education -- we learned that we had to develop ways of communicating with one another so that we were talking about the same things; for example, what mathematics educators mean by semantic structures of arithmetic word problems (Secada & Carey, 1990) is quite different from what language and literacy educators mean. We created this framework as a first step in organizing our own work around complexity of mathematical language found in tasks, and of fostering communication among ourselves. Through this article, we are taking some first steps in sharing that framework with teachers and other educators in the hopes that they, too, find this helpful.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Pick a specific mathematics problem and discuss how the eight mathematical practices of CCSSM might play out in its exploration.

2. What is an example of a mathematics problem that you needed to revise (either before or after using it with students)? Describe the process or nature of your revision.

3. How would eighth-graders in the city in which you teach relate to the tasks found in Figures 1, 2, and 3 for assessment purposes? For instructional purposes?