Methods, Maps, and Meaningful Mathematics

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Abstract

This article is a reflection of one mathematics educator's journey towards being a critical mathematics teacher educator. By illustrating the manner in which she selected, developed and provided an opportunity for bilingual pre-service teachers (BPSTs) to teach a mathematics lesson with a social justice component to fifth graders in a methods course, she reflects on her own growth and its potential for transformational pedagogy with her students and their future students. The decision making process of task selection, introduction of the task to BPSTs, and the support provided to them to teach the lesson, is articulated in order to make visible the challenge in transforming a teaching practice. Mathematics educators who are also seeking to infuse more social-justice focused mathematics activities into their methods courses may relate to ideas in this article, and find support in reading the processes of an early-career teacher educator asking questions of her own praxis.

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Social Justice in a Bilingual Mathematics Methods Course with a Field Component

This article is about my growth and development as a mathematics teacher educator, and how I strive to "walk the walk" when it comes to teaching future elementary school teachers about mathematics for social justice. I borrow the alliteration in the title of this piece from Gutstein's (2013) chapter in *Rethinking Mathematics (2nd ed)*, entitled *Math, Maps, and Misrepresentation*. In that chapter, Gutstein shares the experience of presenting a task to his 8th grade students in which two world maps, the Mercator and Peters projections, portray landmass differently.

The Mercator map, which is present in almost every schoolroom in the US, distorts the landmasses of the southern hemisphere and makes the US, Europe, and Russia appear larger than they are. The Peters map, on the other hand, is designed to preserve relative landmass; countries that are bigger on the globe also look bigger on the map. Gutstein explored with his students how mathematics illuminated the misrepresentation of countries on the Mercator map which then led them to questions about what other information has been misrepresented as part of their education, and what colonial legacies are embedded within something as simple as a map.

The focus of this article is on how and why I, as a mathematics educator, selected this task for my bilingual pre-service teachers (BPSTs) to implement with fifth grade students in a field-based methods course, and what was learned from that experience. The decision-making processes I engaged in as I introduced the task to BPSTs, supported them during the planning session, and then debriefed the activity after they tried it with fifth graders are explored. Sharing the details of this experience will contribute to the development of fellow mathematics educators, who want to better understand what our future elementary teachers get out of being supported to plan and enact an activity designed to infuse social justice into doing mathematics.

Bilingual Teacher Preparation: Towards *Educar Para Transformar*

I teach mathematics methods to BPSTs in a program that is moving towards an *Educar Para Transformar* model of teacher education. As Flores, Sheets, and Clark (2011) describe, the focus of this model is on "transforming program faculty, as well as *aspirantes'* (BPSTs') ideological positions, knowledge, and skills." (p. 13). The ultimate goal in such a model is teachers, or in their term *aspirantes*, who teach for freedom: "Prepared to lead, they advocate and bring about change in self, bilingual learners, schools, and communities." (p.14). Even naming candidates *aspirantes* communicates that they are aspiring towards something bigger than typical teaching as we know it. As we strive toward cohesion in our program around *Educar Para Transformar,* we also engage in personal transformation. For me, this transformation is centered on how I shift my thinking and actions around what teaching mathematics for freedom means. I turn to bodies of work in teaching mathematics for social justice (such as Gutstein, 2006; Gustein & Peterson, 2013; Wager & Stinson, 2012) as a starting point while I continue to evolve in my understanding of what teaching mathematics for freedom with Latin@ populations means.

The SF State program's focus on Spanish-immersion classrooms in urban schools draws many future teachers from around the bay area, including the local San Francisco communities that we focus on serving. Most of our students are Latin@, born and raised in the United States; many grew up speaking Spanish in the home, others are heritage language speakers. Most of our students begin coursework for their teaching credential the semester after they

graduate from college [1], but we frequently have older students returning to school after starting families or teaching as paraprofessionals in schools.

I strongly identify with our students who were born and raised speaking Spanish in some contexts, and English in others. Bilingualism is a spectrum, on which I identify myself as not fully fluent in Spanish, but with sufficient skills to converse, do some teaching, and to identify and pick up academic language for teaching mathematics. In our program, the unofficial Spanish of classrooms situated within urban areas of the United States melds with each of our experiences – as Mexican, Central American, Peruvian, and/or multi-generational US-born Spanish-speakers, some with formal study in the language and others with less formal study. It is in this context that we negotiate not just what it means to teach and learn mathematics, but also the Spanish, English, and mathematical languages that go with it.

Goals of the Mathematics Methods Course

Most of the BPSTs bring strong commitments to social justice for Latin@ families. Many of our students are active in local campaigns around gentrification and displacement impacting the Mission district, an historically Latin@ neighborhood in San Francisco. However, as within any teacher education program, our students do not necessarily share a vision of social justice for teaching mathematics. These commitments and the BPSTs' experiences as mathematics learners make for widely different beliefs of how to teach mathematics; from wanting to model everything step by step for students, to wanting to connect mathematics to the real world. The three key instructional principles I strive to have them leave with are:
1) Doing mathematics is a social activity, in which the teacher facilitates student learning by attending to both content objectives and the identity development of students; 2) Mathematics must be relevant to and responsive to students' needs, lived experiences, home lives, and communities; 3) Teachers have a lot of power over students' attitudes towards math, both in how they model enthusiasm for the subject, and how they socialize students into what is productive engagement with mathematics.

To me, there are aspects of social justice in each of those these principles, but to get at the heart of what Gutstein (2006) calls *reading and writing the world with mathematics*, a teacher needs to start with expanding their understanding of what counts as doing mathematics, and what kinds of knowledge get privileged - traditional, community, or critical mathematical knowledge. Therefore, in methods courses taught by me, I want to make sure that I as the mathematics educator introduce examples of tasks that shift the understanding of what counts as mathematics, and open up spaces for students to draw on their knowledge of situations and sense of fairness as they learn mathematics.

I typically teach this course at the beginning of the second semester, in partnership with a fifth grade teacher, at a local elementary school. This teacher is also a graduate of and mentor teacher in our program. We launch the mathematics methods course with four full days of cycles of learning mathematics, planning activities to teach, teaching mathematics activities to fifth graders, and debriefing afterwards. The BPSTs teach the students in Spanish, so much of our planning time is in Spanish as well as English.

It was my hope that by selecting a task to try out with fifth graders, and engaging the BPSTs in how to implement the lesson, I would support the BPSTs to engage in aspects of teaching mathematics for social justice.

Reflections on the Teacher Educator Planning Process

I came to the decision to have BPSTs teach the Maps task (Gutstein, 2013) through a reflective process. In this section, I will share with the reader how I engaged in a sense-making process around how to select the task and decide to give it time in the course. Though my thoughts on this continue to evolve, I offer these reflections on where I was in that moment in time.

Why did I incorporate this kind of task? The answer to this question is deeply entwined with who

[1] This is typical in California, where the credential is a post-baccalaureate degree.

I am as a mathematician, a teacher, and my commitments to Latin@ students, adults and children. When I plan the methods courses, I typically engage equity issues through discussions about identity, culture, and status in the classroom. I often have students read examples of teaching mathematics for social justice from the *Rethinking Mathematics* book. I realized in my third year as an assistant professor, supporting my students to teach a critical mathematics lesson was missing from my syllabus. I think this is because I had extensive and excellent preparation in how to support general populations of elementary pre-service teachers (PSTs) to learn how to enact mathematical practices that engage students in sense-making and problem solving, but not as much training in how to support PSTs to design and implement tasks that help their students understand and act on a social justice issue. Further, I often felt like I had to privilege content knowledge because within the bilingual program many of our BPSTs had not had adequate opportunity to learn enough mathematics to be able to teach it in deep conceptual ways. My commitment to transformative education was often trumped by my commitment to help the Latin@ future teachers learn more mathematics to boost their teaching confidence, so that they could effectively teach foundational mathematics concepts to future generations of Latin@ students.

In my methods classes, I was not privileging teaching for social justice in the way I was privileging teaching through problem solving (Van de Walle, Karp, and Bay-Williams, 2015), and attending to status issues in the classroom (Featherstone et al, 2011), in part because without a comprehensive curriculum of critical mathematics tasks teachers must create it as they go. For that reason, I often encouraged my students but did not require that they incorporate social justice into their final mini-unit planning assignment.

I find inspiration in mentors and other educators who support their students in infusing social justice into a lesson that they planned as part of their teacher preparation coursework (Aguirre, 2009; Koestler, 2012). The field experience with my bilingual cohort further provided the opportunity to go beyond planning and have them actually teach the lesson. The research on novice teacher learning in which PSTs enact and reflect on their teaching within methods courses suggests that PSTs are better prepared to try out complex tasks in their own classrooms when they have been supported to try out similar versions with a coach or with the support of classmates (Lampert, Franke, Kazemi, et al., 2013). Based on this research, I decided to take two of the four days with fifth-grade students to have the BPSTs lead a mathematics lesson connected to social justice. Reformatting my class time to privilege experiencing and implementing a social justice mathematics task was a key step in my development as a critical educator.

Why am *I* selecting the task, if building agency is part of a social justice pedagogy? As Gutstein (2007) wrote, a main goal of mathematics and social justice is for students to develop agency with mathematics, to be able to generate their own questions and also take actions to solve issues. It is therefore not lost on me that even while building my BPSTs' agency was a goal, our first few forays into teaching mathematics needed my guidance. To me, this meant bringing a task to the BPSTs, letting them experience and analyze it as learners, and then allow for agency in how they worked through it with the fifth grade students. I also needed to take the long view and see it as one small step in a long road of development in their careers as teachers. Therefore I selected the task and gave them the responsibility of planning how to teach it.

A big question in this research is whether or not mathematics for social justice can truly be taught when there is no or little relationship between students and teachers. This continues to be a big question for me. I did not doubt my ability to engage the BPSTs in this task having begun building rapport the prior semester, but I did wonder what BPSTs would be able to accomplish and understand about this lesson given that they would have just spent a total of two hours with the students the prior week. Would I be perpetuating a new bad practice that as a teacher you can bring any activity that you think is about social justice and expect buy-in from your students? Such a stance seems to fly in the face of teaching for freedom, if the teacher has so much control and so little established relationships. Then again, could we claim that students simply would not engage in any kind of task before teachers have established a deep relationship? It seems like either

extreme holds little explanatory power about learning. Even though one of our first-day tasks was a "get to know you task" with the children in a circle and they worked with the same 7-9 children each day, BPSTs did not have the relationship or rapport that Gustein (2006) and others describe as a key piece of engaging in the dialectic praxis of teaching for social justice. This is another reason why this activity must be described as a facsimile, an approximation of a practice in teaching much like the rehearsals described in the work of Lampert et al. (2013). It is the right size to allow the teachers to practice implementing a complex task, but lacks deep relationships between students and teacher. Although I did have that concern, I argue that the activity was useful to help BPSTs develop their own understanding of how an activity such as this one could work, and create a vision for how to implement such tasks with their own students in the future, who they will know in deeper ways.

BPSTs Learn About and Plan the Map Task

I ultimately decided to modify the Gutstein (2013) Map Task for fifth graders. The use of visual aids was a strong point in this task, giving both teacher and students physical materials to manipulate and reference. The mathematics of ratios was just out of reach for most of the fifth graders, but they all had a lot of experience with division by this time. I also thought that even the students who could not quantify difference in representation of landmass could cut out and hold up two different representations of the same landmass (Brazil, for example) and experience the difference in sizes to draw conclusions about difference in representation (i.e., Brazil in the Peters projection would be bigger than in the Mercator given the same sized map). Also, the task could culminate in asking students to say which map they thought was more fair and why, drawing on their personal sense of what is fair. I ran the ideas for the task by my fifth grade partner teacher. He was enthusiastic about it, adding that it had multiple entry and exit points, meaning that students could start to tackle it in different ways, and that the fifth graders may get different but equally valuable lessons from it.

To launch the activity in the methods class, I first located a short video on Youtube[2] that showed various map projections being morphed from the globe. Each video highlighted how some part of the globe is distorted to shift from the surface area of a perfect sphere to a rectangular shape. I also peeled an orange and asked if anyone thought they could arrange the peels into a perfect rectangle. Everyone seemed to understand that distortion was inescapable when mapping the globe onto a two dimensional rectangular map.

I then introduced the task by having the BPSTs think-pair-share differences between the two projections of the maps: the Mercator and the Peters. Their observations ranged from beginning to quantify differences in sizes of continents to reflecting on how they had never seen or only rarely seen a Peters map, questions arose, "*why is that? Who makes Mercator maps, and why are they more prevalent in classrooms?"* The BPSTs not only enjoyed the activity, they immediately saw that there was a relevant issue of fairness in the activity. From there, I supported their transition to using mathematical arguments to make claims about the differences in "fairness" that they were noticing. They used the tracing graph paper to trace shapes of various pairs of countries on the Mercator and Peters projections, and recorded these areas (using graph paper squares as the unit of measure) on a t-chart. Then they calculated the ratios between graph paper landmasses and found how they compared to actual ratios between landmasses, which they researched using their phones and laptops. This also brought up a discussion for us about what sources were reliable or better on the internet, including which reported in square kilometers versus square miles and whether one was more appropriate for the classes we were working with.

Many BPSTs struggled with the concept of ratio in this context. Some of them were not always sure what the answers to the divisions they were doing meant. I facilitated discussions in groups about how to phrase what the results of their calculations meant

[2] From the PBS show Life by the Numbers, we watched the animated portions of this clip https://youtu.be/X4wgFSHZXBg?list=PLSfHj8toBl183XJbJkqLSNWhZroSC4HGl

(e.g., "If you are comparing New Zealand to Brazil, and your calculation gives you 1/32, then this means Brazil is 32 times bigger than New Zealand."). They also were not always sure what having two similar ratios, between the same two countries on each projection, should mean. Mathematically, this should mean that those landmasses are less distorted on the Mercator map, as is the case with countries near the equator. In our discussions of how to calculate relative size of one country's area to the other, we discussed all these issues. I concentrated on facilitating talk by probing thinking, eliciting ideas, and validating contributions made with words, partially formed ideas, gestures, etc. I looked for many ways to validate thinking.

As we talked, many BPSTs took notes on how to calculate the ratio of two landmasses and what the ratio meant, and how to express mathematical relationships using Spanish academic language, for example: "____ es _____ veces mas grande que _____ en el mapa Mercator, " as a sentence frame for "___ is __ times bigger than _____ on the Mercator map." They wanted to ensure they had noted the key mathematical talking points for discussions with the fifth graders, and to be ready to provide academic language support during the lesson. To me, this also signaled that they were taking the mathematical ideas in this task seriously, and knew that the value of the task was both about how students would experience the maps differently (ex. though Mexico is smaller than the United States, it appears even smaller on the Mercator than the Peters map), but also how they would express mathematical evidence about how they were different (ex. "Mexico appears to be about 12 times smaller than the total landmass of the US on the Mercator, but is only 6 times smaller on the Peters."). This was also important for me to witness, since I was not sure how the activity was going to be received, and if it would pose any challenges given their wide range of confidence in their own mathematics skills.

At some point in the learning portion, I found a reasonable place for us to pause, reflect on what we had figured out for ourselves, and to make a decision about the kind of guidance to give the pre-service teachers as they headed into planning with their teaching partners. I needed to ensure everyone felt comfortable with the mathematics of the task, but also encouraged them to not get bogged down in the calculations for this activity but to focus on using mathematical evidence to support arguments for which map was more fair. In retrospect, I felt a strong urge to give them a pep talk, since this kind of activity situated within a mathematics lesson was new to them, and new to the fifth graders, and I wanted them to go into it with energy and excitement even if they were not perfectly clear in their plan for implementation.

BPSTs Lead the Task with 5th Graders

There are many layers of learning on which to reflect in my role as the teacher educator engaging in on the spot observation, coaching, and support of BPSTs working with fifth graders. Here I will share some reflections on what I was learning about the BPSTs in this process.

On the first day, I noticed a marked difference in how various groups of BPSTs implemented the activity with students. It seems like some BPSTs were able to keep the activity open and problem-solving oriented, even while they scaffolded technical skills like expressing amounts as ratios, whereas other BPSTs implemented it procedurally with little input from the students. Watching this unfold I reflected more deeply about unpacking the pedagogies that support the task that is to be implemented in rich ways for students. Even though during planning I had encouraged every group to err on the side of letting the children explore, this seemed to not fit with some BPSTs ideas of what it meant to teach mathematics, even with an open-ended context and having experienced this task as an exploration themselves.

After the first day, the BPSTs reflected and regrouped. Many thought the activity had been challenging to implement. Still, the BPSTs went into the planning phase the next day with fresh energy. Some decided to re-launch, and completely start the activity again. Others decided to use student work as the launching point into the second day. Together the class generated two goals for the day that felt reasonable given where each group had ended the prior day: *discuss what mathematical evidence fifth*

graders found in their exploration, and support the fifth graders to write one or two sentences answering which map they thought was more fair and why. By helping the BPSTs establish these objectives, I can reflect that I was trying to give BPSTs concrete goals from which to organize their approaches. I did not want them to feel lost about the multiple directions the task could go in, so we put some structure around it to have an end goal.

To summarize the second day, more fifth graders were doing focused work than the first day. It seems the one-day familiarity and returning to the task was good for them too, not just the BPSTs. However, BPSTs still noticed and reflected on the challenges of the task. In order to reflect on the teaching in a debrief in the last 30 minutes of the methods class, after an intensive two days with this activity, I asked them to write down simply: "what worked" and "what did not work" today. Some BPSTs wrote about the activity over all, covering the two days, and other noted specifically what came out of just this day. The table below summarizes ideas from the BPSTs' reflections at the end of the second day.

Worked
Talking about the maps, students able to identify many differences
Using talk moves to get a variety of students involved in whole class discussion (revoicing, repeating, explaining to their peer)
Reviewing what we did the prior day, going from there
Everyone understood that the two maps were different, but were of the same landmasses (ie countries, continents)
Didn't Work
Too long - this lesson seems like it needs more than two days!
Not having calculators - the calculation took too long since we did not have enough calculators for our group.
Students' reflections didn't all contain math
Too many new concepts at once for most kids

Table 1
BPSTs' Reflections on What Did and Did Not Work Implementing the Map Task

A variety of reflections around what fifth graders noticed or wrote emerged in the BPSTs' reflections, which exposed complexities the BPSTs had to manage that may not have emerged with a more mainstream mathematics task. One BPST noted that it was interesting how a few fifth graders used mathematics to show that the Peters was indeed more accurate to landmass, but had concluded in a closing group discussion that the Mercator must be the better map because it was in all the classrooms. Another BPST noticed that one fifth grader wrote in her exit ticket, "Yo pienso que los dos mapas son incorrectos, porque nadie puede replicar el mundo actual. Los dos tienen sus imperfecciones / *I think that the two maps are incorrect, because nobody can replicate the real world. The two maps each have their imperfections."* It seems this type of comment was a theme, where fifth graders hesitated to say a particular map was better.

Some BPSTs discussed the importance of a classroom culture in which students can make arguments to defend their position, along with the importance of normalizing these kinds of math activities. The discussion deepened further by leading us to think about how non-mathematical ideas can be valued alongside the mathematical ideas in an activity like this one, in some ways bringing our two-day experience full circle. By returning to the original idea of shifting what ordinarily counts as doing mathematics to including incorporating ideas of fairness, we had come back to our starting points of discussions about mathematical tasks that draw on students' sense of justice as well as mathematics.

Final Thoughts on Growing as a Critical Mathematics Educator

Given how BPSTs focused the lesson debrief on what did not work, I was surprised to hear from most of the BPSTs that this activity was worthwhile, and that activities like the Gutstein Map task needed to be implemented in classrooms more. Some BPSTs offered that despite difficulties this was more meaningful mathematics than they remember learning, and wanted to keep learning ways to design these kinds of activities and make them work in their own classrooms. It seems that an important lesson for me, as the mathematics educator, is that even as BPSTs were critical of their own performance they understood the value of tasks that bring social justice into mathematics, and the value in the process of

thinking through together how to implement such tasks. Feedback of this kind from my students keeps me reflecting on the value of such experiences, even if they are small snapshots of social justice oriented tasks and not embedded deeply in transformative pedagogy. Similar to the research on novice teacher learning of rehearsals of practice (Lampert et al, 2012), it seems that these mini facsimiles of social justice mathematics practice may be useful to give BPSTs the opportunity to envision, enact, and trouble-shoot a teaching practice, so that they can have a stronger starting point to try out such tasks in their own classrooms. However, I conclude that I need to support BPSTs better in seeing that task implementation needs to be situated within pedagogies that focus on developing student agency, voice, and understanding. It is one thing to model how such tasks could work, but it is quite another to unpack it, and to connect it to one's own sense of teaching for liberation. Shifting curriculum is a good starting point, but developing a concrete notion of mathematical pedagogies for freedom is perhaps a longer, more involved process and I am somewhere on that trajectory in my own learning.

As I shift the content of what counts as learning to teach mathematics in my methods courses, I must also shift what it means to show progress in teaching mathematics for freedom. The BPSTs had to let go of some of the control that is normally expected in traditional curricula, and they had to trust that students could generate ideas that were worthwhile. They also had to develop practices that attend to mathematical thinking side-by-side with critical literacy knowledge. This practice stretched the BPSTs in new ways to integrate multiple knowledge bases into a mathematics lesson. I have to be aware of the ways in which the BPSTs are challenged and can rise to those challenges in a task like this. I need to understand how they are growing not just as mathematics teachers, but as critical mathematics teachers.

I also think that I have turned a corner in my own philosophy of teaching. If I am going to *say* something is important, I am going to *model how that value needs to be a priority*, and I am going to *try it out* with BPSTs and children so they can see how it works. I am going to attend to my BPSTs' growth as transformative educators using multiple markers that help me attune to who they are in relation to mathematics, what teaching mathematics for social justice means, and what pedagogies for transformation in the mathematics classroom are from their perspectives. As I grow in this role, I grow as not just a mathematics educator, but a *critical* mathematics educator, *educando para transformar, a mí mismo y a mis aspirantes*.

References

Aguirre, J.M. (2009). Privileging mathematics and equity in teacher education: Framework, counter-resistence strategies, and reflections from a Latina mathematics educator. In B. Greer, S. Mukhopadhyay, A.B. Powell, & S. Nelson-Barber (Eds.), *Culturally Responsive Mathematics Education* (pp. 295-320). New York, NY: Routledge.

Featherstone, H., Crespo, S., Jilk, L., Oslund, J.A., Parks, A.N., & Wood, M.B. (2011). *Smarter together!: Collaboration and equity in the elementary math classroom.* Reston, VA: NCTM.

Flores, B.B., Hernández-Sheets, R. & Riojas Clark, E. (Eds.). (2011). *Teacher Preparation for Bilingual Student Populations: Educar para Transformar*. NY: Routledge.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. New York, NY: CRC Press.

Gutstein, E. (2013). Math, Maps, and Misrepresentation. In E. Gutstein & B. Peterson (Eds.). *Rethinking Mathematics: Teaching Social Justice by the Numbers* (2nd Ed.) (pp 189-194). Milwaukee, WI: Rethinking Schools.

Gutstein, E. & Peterson, B. (Eds.). (2013) *Rethinking Mathematics: Teaching Social Justice by the Numbers (2nd Ed.)*. Milwaukee, WI: Rethinking Schools.

Koestler, C. (2012). Beyond apples, puppy dogs, and ice cream: Preparing teachers to teach mathematics for equity and social justice. In A. Wager & D. Stinson (Eds.). *Teaching Mathematics for Social Justice: Conversations with Educators* (pp. 81-98). Reston, VA: National Council of Teachers of Mathematics.

Lampert, M. Franke M., Kazemi, E., Ghousseini, H.,

Chan Turrou, A., Beasley, H., Cunard, A., Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education, 64*(3), 226-243.

Van de Walle, J.A., Karp, K.S., & Bay-Williams, J.M. (2015). *Elementary and Middle School Mathematics: Teaching Developmentally, (9th Ed.)*. New York, NY: Pearson.

Wager, A.A. & Stinson, D.W. (Eds.). (2012). *Teaching Mathematics for Social Justice: Conversations with Educators.* Reston, VA: National Council of Teachers of Mathematics.