Developing Mathematics Literacy for Bilingual Learners: A Framework for Effective Learning

Zulmaris Diaz, J. Joy Esquierdo, Olga Ramirez, and Isela Almaguer

Abstract

A framework is proposed for how bilingual learners develop knowledge, language, and mathematics literacy. The framework centers on principles of learning, effective pedagogy, and second language acquisition theories, and these elements are incorporated in a mathematics lesson depicted in this article.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Does the term *bilingual learners* seem different from *English Language Learners*? Explain your reasoning and any possible influences from your experiences in teaching.

2. How do you define *mathematics literacy?*

3. What challenges might bilingual learners face when developing mathematics literacy?

4. Discuss any experience or knowledge that comes to mind connected to the components in Figure 1, page 12.

Zulmaris Diaz (diazz@utpa.edu) is an Assistant Professor at The University of Texas-Pan American. Her research focuses on teacher education, specifically in the area of English as a Second Language (ESL), and the development of math literacy in bilingual children.

J. Joy Esquierdo (esquierdo@utpa.edu) is an Assistant Professor at The University of Texas-Pan American. Her research focuses on the academic performance of bilingual students in various areas such as gifted education, content literacy development, cognitive development, and best teaching practices for bilingual learners.

Olga M. Ramirez (oram@utpa.edu) is a Full Professor at The University of Texas-Pan American. Her research focuses on family math, preparation of elementary and middle school teachers, diagnosis of mathematics methods, and professional development of teachers.

Isela Almaguer (almagueri@utpa.edu) is an Associate Professor at The University of Texas-Pan American. Her research focuses on the teaching and learning of English Learners (ELs), instructional practices involving the literacy development of second language learners in dual language settings, and technology integration in teacher education.

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The belief that mathematics is an easier subject than others to teach to students who are English Language Learners (ELLs) has misconceptions. While some view mathematics as a subject with minimum linguistic requirements that involves only numbers, many mathematics teachers would disagree. Mathematics involves specialized vocabulary, oral and written language, multiple representations of concepts, and same terminology for different concepts (Echevarria, Vogt, & Short, 2001). Moreover, research indicates that knowing how to complete computational problems and repeat definitions verbatim is insufficient for mathematical literacy.

Martin (2007) defines *mathematics literacy* as the ability to "reason, analyze, formulate and solve problems in a real-world setting" (p. 28). For students learning English and mathematics concurrently, becoming mathematically literate presents certain challenges. They need to learn not only English, but also the language of mathematics to construct meaning in mathematics (Ron, 2005); further, they need to communicate orally and in writing so as to explain solutions, provide conclusions, or present arguments (Moschkovich, 2002).

Another challenge faced by ELLs as they develop math literacy is the achievement of the Communication Standard outlined by the National Council of Teachers of Mathematics (2000), which suggest "instructional programs from prekindergarten through grade 12 should enable all students to-

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely" (p. 59).

For these reasons, we present a framework designed to support the development of mathematics literacy for ELLs. The framework will be complemented with an experimental activity that examines a lesson in a mathematics classroom through the lens of our framework.

Bilingual Learners

Throughout the general literature, students who are learning English are referred to as English Language Learners (ELLs), yet in this article we will use a more encompassing term. We refer to such students as Bilingual Learners (BLs) to emphasize that they are learning content at the same time they are developing two languages, English and their mother tongue. It is crucial when teaching BLs to keep in mind that the main focus needs to be on the cognitive development of the students and, as they are exposed to English, their linguistic abilities will develop concurrently. Teachers may sometimes become so focused on increasing language proficiency in the students' second language -- in most cases, English -- that the development of mathematics literacy can lose priority.

Framework for Teaching BLs New Content Literacy

The proposed mathematics lesson framework (in Figure 1) resulted from an extensive review of the literature on how BLs best develop content knowledge and skills. It centers on three fundamental elements: three principles of learning, effective pedagogy, and second language acquisition theories, all of which will result in a learner-centered classroom environment that supports the development of grade-level content literacy (Baker, 2006; Cook, 1992, 2002; Cummins, 1981, 1984; Krashen, 1982; Padrón & Waxman, 1999; National Research Council, 2000).

Three Key Principles of Learning

When developing the proposed mathematics lesson framework, we focused on three major principles of learning presented by the National Research Council (2000):

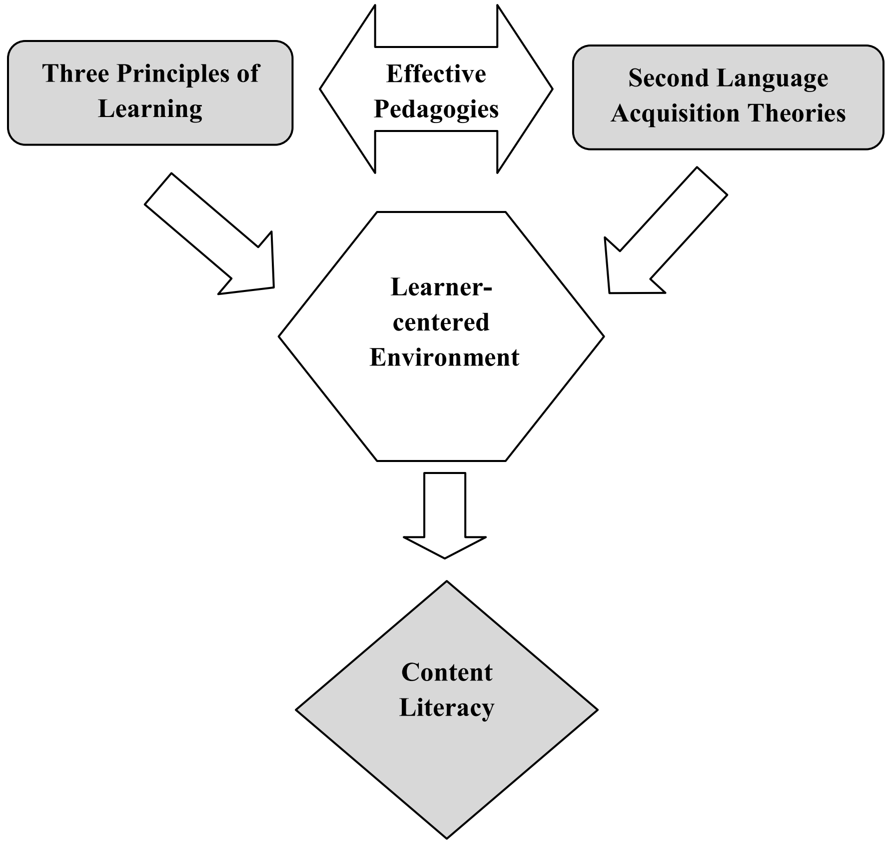

Figure 1. Framework for teaching Bilingual Learners new content literacy (modified from Diaz et al., 2010)

Principle of Learning #1**:** All students start school with preconceived concepts of how the world functions. They bring experiences from home and from their surrounding community, which help them construct new knowledge in the classroom.

Principle of Learning #2**:** In order for students to develop the ability to make inquiries, they must have developed basic factual knowledge and the ability to manipulate that knowledge (see Figure 2 to the right for an illustration).

Principle of Learning #3**:** For bilingual learners to become lifelong learners, they need to take a metacognitive approach to their learning. Metacognition occurs when a student makes a conscious effort to control and monitor his/her learning through the use of various learning strategies (Brown, Bransford, Ferrara, & Campione, 1983).

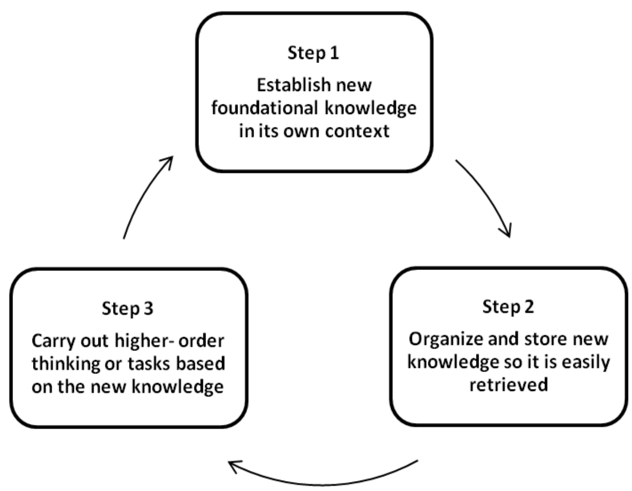

Figure 2. Learning principle #2 (Esquierdo, 2010)

Pedagogy Supportive of Language Acquisition and a Learner-Centered Classroom

When providing content instruction, mathematics teachers must be sure to employ solid pedagogy that upholds the theoretical views of language acquisition. Padrón and Waxman (1999) propose five research-based instructional practices (explained in Table 1 below) that support language acquisition, development, and knowledge for BLs: (1) Culturally Responsive Teaching; (2) Cooperative Learning; (3) Instructional Conversation; (4) Cognitively Guided Instruction; and (5) Technology-Enriched Instruction.

These instructional approaches are just a few of the research-supported strategies used to provide meaningful and effective instruction to BLs. They focus on a *learner-centered* environment, where "the students' own desire to know, to discuss, to problem solve, and to explore individually and with others" serves as the foundation of instruction rather than imparting "learning that is dictated, determined, and answered by the teacher" (Glickman, 1998, p. 52).

Table 1. Instructional practices that can support language acquisition

Pedagogy	Description
Culturally Responsive Teaching	Instruction that builds on the languages and cultures that children bring from their home and community (Slavin & Cheung, 2005).
Cooperative Learning	Instruction that involves the use of small groups as a means to optimize students' own and each other's learning. Some benefits: (1) enhances instructional conversations; (2) develops social, academic, and communication skills; and (3) develops proficiency in English (Calderón, 1991; Christian, 1995; Johnson & Johnson, 1991; Rivera & Zehler, 1991).
Instructional Conversation	Extended instructional discourse between the teacher and students (Duran, Dugan, & Weffer, 1997). Provides opportunities for extended academic conversations and allows BLs to reformulate previous concepts and attach new vocabulary to them (Christian, 1995).
Cognitively Guided Instruction (CGI)	Instruction allowing students to articulate their thinking which in turn provides teachers with a better understanding of how children learn mathematics (Carpenter, Fennema, & Franke, 1996).
Technology-Enriched Instruction	Instruction utilizing technology to help connect learning in the classroom to real-life situations (Means & Olsen, 1994) and allows students to access information in their native language as well as in their second language. Examples include the use of virtual manipulatives, web-based picture libraries, multimedia, calculators, etc.

Second Language Acquisition Theories

Teachers ought to be cognizant of students' acquisition of a second language. We will examine some principles of second-language acquisition with the intention to help teachers plan mathematics instruction for BLs. These principles take into consideration the learner as the center of his or her language and learning development and how all he/she brings into the classroom promotes language proficiency and academic achievement, specifically mathematics literacy.

Baker (2006) suggests that it is imperative that the education community stop viewing BLs as "two monolinguals in one person" (p. 10), so there is a need for a paradigm shift such that individuals are viewed as having *multi-competence* (Cook, 1992, 2002) in both languages. It is well known that language comprises four domains: listening, speaking, reading, and writing. Within these domains, there is much variation in language development and ability, spanning these stages (Baker, 2006; Krashen & Terrell, 1983):

> (a) simple or what second language acquisition scholars will call pre- and early- production stage: the person has limited comprehension of the language and uses short phrases to communicate;
>
> (b) basic or speech emergent stage: the person has an increased comprehension of the language, is less hesitant to speak and uses simple sentences to convey meaning;
>
> (c) fluent or intermediate fluency stage: speech is at greater length with the use of more complex sentence structures; and
>
> (d) accomplished or advanced proficiency: the person uses complex grammar and specialized academic vocabulary.

For BLs, the level of fluency within each of the language domain will depend largely on the need and use of a language (Grosjean, 1998). In fact, it is almost impossible for a bilingual person to be equally competent in both languages (Fishman, 1971). For example, some BLs might have a fluent or intermediate level of proficiency in speaking English when it is used in a social context (e.g., shopping, interactions with family, etc.), but demonstrate basic or speech emergent skills in reading and writing when used in formal contexts (e.g., academic lectures, work, etc.). Students might have an accomplished or advanced level of proficiency in speaking and listening to Spanish in formal context, but demonstrate fluent or intermediate skills when reading and writing Spanish in formal context.

Consequently, BLs need to be given opportunities and access to rich language environments in order to develop multi-competence in both languages at the social and formal context, or what Cummins (1984) identifies as the two key dimensions of language proficiency: (1) basic interpersonal communicative skills (BICS), language skills that are acquired easily through daily living; and (2) cognitive academic language proficiency (CALP), the language proficiency learned in an academic setting (Cummins, 1981).

It is important for teachers not to be deceived by BLs' language fluency in the social context since they might not have the same language proficiency in the academic or formal context. If bilingual learners are not exposed to the language of the classroom, the "vocabulary and the rhetoric style that make up the academic" language (Gandara & Contreras, 2009), specifically in our case the language of mathematics, they will encounter difficulties when having to read and analyze mathematical texts, including having to support, explain, and articulate their results and ways of thinking mathematics.

Under the premises that language is acquired through social interaction in different contexts, Krashen (1982) has concluded that the key to second-language acquisition is not the quantity of exposure to the second language, but the quality of instruction in the second language. That is, when a teacher is aware of the bilingual learner's second-language development, the focus goes from a *more coverage* approach to a *more appropriate* experience. The acquisition of language competency and content literacy in the second language is a result of comprehensible input, the spoken or written message that is delivered at the student's level of comprehension, and an accommodating affective environment, where the students' level of anxiety is low.

Learner-centered Environment

The main focus of a learner-centered environment is learning with understanding, while taking into consideration the needs, abilities, and interests of the learner. A learner-centered classroom promotes active exploration and construction of meaning, while moving away from the passivity of listening to the teacher's lecture and reading the textbook. It builds on the idea that the learner is responsible for his/her own learning.

The proposed framework serves as a catalyst for the creation of a learner-centered environment. Two of the main elements of the framework ask teachers to take into consideration the learner – specifically, how BLs learn based on the three principles of learning – and understand how they acquire the second language and develop language proficiency. Moreover, using effective pedagogies in the classroom help bilingual students become motivated about their own learning. Each of these elements contributes to the evolution of mathematics literacy collectively; they do not work in isolation. In other words, a teacher cannot simply assume that, for example, using the principles of learning component of the framework will guarantee that bilingual learners develop mathematics literacy. All three components of the framework need to be considered when planning and delivering instruction to BLs so that content literacy can be acquired.

A Closer Look in a Mathematics Classroom

The following scenario is a hypothetical sixth-grade classroom applying ratios through a real-world experience. The teacher, Mr. Cruz, starts his lesson asking the students about their homework. For homework, the students had to go to the store to choose a liquid product (e.g., juices, cleaning products, milk) and record the different size containers of the chosen product and their respective prices (e.g., a gallon of milk is $4.00; a ½ gallon is $2.50, a quart is $1.40, and a pint is $0.75).

Mr. Cruz asks the students, "So what did you find out?" "I found out that the bigger the container, the higher the price," offers Michael. María raises her hand and says, "I recorded the prices for milk and noticed that one gallon of milk costs $4.00, and ½ a gallon costs $2.50, and a quart costs $1.40." Mr. Cruz adds, "That's right. The prices will vary according to the container sizes. Today we will compare how prices of milk vary depending on their container size by using ratios. Ratios are used in our everyday life. Let's look for example at the different size containers of milk and let's use the prices María recorded." The teacher then places on the board a picture of a gallon, ½ gallon, and a quart of milk and writes down the prices María wrote down as homework. He then asks "which one do you believe is a better buy and why?"

Mario answers, "I think the quart is a better buy because it's cheaper." "No but you get less fluid ounces. I think it is the gallon," responds Jesús. Lucia adds, "Yo creo que el medio-galón tiene el mejor precio porque tiene más cantidad pero solo cuesta $2.50" [*I believe that the half-gallon is the better price since it has more milk and it only costs $2.50*]. Mr. Cruz continues with the lesson by commenting, "Okay, let's see which one is a better buy. To find out, we need to calculate the price per ounce. To do this, we need to take into consideration the ratio between the price and the fluid ounces of the container." Using an organized table, Mr. Cruz guides the students on how to study and calculate ratios (see Table 2).

Table 2: Price for one gallon of milk

Capacity of Container *Capacidad del envase*	Equivalent Fluid Ounces *Equivalencia en onzas líquidas*	Price of Container *Precio del envase*	Price per Ounce *Precio por onza*
1 gallon *1 galón*	128 fluid ounces *128 onzas líquidas*	$4.00	$4.00÷128= $0.03125

Mr. Cruz continues, "Now get with your partner and find out what the price per ounce would be if the gallon would cost $4.50. As students work in partners to respond to Mr. Cruz' subsequent question, he walks around guiding them when needed and asking them to explain how they solved the problem. In the background, you can hear the students using English and Spanish to solve the problems and also discussing the different steps they take when calculating division operations with and without the use of a calculator. Mr. Cruz takes advantage of the moment and asks Lucia, who recently emigrated from México, to explain to the class how she divides since she solves the problems differently.

After the students have completed the problems he posed, Mr. Cruz asks them "What did you find out?"

Marco responds, "if the gallon costs $4.00, the price per ounce is $0.03125, but if the price per gallon costs $4.50, the price per ounce is $0.03516." "Very well, now let's use the calculators and see what the price per ounce is when the ½ gallon costs $2.50," says Mr. Cruz. The class continues with the same study format and the teacher guides the students by showing them how to link what they know about determining the price per ounce of milk when given the price per gallon to finding the price per ounce of milk when a half-gallon costs $2.50 such as in Table 3. As the students work together they use both languages to complete the assignment and to help each other.

Table 3: Price for half-gallon of milk

Capacity Container Capacidad del envase	Equivalent Fluid Ounces Equivalencia en onzas líquidas	Price of Container Precio del envase	Price per Ounce Precio por onza
$\frac{1}{2}$ gallon $\frac{1}{2}$ galón	64 fluid ounces 64 onzas líquidas	$2.50	$2.50 ÷ 64 = $0.03906

After the students have completed the work given, Mr. Cruz asks them "What did you find out?" Angela responds that "if half-gallon costs $2.50, the price per ounce is $0.039063." "So then, which one do you think is the better buy?" asks Mr. Cruz. All students shout "the gallon!" Mr. Cruz continues, "So far, the gallon of milk is our better buy, but we cannot make assumptions. We need to calculate all the ratios. Continue working with your partner and find out the price per ounce when the quart of milk is sold for $1.40." After the students complete all the calculations, Mr. Cruz asks them, "Which one is the better buy and why?" The students answer in unison "the gallon of milk, because the price per ounce is $0.03125." Subsequently, Mr. Cruz asks the students to summarize how the price per fluid ounce is determined. As the students explain their thinking, he encourages them to notice that the price of the container must be divided by the number of fluid ounces in the container. Then Mr. Cruz asks the students to write in their journals a "word square" (Winsor, 2007) for the term "ratio" (see Table 4 below).

Table 4: Word square for the word "ratio"

Ratio	Razón
A **ratio** is a relationship or comparison between two numbers **Razón** es una relación o comparación entre dos números semejantes.	Ratio = $\frac{PricePerContainer}{NumberofFluidOunces}$ Razón = $\frac{\text{Precio del envase}}{\text{onzas líquidas}}$

After having students write in their journals, Mr. Cruz asks the students to work in pairs and use the findings from their homework to calculate the ratio of the prices of each of their recorded liquid products and to decide which size container is a better buy. Then, they are to write a small paragraph explaining which size container is a better buy and why, and they are to present the findings to the class. At the end of the lesson, Mr. Cruz reminds the students that for homework they will have to compare three different brands of cereal and decide which one is the best buy.

Connection of Lesson with Framework

Three Principles of Learning

At the beginning of this lesson, Mr. Cruz makes use of his understanding of the first Principle of Learning when he asks the students to share what they found out from their homework and when he makes connections to their prior experiences and knowledge. When Mr. Cruz asks the students to work with partners to use the findings from their homework to calculate the ratio of the price per ounce for each of the liquid containers and cereal brands and to decide which container or brand is the best buy, this demonstrates how this teacher makes use of the second Principle of Learning. The students will also be required to apply the same knowledge to a different context when they will have to compare three different brands of cereal. Moreover, Mr. Cruz makes use of Learning Principle #3 when he helps the students understand the new concept by organizing the new information into a table both in English and Spanish. He also makes the connection between English and the students' native language when he asks the students to create a word square for the new term. These two approaches are cognitive strategies which help promote the development of metacognitive strategies for BLs.

Pedagogy

Throughout the lesson, the students are working cooperatively. They are working in pairs instead of larger groups because students who are not proficient in English, females, and minority students tend to participate less in cooperative learning activities when groups are larger (Webb, 1984). Therefore, in Mr. Cruz's classroom, by working in pairs, all students contribute equally and are provided with opportunities for mathematical discourse, which allows BLs to process the new information further and develop language as they discuss findings with each other. As the teacher explains the concepts and walks around when students are working in pairs, he is making use of instructional conversations to promote language development and mathematical literacy. Likewise, there are hints of CGI when the students explain how they got their answers for finding ratios and when Lucia describes the method she learned in México of solving division problems. Teachers need to understand mathematical cultural differences, accept them, and allow students to use their own strategies (Midobuche, 2001). Moreover, students had ample opportunities to participate in academic conversations with their partners, the whole class, and the teacher. They also had the opportunity to use technology – in this case, calculators -- to solve problems.

Second Language Pedagogical Strategies

In this particular lesson, the teacher works under the premise that language is acquired through social interaction, with comprehensible input in an accommodating affective environment. The students are acquiring the English language as they experience and discuss the mathematics content in this lesson. They are not learning English rigidly through the review of rules and grammatical structures, but are learning and applying their English and Spanish skills as they attain the new vocabulary and knowledge introduced in the lesson and modeled by the teacher. Additionally, Mr. Cruz uses Krashen's (1982) notion of comprehensible input by providing visuals, and modeling allowing for language and concept transfer through the use of word squares (Winsor, 2007). All of these strategies provide support to the BLs' comprehension of the mathematical content and the development of the second language. Most importantly, the lesson is designed to lower the affective filter and allow BLs to feel comfortable taking academic and linguistic risks.

Learner-centered environment

The setting of a learner-centered classroom environment is created by employing the three key Principles of Learning (NRC, 2000), effective pedagogy, and second language acquisition theories. One fundamental trend among these three major areas is that providing a learning-safe, risk-free classroom helps the development of mathematics literacy skills in BLs to flourish. In the learning framework espoused in this paper, it is clear that permitting BLs to discuss the mathematics requirements of the lesson in both English and Spanish is empowering and fundamentally important in supporting a learner-centered environment.

Conclusion

This article proposes a framework that encompasses how BLs develop knowledge, language, and mathematics literacy. It provides a functional structure for teachers on how three principles of learning, effective pedagogy, and second language acquisition theories, collectively, can lead to the development of mathematics literacy for BLs. It is based on the premise that teachers must view bilingualism as a strength and not as an obstacle to teaching and learning.

It is crucial that teachers keep aiming for BLs to perform higher-order thinking skills and tasks. We posit that when BLs have the opportunity to explore actively and construct meaning by engaging in higher-order thinking projects in a learner-centered environment, they expand their understanding of the language (first and second language), content literacy, and most importantly, transfer knowledge from one language to another. They also gain the ability to apply the information to an assortment of contexts and use language as a tool. Thus, this article provides an important message to teachers of BLs that promotes exploring, inquiring, and applying new mathematics knowledge in and out of school contexts with opportunities to use both English and their mother tongue to "communicate their mathematical thinking coherently and clearly to peers, teachers, and others” (NCTM, 2000, p. 59).

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Looking back at Figure 1, what did you learn about each component of the framework?

2. Which of the three Principles of Learning most resonates with your approach to teaching? Explain.

3. Review a mathematics lesson you have previously designed and/or delivered to BLs. Does it encompass any components of the framework? What modifications can you make to the lesson plan so that it can utilize the major components of this framework?

4. Create a mathematics lesson for BLs that takes into consideration the framework in this article.

5. Analyze a video lesson through the lens of the proposed framework. For example (from http://www.learner.org/catalog/browse.html?discipline=5), suggested Annenberg/CPB Mathematics Videos are "Ladybugs" or "Marshmallows" (from Teaching Math: A video library, K-4) or "The Largest Container" (from Teaching Math: A video library, 5-8).

6. How can teachers optimize mathematics learning for BLs?

"DARE to Reach ALL Students!"

TODOS Mathematics for ALL