Teaching Mathematics to English Language Learners Using Robert Moses' Five-Step Approach

Ruth Ahn, Ji Yeong I, and Robin T. Wilson

Abstract

An eight-week summer intervention program in a low-performing middle school in Southern California applied Robert Moses' Five-Step Approach outlined in Moses & Cobb (2001). The Teachers Radically Enhancing Education (T.R.E.E.) Project brought hands-on, experiential mathematics teaching to 20 Latino English Language Learners who failed one or more courses in the previous academic year. At the end of the eight-week program, the 20 students showed improvement in mathematical performance and behavior. An original activity plan created by the participating pre-service teachers based on the Five-Step Approach is included.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions:

1. As a teacher, what are major challenges you have experienced in teaching mathematics to students from culturally and linguistically diverse backgrounds? Describe what has (and what has not) worked well in addressing these challenges.

2. Which mathematical concepts are most frequently emphasized in K-8 mathematics standards? With which of those mathematical concepts do you think students have most trouble? Why do you think that is the case?

Ruth Ahn (ruthahn@csupomona.edu) is an Assistant Professor of Education at California State Polytechnic University, Pomona. Her research interests include preparing teachers to teach culturally and linguistically diverse populations and student-centered teaching and learning.

Ji Yeong I (jihtc@mail.missouri.edu) is a mathematics education Ph.D. student at the University of Missouri. She taught mathematics and science at private and public high schools in California.

Robin T. Wilson (robinwilson@csupomona.edu) is an Associate Professor in the Department of Mathematics and Statistics at California State Polytechnic University, Pomona. His research interests include knot theory, 3-manifold topology, and mathematics education.

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In response to achievement disparities of ethnic and linguistic minority K-12 students (NCES, 2009), a pre-service teacher learning community, Teachers Radically Enhancing Education (T.R.E.E.) Project, was created at a university in Southern California to prepare pre-service teachers to teach abstract mathematical concepts effectively to diverse students. Among multiple frameworks used for this project, Moses' Five-Step Approach (Moses & Cobb, 2001) became the guiding framework in creating various activity plans based on sixth-grade essential mathematical concepts identified by the National Council of Teachers of Mathematics (2006) and the California Mathematics Standards (California State Board of Education, 1997).

This method demonstrates experiential learning, in which students experience concepts through familiar physical events before learning academic jargon and abstract symbols (see Figure 1). This approach reflects the ideas of the experiential learning model associated with Dewey (1938) and grounded in Quine's (1990) idea that scientific language comes from a regimentation of our ordinary discourse (Dubinsky & Wilson, under review). Taking students through the process of having a common experience, to discussing the experience in everyday language, then "mathematizing" the discourse gives all students an opportunity to use the experience as a frame of reference to engage in discussion about mathematical concepts, regardless of their previous background.

The goal of the Five-Step Approach is for the students to be guided by the instructor through the process of mathematizing the experience. As explicated by Dubinsky and Moses (2011), the first step in Moses' Five-Step Approach consists of a concrete participatory experience for students. In the second step, students draw their own pictures of the event. During this step, teachers help students identify the important features of the experience that they will need in the process of mathematizing the event.

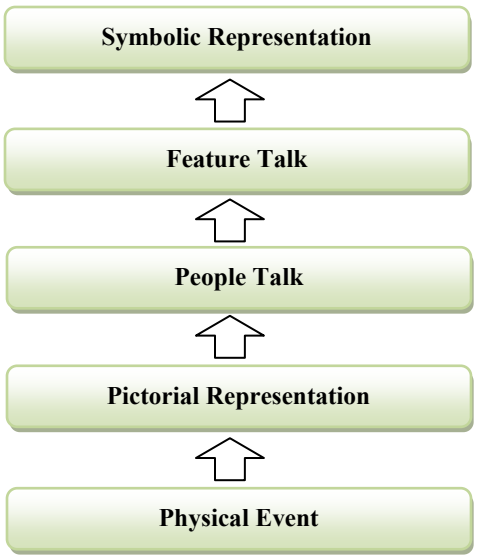


Figure 1. Robert Moses' Five-Step Approach

The third step ("people-talk") involves talking about the event and the important features with the students with everyday language that the students find familiar. The fourth step ("feature-talk") involves students moving from talking about the experience in ordinary discourse to using a more mathematical language. The fifth (final) step involves introducing the iconic representations for the mathematical symbols to complete the mathematization of the experience (Dubinsky & Moses, 2011).

Support for ELLs

The T.R.E.E. Project focused specifically on the effectiveness of Moses' Five-Step Approach with teaching mathematics to English Language Learners (ELLs). The Five-Step Approach may be a promising tool for ELLs because the process invites all students to first create their own symbolism and language to describe the mathematical objects and concepts involved in the lesson, and by doing this it gives ELLs ownership over the formal mathematical language and symbolism.

In the first step, physical events not only engage students but also support ELLs by creating multiple access points in addition to oral explanation for students to use to develop their understanding of abstract concepts. In the second step, ELLs can discuss the mathematics of the experience without formal mathematical language and symbolism (that will be introduced in steps 4 and 5). This step lowers ELLs' affective filter and offers a non-threatening opportunity to check if they are on the right track before they actually speak out.

Teachers may want to proceed to the third step in a small group setting so ELLs may participate without the added stress of speaking in public and have the opportunity to develop both their English skills and mathematical understanding by interacting with peers (Garrison & Mora, 2005). Furthermore, students' talking in everyday language or even in their first language may help teachers decide when formal mathematical terms should be introduced or connected to ordinary language.

In the fourth step, students learn the formal mathematical language for the concepts and objects involved. Therefore, the scaffolding of academic language that is implicit in the Five-Step Approach is well-suited for not only ELLs but also native speakers that struggle with the barriers of the language of mathematics. The fifth step comes after students go through the other steps by doing, drawing, and talking. This step is to guide students through the process of transitioning their acquired knowledge into a formal mathematical symbolism. This is the step where students are introduced formally to mathematical language and symbols such as +, =, or (-5) + (-2) to connect with what they experienced in the first four steps.

Teachers may fail to utilize classroom activities effectively if they believe simply conducting activities directly leads to students grasping the target concepts (Ahn, I, & Walker, under review). Jumping to the mathematical statement without proposing sufficient scaffolding and connections misguides students although they may have fun doing the activity. An appropriate sequence of scaffoldings is crucial when physical activities are used in the classroom. In this sense, the multiple steps of scaffolding in the Five-Step Approach establish a solid understanding and a safe learning environment, particularly for ELLs.

As an illustration of the five steps of Figure 1, consider this activity sequence for teaching fractions: (1) teachers brought pies of the same size and had student cut them into differing number of pieces; (2) students drew about their experience on cutting pies; (3) teachers talked with students about which pieces were bigger than others and how to share the pieces equally with others; (4) teachers had students compare and combine slices to explore the notion of "equivalent fractions" and queried which fraction representations were simplest. To introduce the concept of "equivalent fractions," teachers drew from the language that students brought up during the third step and such as "same," "equal," or "similar"; and finally, (5) teachers asked students to write and draw about these equivalent fractions in their journal.

The following sections present an original activity plan based on Moses' Five-Step Approach in teaching mathematics, created by the participating pre-service teachers. In addition, we will briefly discuss the project's results after the eight-week summer intervention program in sixth grade ELLs' understanding of basic fundamental mathematical concepts. It is beyond the scope of this paper to discuss the effect the T.R.E.E. Project had on pre-service teachers' thinking and teaching, but this is addressed by Ahn, I, & Walker (under review).

Experiential Learning in Mathematics

In applying this framework, we deliberately tapped into multi-sensory methods such as visual, auditory, kinesthetic, and tactile (VAKT) approaches (Rose & Zirkel, 2007), embedding them in the hands-on experiences of Steps 1 and 2. An emerging body of research has focused on the importance of providing hands-on, experiential learning opportunities that use VAKT approaches when teaching abstract mathematical concepts (Garrison & Mora, 2005; Gutstein, Lipman, Hernández, & Reyes, 1997; Lo Cicero, Fuson, & Allexaht-Snider, 2005; McLaughlin and Talbert, 2001; Ricks, 2010). Boakes (2009) discussed the use of tactile activities -- namely, origami -- as a teaching tool in strengthening spatial visualization skills and building general geometric understanding among 56 seventh graders in the U.S.. After instruction, students in the experimental group showed significant differences from control groups on card rotation tests. Boakes further explained that while gains in geometry knowledge were

similar between the two groups, the results showed potential benefits to teaching mathematics by the use of paper folding.

Similarly, Pearn (2007) explained the use of paper folding in conjunction with fraction walls and number lines when teaching fractions to fifth through eighth grade students in Australia. Here, as students physically folded paper strips in response to fraction questions, teachers guided them to talk about how they came to their folding decisions as they worked on the problems. Next, students compared each other's strips on the fraction wall and the number line. These visual and tactile dimensions of experience helped students understand the relationship between fractions and their location on the number line while developing mathematical vocabulary. Furner (2009) discussed an approach to engage ELLs from Central America, by having them create base-20 counters with corn kernels and popsicle sticks as an aid in representing numbers in the ancient Mayan style, and deepening understanding of place value.

T.R.E.E. Project

Design

The Project was designed in response to regional and statewide need where schools with over 50% ELLs and 90% Latinos are not uncommon. The project aimed to transform pre-service teachers' thinking about how to effectively teach ethnically and linguistically diverse learners within a pre-service teacher learning community by applying Moses' framework. The T.R.E.E. Project consisted of 10 pre-service teachers, the researchers (first and third authors), and the lead teacher (second author). Under the close guidance of the researchers and lead teacher, during the eight-week summer program, the team met over four hours daily, Monday through Thursday, teaching abstract mathematical concepts to ELLs by applying the Five-Step Approach. All of the teaching integrated VAKT activities: half of the teaching provided kinesthetic learning opportunities outside the classroom, while the other half provided visual and tactile activities inside the classroom, deliberately avoiding a traditional linear "lecture" style. In addition to these teaching hours, the T.R.E.E. Project involved a 30-minute debriefing time after each day where pre-service teachers discussed and made sense of pedagogical and behavioral issues experienced. These discussions continued with expanded opportunities for interaction on an online discussion board.

Participants

Ten pre-service teachers (seven female and three male) enrolled in teacher education courses volunteered (and received from a grant a modest stipend) in this eight-week summer session in which they taught mathematics to 20 sixth-grade ELLs. Of the 10 pre-service teachers, five were Latino/a, three were Asian, one was white and one was biracial (White/Latina). The 20 ELLs (13 girls, 7 boys) who were selected by their school principal to participate in the summer intervention program either received a failing grade in mathematics or scored "Far Below Basic" or "Below Basic" on the state standardized mathematics test. Each student was assigned to a four-student group that was taught by two pre-service teachers. Group selection was based heterogeneously on test scores, grades, and gender. The middle school site was identified based on the principal's willingness to participate in the program and the school's status as a "Program Improvement School" in its fifth year under the No Child Left Behind Act. According to www.cde.ca.gov/ta/ac/sa/ (School Accountability Report Card), 85% of the school's students were Latino, 47% were ELLs, and 79% were socio-economically disadvantaged.

Assessment

Brief pre- and post-tests were given at the beginning of week 1 and end of week 8, respectively, and focused on sixth grade essential mathematical concepts in NCTM (2006): basic operations, negative numbers (integers), fractions (performing operations and writing in simplest form), one-step linear equations, and rate. The structure of questions and directions in both tests followed the sixth-grade California Standards Test (www.startest.org/cst.html) closely in order to bring standardization to the assessment. While the pre-test contained 30 questions, the post-test was reduced to 18 questions, dropping the geometry unit. This decision was made as a result of adapting to students' progress, especially when it was determined that they needed more time to work on other fundamental concepts such as basic operations, negative numbers, and fractions. The remaining questions were identical in concepts and similar in the use of wording and choice of numbers to their corresponding pre-test questions. It is important to note that because many of the students refused to take any kind of test by putting their faces down on

their desks, it was decided to assess them with fewer questions per concept: five on basic operations, three on negative numbers, five on fractions, two on rate, and three on equations.

Example: Integer Addition Game

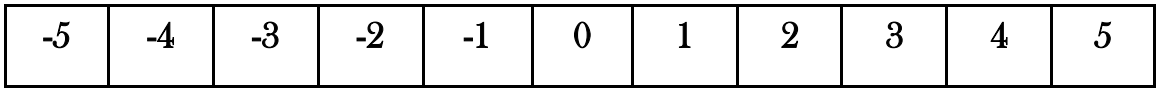
As mentioned earlier, each activity using the Five-Step Approach begins with a physical event that the students "mathematize" through a process that Moses has broken down into steps for teachers and practitioners to follow. We will describe one activity developed by pre-service teachers in the T.R.E.E. Project that adheres to this model. The physical experience that was the basis for this activity is similar to the typical hopscotch game and was originally designed for groups with four to six students. All students in the T.R.E.E. Project performed this outdoor activity at least twice, with the goal of teaching students how to add and subtract positive and negative integers. Usually two pre-service teachers worked with one student group. One teacher helped students individually while the other led the game for the whole group.

Prior to the activity, teachers and students constructed a hopscotch-like board on the ground using chalk or tape (see Figure 2). They then labeled the board with positive and negative integers to model the number line centered at '0'. Next, teachers took two different colored dice and explained to the students that one of the colors (e.g., black) represented the 'positive direction,' and the other color (e.g., red) the 'negative direction.' Each player began at '0' and rolled the dice one by one, and the students moved according to the instructions given by each roll of the dice. The students took turns until one student or team reached the designated end-points of the board or moved past them. Teachers divided the students into two groups with the same number of students and let them compete with each other.

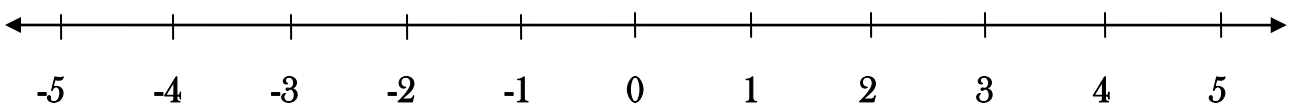
The game was designed to model addition of positive and negative numbers, with the dice representing the distance to travel and the direction. As the students played the activity in various ways, the teachers guided them to find patterns or other relationships. For example, when adding a positive number and a negative number, one of the scaffolding techniques that the teachers used with the students was to notice that when starting at the origin, if they moved farther to the left than to the right, then their ending position would be negative, and if they moved farther to the right than to the left, their ending position would be positive because equal-sized forward and backward movements were "opposites" that offset each other.

After they played the game, the teachers had students draw pictures about their experience on a piece of paper or on the ground with chalk as the second step of the Five-Step Approach. Teachers did not teach anything at this moment, but tried to remind students of the activity they just did and helped fill in details.

The third step in the Five-Step Approach involves what is referred to as "People Talk" where the teachers use language that the students use in their ordinary life, rather than mathematical or academic language, to discuss with their students what they observed and pat-



Game Board 1



Game Board 2

Figure 2. Hopscotch game boards

terns they may have noticed during the experience. For example, the teacher might use "farther to the right" instead of "greater." For this particular activity, "the number to the right of 0" or "the number on the blue die" can be used for "positive number." This discussion continued until the teachers were convinced that the students understood the underlying mathematical concepts.

The fourth step in the process involves "Feature Talk" where the teachers introduced the students to the targeted mathematical terminology and symbolism used to describe the main features of the event. For this activity this step involved introducing the vocabulary of "negative" and "positive" for "movement to the left" and "movement to the right" respectively, and introducing the word addition for the operation "followed by."

For the final step in the Five-Step Approach, students were reminded of the formal mathematical symbolism for positive (+), negative (-), and addition of integers ((-5) + (-3)). They were then asked to describe patterns and rules they formulated during earlier steps in terms of the formal mathematical language and symbols.

One benefit of this particular game was that once students became familiar with the activity with addition of a positive integer and a negative integer, the game was used to teach addition of two negative numbers and subtraction of two integers as well. Figure 3 shows the visual sequence of how this activity was taught.

Results

Based on the areas in which gaps were identified in student background knowledge, the eight-week schedule was slightly modified and implemented as follows: Weeks 1 through 3 was basic operations; week 4 was negative numbers; weeks 5 and 6 were fractions; week 7 was equations; and week 8 was review and a fieldtrip.

The Five-Step Approach was not only used for teaching fractions (a major priority identified by the National Mathematics Advisory Panel, 2008), but also became the basis for teaching *all* of the essential concepts taught during the summer. After strategically applying Moses' Five-Step Approach in our teaching, it was found that the students had improved performance on CST assessment items in the targeted sixth-grade mathematical concepts including basic operations, negative numbers, fractions, rate, and solving one-step equations. Although there was gain across the board, the greatest improvement was in the area of fractions: from 24% at

Step 1.1 **Using sidewalk chalk (or tape), construct a hopscotch-like board (in the style of Figure 2) on the ground and label it from -10 to 10.**	**Step 1.2** **Take two differently-colored dice. Designate one die to be 'left' and the other to be 'right.'**	**Step 1.3** **Players begin the game standing at '0'. Each turn for a player consists of one roll of each color die.**	**Step 1.4** **Students move as the dice show and take turns until one student or team wins by going beyond either end (10 or -10) of the board.**
Step 2 **Ask students to draw their experience.**	**Step 3** **Have students discuss what they found and then guide them to make connections to integer addition.**	**Step 4** **Help students organize patterns or rules they found and express them in formal mathematical language.**	**Step 5** **Introduce the formal mathematical symbols relating to the formal language in Step 4.**

Figure 3. Hopscotch game (the first step of the Five-Step Approach spans steps 1.1-1.4)

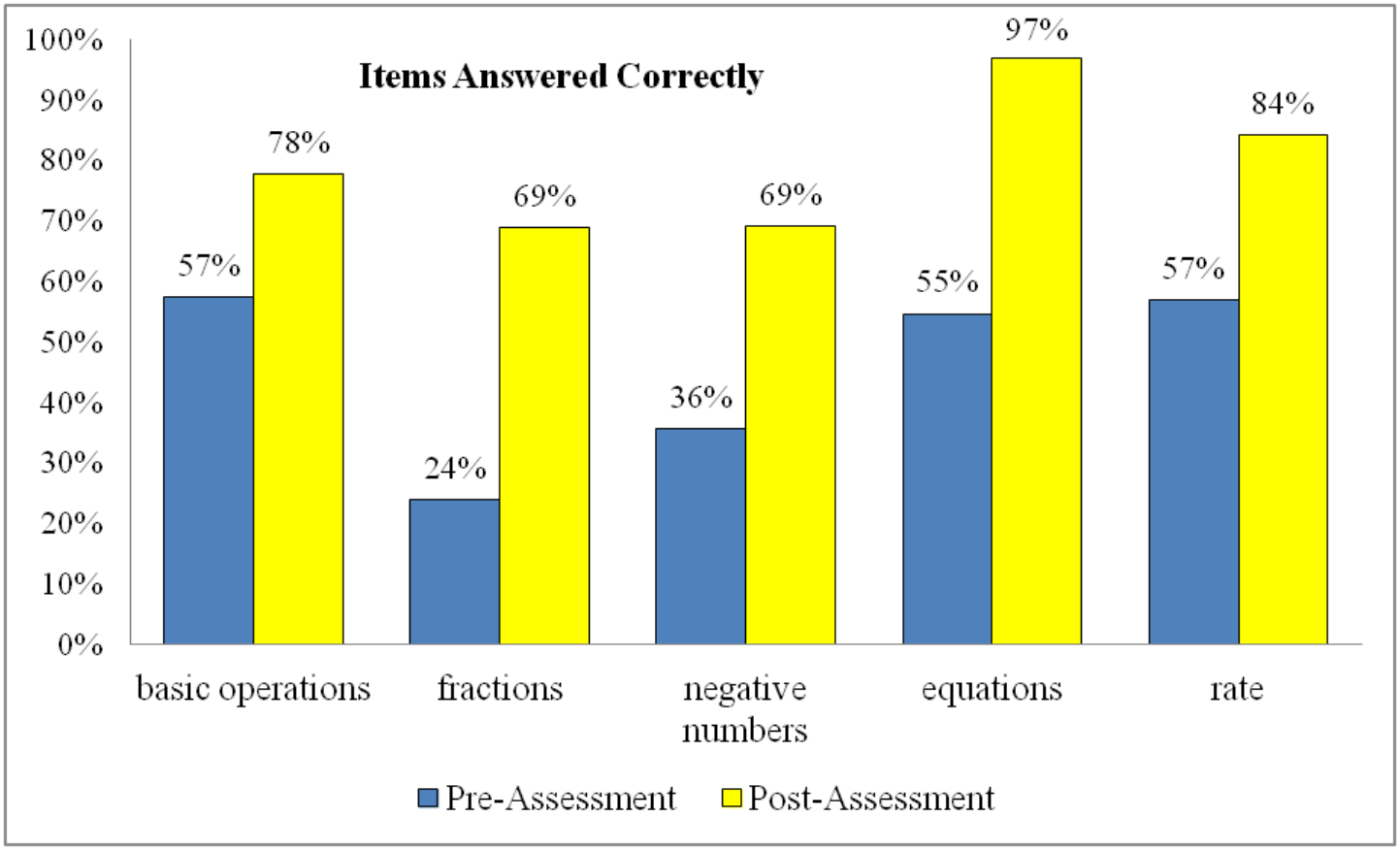


Figure 4. Pre-post results from California Standards Test items

the beginning to 69% at the end of the program, a gain of 45 percentage points (see Figure 4).

The T.R.E.E. Project illustrates an application and extension of the Five-Step Approach in teaching mathematics to ELLs. We are encouraged not only by the test results (Figure 4), but also by positive changes we observed in the students' behavior. By the end of the program, those students who had initially exhibited behaviors of "shutting down" no longer put their faces down or refused to take the tests. With its safe environment and scaffolding, Robert Moses' Five-Step Approach appears to be a promising vehicle for teaching mathematics to students, particularly ELLs.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions:

1. In a university pre-service teacher education program, students are taught various theories and frameworks. This project used Robert Moses' Five-Step Approach from the Algebra Project to teaching mathematics. Why do you think it is important to have a guiding framework such as this when you plan and teach your lessons?
2. If you were given a textbook by your school district and asked to teach 50 mathematics concepts with the majority of your students being ELLs, what would you do? How would you go about identifying a sequence of lessons?
3. In reflecting on your own mathematics education, how did your teachers teach you mathematics? Did they provide concrete experiences first or the abstract symbolism in formulae or equations? In other words, did you first encounter Step 1 (experiences) or Step 5 (formulae with symbols such as +, -, =)?

4. Write a sample activity plan for teaching the addition of fractions with equal denominators by following Robert Moses' Five-Step Approach. Share with others.

5. Explore other benefits and insights games can have by reading articles such as the Jiménez-Silva, Gómez & White-Taylor article in the 2010 issue of *TEEM.*

6. Brainstorm with colleagues outside your department different ways how the Five-Step Approach may be used across Language Arts, Social Studies, Science, Music, etc.

"DARE to Reach ALL Students!"

RICHARD TAPIA WINS NATIONAL MEDAL OF SCIENCE

(Photo courtesy of the White House)

This fall, Richard A. Tapia was among 12 scientists to receive the top award the US offers its science researchers -- the National Medal of Science. Tapia was honored "for his pioneering and fundamental contributions in optimization theory and numerical analysis and for his dedication and sustained efforts in fostering diversity and excellence in mathematics and science education." Tapia is known nationwide as a champion of underrepresented minorities in the sciences and one of his many hats at Rice University is Director of the Center for Excellence and Equity in Education. In 2005, Tapia received Rice's highest academic title by being named University Professor, one of only six professors in Rice history to receive this honor. In 1992, he became the first native-born Hispanic elected to the National Academy of Engineering. The medal Tapia received from President Obama is yet another item in a long list of honors to this educator born in a family of modest means where no one had gone to college.