**Equitable Practices in Mathematics Classrooms: Research-Based Recommendations**

**Judit Moschkovich**

**Abstract**

This paper is based on the Iris M. Carl Equity Address the author delivered at the 2012 annual meeting of the National Council of Teachers of Mathematics. That invited keynote considered the question of equitable teaching practices in mathematics classrooms for students from non-dominant communities. Although research cannot provide quick answers to this question nor can it provide a recipe for equitable teaching practices, there are research-based recommendations that can guide researchers, teachers, and administrators in developing their own approaches to supporting equitable practices in mathematics classrooms. Several resources are provided for considering this question: a definition of equity, a definition of equitable practices, a framework for organizing research findings relevant to equitable practices, and questions to consider when designing equitable mathematics instruction. This discussion is informed by a sociocultural and situated perspective on mathematical thinking, on language, and on bilingual learners (for details of that framework, see Moschkovich, 2002, 2010).

**Discussion And Reflection Enhancement (DARE) Pre-Reading Questions**

1. What dimensions would you include in a definition of equity?
2. What recommendations for equity in mathematics classrooms have you come across? Are you aware of research that supports these recommendations?
3. What do you think are characteristics of mathematics classrooms that support academic achievement for students from non-dominant communities?
4. What do you know about mathematics learners who are bilingual or learning English?

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# Equitable Practices in Mathematics Classrooms: Research-Based Recommendations

**Judit Moschkovich**

## Equity

How should we approach (if not define) equity? Gutiérrez (2009; 2012) proposes that there are four dimensions reflected in research addressing equity: access, achievement, identity, and power. In Gutiérrez's view, *access* relates to the tangible resources that students have available to them to participate in mathematics, including quality teachers, adequate technology and supplies, rigorous curriculum, classroom environment that invites participation, reasonable class sizes, tutoring, etc. *Achievement* focuses on tangible results for students at all levels of mathematics. Achievement involves course taking patterns, standardized test scores, and participation in mathematics courses at different academic levels (from elementary to graduate school). Studies focusing on *identity* examine whether students find mathematics meaningful to their lives and have opportunities to draw upon their cultural and linguistic resources (e.g., other languages and dialects, algorithms from other countries, different frames of reference). This dimension pays attention to whose perspectives and practices are valued. The *power* dimension can involve examining voice in the classroom, for example who gets to talk and how contributions are taken up (or not).

One way to summarize this approach to equity is to say that students from non-dominant communities need *access* to curricula, instruction, and teachers shown to be effective in supporting the *academic achievement, identities,* and *practices* of these students. I use the phrase "students from non-dominant communities" not to describe students who are in the majority or minority in terms of numbers, but instead to describe students who are not from the culturally dominant communities (middle-class, white, Anglo, English speaking). This phrase thus refers to poor and working class students; in U.S. schools these students are predominantly students of color and many are English learners. The issue is not numbers (majority or minority) but instead dominant and non-dominant cultural practices (Gutiérrez & Orellana, 2006).

How can curricula, instruction, and teachers support the academic *achievement, identities, and practices* of these students? First, students need access to important mathematics. Curriculum policies should follow the guidelines for traditionally underserved students (AERA, 2006), such as instituting systems that broaden course-taking options and avoiding systems of tracking students that limit their opportunities to learn and delay their exposure to college-preparatory mathematics coursework. Second, students need access to environments that have been documented as supporting the academic achievement of students from non-dominant communities.

The general characteristics of such environments in the United States are that curricula provide "abundant and diverse opportunities for speaking, listening, reading, and writing" and that instruction "encourage students to take risks, construct meaning, and seek reinterpretations of knowledge within compatible social contexts" (García & Gonzalez, 1995, p. 424). And third, students need access to teachers who have been documented as being successful with students from non-dominant communities. Some of the characteristics of such teachers are: a) a high commitment to students' academic success and to student-home communication, b) high expectations for all students, c) the autonomy to change curriculum and instruction to meet the specific needs of students, and d) a rejection of models of their students as intellectually disadvantaged (García & Gonzalez, 1995).

## Equitable Teaching Practices

I define equitable teaching practices for students from non-dominant communities in mathematics classrooms as those practices that: (a) support mathematical reasoning and mathematical discourse---because we know these lead to conceptual understanding and learning mathematics, and (b) broaden participation for students from non-dominant communities---because we know that participation is connected to opportunities to learn.

To support mathematical reasoning, classroom practices need to provide opportunities for students to participate in different kinds of mathematical practices and use multiple resources to do and learn mathematics. To broaden participation, classroom practices need to provide opportunities for students to use multiple resources to participate in classroom work. Equitable classroom practices, then, are fundamentally focused on honoring student resources, in particular the "repertoires of practices" (Gutiérrez & Rogoff, 2003) that students from non-dominant communities bring to the classroom. Lee (2003) argues that we should "neither attribute static qualities to cultural communities nor assume that each individual within such communities shares in similar ways those practices that have evolved over generations (p. 4)."

To avoid reducing cultural practices to individual traits that are static or that all members of a group share, Gutiérrez and Rogoff (2003) propose that we focus not on what an individual does or says but, instead, on what they call "repertoires of practice." These are a collection of multiple and varied practices because learners have access to multiple practices. Any individual is likely to have had multiple experiences with different practices from many different communities, not only their families, but also through their friends, school, sports, mass media, etc. These "repertoires of practice" are not static because individuals develop and communities change.

An example of a linguistic practice that is important in the classroom is intonation. For example, intonation patterns vary, not only across national languages (e.g., English and Spanish), but also among varieties of a national language (e.g., Spanish). In the case of Chicano English:

> "Perhaps the most prominent feature distinguishing Chicano English from other varieties of American English is its use of certain intonation patterns. These intonation patterns often strike other English speakers as uncertain or hesitant" (Finegan & Besnier, 1989, p. 407).

In order to honor the resources that students bring to the classroom, teachers need to learn what practices (cultural, linguistic, mathematical, etc.) are common among students from non-dominant communities, including students who are bilingual, and/or learning English, or use non-dominant language varieties (although these are usually called "dialects," I use the phrase "language varieties" because the label "dialect" can reflect a deficit view of those language varieties). There are many ways to learn about the practices that are common in students' home communities. Getting to know the local communities, attending local events, or visiting students' homes are arguably the best windows into students' lives outside of school. Reading (fiction, non-fiction, books on multicultural approaches to education, articles on social justice approaches to mathematics teaching, etc.) is a more indirect way to learn about students' home practices.

## Framework for Equitable Classroom Practices

To frame the many connections among language, culture, and mathematics learning/teaching, I will use Brenner's (1998) framework for cultural relevance for instruction and curriculum (see also Nelson-Barber & Moschkovich, 2009). This framework identifies three areas central to ensuring that curricula and instructional practice are culturally relevant for students: *cultural content*, *social organization*, and *cognitive resources*. Brenner's three-part framework can be used as a broad guide for designing curricula, instruction, and assessments. The three dimensions can be used to organize the results of relevant research.

**1. CULTURAL CONTENT**
- Do mathematical activities connect to those in local community?

**2. SOCIAL ORGANIZATION**
- Do classroom practices facilitate comfortable and productive student participation?
- Do classroom practices fit with learners' communication practices in home/community?

**3. COGNITIVE RESOURCES**
- Does instruction enable children to use their prior knowledge and experiences as resources?

*Figure 1.* Dimensions for equitable classroom practices (Brenner, 1998).

Brenner's (1998) framework includes the following questions: Do mathematical activities connect to those in local community? Do classroom practices facilitate comfortable and productive student participation? Do roles and responsibilities fit with learners' communication practices? Does instruction enable children to build on their existing knowledge and experiences as resources? These questions for each dimension are useful for considering the complexity in what constitutes comfortable and productive participation for learners, as well as the multiple communication practices that students have experienced, both at home and in school.

As Brenner sees it, examining materials and instructional techniques for their *cultural content* can reveal the extent to which mathematical activities utilized in instruction relate to mathematical activities operating in local community practices, no matter what communities students come from. Similarly, ensuring that classroom *social organization* takes into account a variety of possible roles, responsibilities, and communication styles and includes multiple and hybrid repertoires of practice (Gutiérrez & Rogoff, 2003) will more likely support comfortable and productive student participation.

Classrooms that make use of the *cognitive resources* students bring from previous instruction and from home—a variety of ways of thinking used in their communities to solve problems—make the most of students' existing knowledge and lived experiences (Moll & González, 2004). Language is one such cognitive resource. Teachers' ability to recognize and appreciate students' particular cognitive resources ultimately has a bearing on how they interpret student talk and activity in the classroom.

### Connecting Mathematics to Local Communities

The central question for this dimension is whether mathematical activities in the classroom connect to the local community. Connecting school mathematics with children's own experiences and intuitive knowledge has been an important theme in efforts to improve formal mathematics instruction (e.g., Lipka, Webster, & Yanez, 2005; Trumbull, Nelson-Barber & Mitchell, 2002). Several projects in mathematics education have focused on documenting community mathematical activities in different settings. For example, publications from the following projects provide the details of mathematical activities in different communities: "Funds of Knowledge" (Civil, 2002, 2007; González et al., 2001), "El Mercado" (Fuson et al., 1997), and work in Alaska (Lipka, 1998; Lipka & Adams, 2004; Lipka, Webster, & Yanez, 2005) documented local mathematical activities. Even when teachers are working in communities where researchers have not yet documented the local mathematical activities, these publications provide ways to learn about students and their communities through home visits, reports form students, conversations with parents, and other approaches (González et al., 2001). Work in mathematics for social justice (Gutstein, 2003; Gutstein & Peterson, 2005; Powell & Frankenstein, 1997) also provides mathematical tasks that can be readily connected to students and their communities.

When working with students who are immigrants it is important to consider differences in symbols and algorithms (Orey, 2004; Perkins & Flores, 2002; Secada, 1983). For example, in some countries a period is used for marking the thousands place, not for decimals as in the United States (writing 1.234 instead of 1,234), and the comma is used to mark decimals (writing 10,03 not 10.03). Mathematics educators have also documented algorithms common among immigrant students, for example the "Rule of three" or "Regla de tres" to solve proportion problems, and several different approaches to long division (Civil & Planas, 2010; Corey, 2004; Perkins & Flores, 2002):

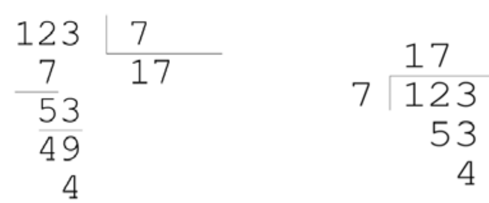


*Figure 2.* Alternative algorithms for dividing 123 by 7.

### Social Organization of Classroom Practices

The central question for this dimension is whether classroom practices facilitate comfortable and productive student participation and fit (as much as possible) with learners' communication practices at home or in their communities. To address this dimension, teachers need to understand children's home language practices. Teachers can learn to value and build on student's linguistic skills while also explicitly modelling the discourse styles expected in school. The rules about who can talk when, about what, and how, and communication routines are established in every classroom. The practice of incorporating students' own ways of using language into the classroom is recognized as one aspect of the success of some classrooms. For example, one successful approach to integrating community language practices that resulted in gains in reading scores is the Kamehameha school integration of "talk story" style of overlapping participation into native Hawaiian children's classrooms (Au, 1980). Another example is Lee's work with African-American high school students' ways of talking (Lee, 1993).

The question to ask about language practices in the classroom is whether a classroom facilitates participation for students from non-dominant communities in terms of the roles, responsibilities, and styles of learners' communication practices. Answering this question means having substantial information about and deep understanding of children's home practices and the local community (Moschkovich & Nelson-Barber, 2009). This entails knowing not only local activities that may be used in the mathematics classroom but also students' language practices at home and other community settings. It is important to remember that there may be differences between home and school participation structures. For example, a participation structure common in many homes of students from traditional communities, "intent participation," is a style that involves lots of watching and little talk (Rogoff, Paradise, Arauz, Correa-Chavez, & Angelillo, 2003), in contrast to school instruction that involves large amounts of talk.

What are typical communication practices for students who use two languages? Common practices among mathematics students who are bilingual or learning English include using arithmetic facts in first language, doing arithmetic computation in their first language and then translating the answer, and code-switching, using two languages during one conversation. The social organization of the classroom should include these language practices and these practices should be seen as cognitive resources for doing mathematics in the classroom.

### Children's Prior Knowledge and Experiences as Cognitive Resources

The central question for this dimension is whether instruction enables children to build on their prior knowledge and experiences as resources for mathematical reasoning. There are many different types of cognitive resources. There should be many opportunities for students to participate in mathematical talk in multiple ways. But talk should not be the only resource: students should also have opportunities to draw flexibly on multiple resources, such as drawings, written text, mathematical representations, gestures (Fernandes & McLeman, 2012; Moschkovich, 2002), and manipulative objects, etc. As described above, instruction should support students in using multiple languages and dialects, as well as express their mathematical thinking in everyday ways. Other cognitive resources include stories (for example in story problems) and physical activity (using a motion detector, or walking on a number line).

### Equitable Practices for English Learners

Although it is difficult to make generalizations about the instructional needs of all students who are learning English, research suggests that high-quality instruction for English Learners (ELs) that supports student achievement has two general characteristics: a view of language as a resource rather than a deficiency, and an emphasis on academic achievement, not only on learning English (Gándara & Contreras, 2009). Mathematics teachers who work with ELs need to know some things that are specific to their students. They also need to be aware of mathematics notation in other countries. Lastly, they need to know some things about language in general and about bilingualism in particular.

First, mathematics instruction should be informed by knowledge of students' experiences with mathematics instruction, language history, and educational background (Moschkovich, 2010). Teachers need to know the details of a student's history with formal schooling, for example which grades they attended, where, and in what language (or languages). They should have some information about their language history, for example are they literate in their home language, what is their reading and writing competence in the home language. Some students may not have had any formal instruction in the language spoken at home. Another important information is the students' history with school mathematics instruction: when they had mathematics classes, in what language, and for which topics.

Mathematics teachers who work with ELs also need to know a few things about language and bilingualism. A few assumptions about language that come from research in linguistics (for more details, see Wong-Fillmore & Snow, 2000) include the following: a) language involves meaning, action, purpose, and discourse practices (not just vocabulary or single words); b) we learn language by using it to communicate (rather than by memorizing definitions and lists of words), and c) learning a second language is long-term process (at least several years).

Teachers also need to be familiar with the findings from current research on bilingual mathematics learners (for a short summary of this research, see Moschkovich, 2009; for a longer version, see Moschkovich, 2007b). Native-like control of two or more languages is an unrealistic definition of bilingualism that does not reflect evidence that the majority of bilinguals are rarely equally fluent in both languages. Teachers need to know and build on the fluencies their students bring rather than comparing bilinguals to monolinguals or focus on how bilingual students miss the mark in comparison to monolinguals. Because bilinguals have a wide range of proficiencies in two languages, teachers should not expect mathematics students to know mathematical terms in a first or second language unless they have had mathematics instruction in that language. Bilinguals have a wide range of proficiencies in modes (listening, writing, speaking, and reading) in their two languages. Teachers should not assume that proficiency in one mode implies proficiency in another mode and should provide mathematics assessment and instruction across all modes. Switching languages is not a sign of a deficiency. In fact, this skill is a complex cognitive and linguistic resource (Moschkovich, 2007a, 2007b, 2009; Valdés-Fallis, 1978; Zentella, 1981). Teachers should not imagine that switching languages is related to mathematical thinking or understanding in any simple way.

## In Closing

There are many ways to define equitable practices in mathematics classrooms. I am certain that the definition and framing I have provided here leave out important aspects and work that is relevant. However, my intention was not to provide the perfect definition, but instead to establish some common ground. It is my sincere hope that the resources I provided here prove useful for designing equitable mathematics instruction.

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[1] Editors' Note: A second edition of Gutstein and Peterson's *Rethinking mathematics: Teaching social justice by the numbers* has been released in 2013 that includes the first edition material plus roughly the same amount of new material.

of Teachers of Mathematics, available at http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/Research_brief_12_Using_2.pdf

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**Discussion And Reflection Enhancement (DARE) Post-Reading Questions**

1. Consider the four dimensions of equity (access, achievement, identity, and power) and discuss how each plays out in your classroom or in your school.
2. Consider the three dimensions for equitable practices: cultural content, social organization, and cognitive resources. Discuss how these play out in your classroom and what you could do to improve one of these dimensions in your teaching.
3. What teaching characteristics are successful with students from non-dominant communities? Which characteristic do you think is the most essential and why? Which characteristic do you think is the most challenging and why?
4. What is an example of an alternative algorithm or notation that is important for teachers to recognize as equally valid if they see their students using it?
5. Did any of the claims about how bilingual mathematics learners use language surprise you? What could you do to learn more about bilingual learners?
6. Describe three things you could do to learn more about the cultural, linguistic, and mathematical practices of the students in your classrooms.

"DARE to Reach ALL Students!" TODOS Mathematics for ALL

## *Acknowledgement of TEEM Reviewers*

We thank these reviewers for their service during the period January 2012-July 2013 (since the previous listing of referees published in the third issue of *TEEM*), and we are always interested in having more people available to review (normally one manuscript per year). If willing, please email teem@todos-math.org with your (1) contact information, (2) grade levels of teaching or research experience, and (3) thematic areas of greatest interest and expertise. And if there is someone you would like to nominate, please relay the necessary information for the editors to be able to send an invitation.

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## "DARE to Share!" - Tell us how you've used *TEEM* articles! - Invitation to Readers

*TEEM* invites readers to write in with experiences they have had applying or extending the articles they see in the journal, and we may publish them in the journal or its website.

For example, while preparing a breakout workshop on culturally relevant mathematics for the 2013 NCTM High School Institute, *TEEM* co-editor Larry Lesser reread Olga Ramirez and Cherie McCollough's paper "La Lotería: Using a Culturally Relevant Mathematics Activity with Pre-service Teachers at a Family Math Learning Event" from the fall 2012 TEEM issue *4*(1), 24-33 and was inspired to write these additional mathematics questions (and solutions) for the context of the board game "La Lotería."

1. What's the smallest number of cards (out of 54) that the dealer could call before your 4x4 board MUST win? (*Hint:* first consider what is the largest number of uncovered spaces your board could have where you don't have a win yet, but the very next card called MUST give you a win)
2. What's the probability that neither of the first two cards called are on your 4x4 board?
3. What's the probability that you have a win after the dealer calls *exactly* 4 cards?

**Solutions:**

1. Suppose 12 cards have been called that are on your game board, as shown by X's:

| | | | |
|---|---|---|---|
| X | X | X | |
| | X | X | X |
| X | X | | X |
| X | | X | X |

   As you can see, it is possible to have this many spaces covered and yet not have one of the winning combinations (of Figure 1 in Ramirez and McCollough, 2012). Assume the 4 uncovered spaces on your game board are the only 4 cards that have not yet been called in the deck of 54 cards. Then, this means that 50 cards have been called so far, and the very next card must produce a win, and so the answer to the question is 51.

2. The probability that the first card IS on your board is 16/54, so its complement is 1 – (16/54) = 38/54. Since cards are drawn independently without replacement, the probability that both the first and second cards are not on your board is (38/54)*(37/53), which is approximately .49.
3. The probability of this (very unlikely) event can be obtained in more than one way. Ramirez and McCollough (2012, pp. 26-27) enumerate the 12 ways a person can win and so we can divide 12 by the total number of ways the dealer can choose 4 cards from 54. In other words, 12/ C(54,4), and this is less than 1 in 26,000. Another way to look at it is to find the probability that the first four cards called by the dealer happen to all be on the player's game board, and then multiply that answer by the probability that set of 4 cards happens to be one of the 12 ways of winning. And so, we obtain (16/54)(15/53)(14/52)(13/51)(12/ C(16,4)), which yields the same (tiny) answer!