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Investigating Mathematics Teaching Practices in Latin America: Reflections on Preparing U.S. Teachers of English Learners

M. Alejandra Sorto

Abstract

This article analyzes teaching practices in mathematics from two elementary classrooms in Costa Rica, a Spanish-speaking Latin American country. Insights and examples from the analysis may inform mathematics teacher educators and teachers of English learners in the U.S.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Many Latino/a students come from and previously studied mathematics in Latin American countries. What do *you* think math lesson looks like in those countries?
2. How do you think teaching practices in Latin American countries are similar to or different from those in the U.S.?
3. In your everyday practice, have you had any students commenting about the way they learned mathematics in their native country? If so, what was your reaction?

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International Comparisons of Mathematics Teaching

The inspiration for this paper comes from lessons learned observing the teaching of mathematics in other countries. By focusing on comparing and describing teaching practices and mathematical discourse, the work described in this article will expand our knowledge of countries about whose teaching practices we know very little of while addressing two of the most pressing challenges in mathematics education: educating *all* children and producing *qualified* bilingual teachers (NCTM, 2000; No Child Left Behind, 2002).

An extensive international comparison about the teaching of mathematics is the TIMSS 1999 Video Study (Hiebert et al., 2003), which built upon the TIMSS 1995 Video Study (Stigler & Hiebert, 2004). The 1995 Video Study examined teaching practices of Japan, Germany, and the United States in 8th-grade classrooms. One major finding was that Japanese teachers taught differently than those in the U.S. by having students engage in solving, presenting, and discussing problems. The 1999 Video Study was extended to high-achieving countries Australia, Hong Kong, Switzerland, the Netherlands, and the Czech Republic. This latter study found that countries using methods other than the Japanese methods could still produce high-achieving students in mathematics. One common finding in the higher scoring countries was the *effective* implementation of tasks or problems requiring higher-order thinking and reasoning.

In a parallel 13-country study on third and fourth-grade student mathematics achievement in Latin America (UNESCO, 1998), the top-scoring country was Cuba, followed by Chile and México. The UNESCO also identified, based on a multivariate analysis of associated factors, seven countries (Argentina, Chile, Costa Rica, Cuba, Colombia, Bolivia, Venezuela) with 'outstanding schools' — "schools whose students demonstrated achievement in mathematics above that which would be expected, given the educational level of their parents" (UNESCO, 2002, p. 8). Carnoy, Gove, & Marshall (2007) followed up the achievement study with a comparison study that included a video examination of teaching practices in 10-12 third grade mathematics lessons each from Cuba, Chile, and Brazil. The comparison results are consistent with the TIMSS results. Cuban and, to a lesser extent, Chilean (private school) teachers engage students successfully in tasks that require students to reason and think, such as:

> If asked to indicate whether or not 430 is divisible by 10, Cuban students would be expected to explain that the zero in the units place is an indicator that 430 is a multiple of ten and is therefore divisible by 10 (Carnoy, Gove, & Marshall, 2007, p. 134).

The purpose of the TIMSS and Carnoy, Gove, & Marshall (2007) studies was to describe a pattern of teaching in high-achieving countries and, in the case of the latter study, to understand better the sources of between- and within-country variation that statistical models can detect but not illuminate (McEwan & Marshall, 2004). However, we can take further advantage of these two sets of studies and link them in natural ways to have a greater impact on the improvement of teaching in the U.S. classrooms. A natural connection is to look at what U.S. and Latin American classrooms have in common: children born in Latin America whose native language is Spanish. According to Jefferys (2007), México and Central American countries (after Cuba) are the leading regions of Spanish-speaking countries from where legal permanent residents come. School-age children of these populations also have the lowest mathematics achievement test scores (NCES, 2004). Even though México and some countries in Central America have (relatively) high or outstanding achievement, not much is written about what their mathematics lessons look like or how their practices compare to other countries.

The main goal of this paper is to illustrate the potential of the examination of teaching practices in mathematics classrooms of our Latino students' native countries by 1) *uncovering different ways children learn mathematical concepts* in their native countries; and 2) *providing authentic classroom situations* reflecting the quality of mathematics instruction for future use by prospective and practicing teachers as well as teacher educators.

Teaching Practices in Central American Countries

In 2006-2007, a large-scale study (involving 385 teachers at 97 randomly-selected schools) was conducted in two Central American countries: Costa Rica and Panama

(Carnoy, M., Gove, A., & Marshall, J. H., 2007). The purpose of this study was to document qualitative differences in both countries' educational systems in order to explain the large differences between countries in student achievement. Different measures were used for this purpose including teacher knowledge questionnaires and videotaped lessons. The results showed that Costa Rica's teachers performed better in the teacher knowledge measures. The lessons, especially at the third grade level, are characterized as being more conceptually focused, having higher levels of cognitive demand for the student tasks, and having longer lessons that allow for exploratory and discovery activities (Sorto, M. A., Marshall, J. H., Luschei, T. F., & Carnoy, M., 2009). An observation derived from video analysis of the lessons is that high quality teaching was achieved in rural classrooms even with very limited instructional resources. For example, in most rural schools in both countries, there was no evidence of any manufactured set of manipulatives, technological equipment, textbooks, prepared handouts, or educational posters. However, there were creative teachers that used what was available in the environment to teach effectively.

To illustrate further this observation, a more qualitative analysis of these practices is necessary. The deeper analysis is an attempt to extract more out of the rich source of data in terms of the mathematical discourse and the role of the teacher's pedagogical choices. Hence, two lessons from Costa Rica with the same instructional goal but different pedagogical approaches were selected, transcribed and translated to explore a mathematical discourse that relates to issues of language for Latino/a students when engaged in tasks with high-level cognitive demand.

Third Grade Lesson 1

This lesson corresponds to a rural public school and the number of students in this particular classroom was large (36 to 40), especially for the physical space. The teacher had the students sitting on benches arranged in continuous rows with enough space for her to circulate from the front row all the way to the back in a snake-like path. This arrangement also allowed students to interact with their partners at each side. The classroom had only one desk in front, a blackboard, and chalk. Teaching materials consisted of a large protractor and wooden sticks use to make popsicles and that are sold at local convenience stores. The main goal of this lesson was to identify angles, sides, and vertices in regular polygons. What follows are two episodes from the lesson (student names in this article are pseudonyms):

T1: Vamos a tomar tres paletitas y vamos a ver que figura nos sale. [*We are going to take three sticks and we are going to see what figure comes out].*

Students work individually on their desks with three sticks of the same length, they all form a triangle.

T1: ¿Que nombre le pondríamos a esta figura? [*What name would you give to that figure?*]
Sonia: Yo le puse triangulo [*I named it triangle*]
T1: Un tri-angulo. [*A tri-angle*] (The teacher emphasizes separation of letters.)
T1: ¿Que características tiene ese triangulo que me han formado ustedes allí? Obsérvenlo bien y luego me dicen porque se llama triangulo. ¿Que le ven ustedes a esa figurita que formaron para que se llame triangulo? [*What characteristics does that triangle that you have made have? Observe carefully and then tell me why it is called a triangle. What do you see in that figure that you have made for us to call it a triangle?*](Several students raise their hands saying "yo, yo" [*me, me*] and the teacher picks Karla in the front row.)
Karla: Tiene tres lados. [*It has three sides.*]
T1: Ahora con el dedito índice yo quiero que me señale esos tres lados. [*I want you to point out those three sides with your finger*]
T1: ¿Que otras características ven en ese triangulo? [*What other characteristic do you see on that triangle?*]
Mario: Tiene tres vertices. [*It has three vertices.*]
T1: Ha, tiene tres vértices, y ¿cuales son los vértices?, a ver señálemelos. ¿Y que son los vértices? Yo se que ya me lo dijeron, pero quiero que me lo recuerden. [*Ah, it has three vertices, and which ones are the vertices? Let's see - show them to me. What are the vertices? I know you already told me, but I want to be reminded.*]
Susana: Son las esquinitas. [*They are the little corners*]
T1: Las equinitas…si, pero anteriormente me lo dijeron en forma diferente. [*The little corners…yes, but you told me something different before.*]

Students had worked at the beginning of the class defining angle, sides, and vertex using only two sticks.

María: Es donde se unen los lados. [*It is where the two sides join.*]
T1: Ha, son las equinitas, o sea, donde se unen los lados. [*Ah, they are the little corners, that is, where the two sides join*]
T1: ¿Que mas ven? [*What else do you see?*]
Karla: Una línea horizontal y dos líneas inclinadas. [*One horizontal line and two slanted lines.*]
Julio: ¡Forma un ángulo! [*It makes an angle!*]
T1: ¿Un? [*One?*](Asking directly to Julio.)
Several: Dos [*Two*]
Julio: ¡Tres! [*Three*]

Students continue the lesson adding one more stick and making figures with four sticks. Here the majority of students formed squares but some formed non-square rhombi.

The class investigated the common characteristics of both figures and deduced the differences as well. One episode of this part of the lesson is worth mentioning because of the use of the expression "lados rectos" potentially to mean right angles and not necessarily straight sides (see further analysis of this episode in the next section):

T1: Vamos a ver si todos están de acuerdo con Karina. Ella dice que su figura tiene cuatro lados rectos. ¿El de ustedes también? [*Let's see if everyone agrees with Karina. She says that her figure has four straight (right) sides, do yours too?*]

Simon: No, el mío no porque yo forme un rombo. [*No, mine doesn't because I made a rhombus*.]

The last part of the lesson involved adding one more stick to form a pentagon and investigating its characteristics.

Third Grade Lesson 2

This lesson corresponds to an urban public school, the number of students in this classroom was also large (33 to 35) but with more physical space than the classroom in Lesson 1. The teacher had the students sitting in individual desks arranged in small groups. This classroom, unlike the rural classroom in Lesson 1, had a whiteboard, overhead projector, markers, and posters. Teaching materials consisted of large construction paper for each group, color markers, and rulers. As in Lesson 1, the main goal of this lesson was to identify angles, sides, and vertices in regular polygons. What follows is one episode of the lesson:

T2: ¿Quien recuerda que es un polígono? [*Who remembers what a polygon is?*]

Oralia: Una figura cerrada [*A closed figure*]

T2: Cuando estudiamos polígonos, dijimos que estaban compuestos por tres elementos, ¿Cuales son esos elementos? [*When we studied polygons we said they were composed of three elements. What one are those elements?*]

Alan: Líneas rectas [*Straight lines*]

Sam: Agudo [*Acute*]

T2: No, recuerden que los tres elementos son vértices, lados y ángulos. [*No, remember that the three elements are vertices, sides, and angles.*]

The teacher draws on the board a triangle, rectangle, pentagon, and hexagon, and then asks students to do the same on construction paper. While some are still drawing, the teacher gives the following color code: vértices - rojo, lados - azul, ángulos -verde. She then creates a triangle by making red dots at the vertices, coloring the sides in blue, and drawing green arcs for the angles. Then she sends one student at the time to the board to create the other polygons. Students cut and pasted the figures and were asked to leave space between each.

T2: Ahora quiero que me escriban abajo de cada figura lo siguiente: Numero de lados, numero de vértices, numero de ángulos. [*Now, I want you to write below each figure the following: number of sides, number of vertices, number of angles*]

T2: Yo les enseño como hacerlo para el triangulo. Numero de lados 3, numero de vértices 3, numero de ángulos 3. Ahora háganlo ustedes para el resto de los polígonos. [*I will show you how to do it with the triangle, number of sides 3, number of vertices 3, number of angles 3. Now you do it for the rest of the polygons.*]

Comparison of Teaching Practices in the Two Lessons

These particular examples show us why examining practices and mathematics discourse in Latino countries is worth investigating for three reasons. First, it *reveals* how children learn and how knowledge is acquired as a function of the instructional technique. In Lesson 1, children are given the opportunity to manipulate models of abstract objects, observe them, draw their own conclusions, and make generalizations. It further reveals, among other things, that the skill of recognizing angles in closed figures like a triangle is not trivial. First, even though the teacher emphasizes the word "tri-angle" from the beginning of the lesson, angles is the last thing the students "see" as a characteristic. One possible explanation for this phenomenon might be that the curriculum sequence places measuring angles after classification of figures by sides. Another reason could be the level of abstraction– it is harder to physically "see" an angle than see sides. Second, students first saw only one angle in a triangle, then two, and finally all three. This little episode shows us how the children construct their knowledge and make connections with previous knowledge, in this case, from knowing about angles involving two segments to angles formed by closed figures. By contrast, Lesson 2 gives the opportunity only to know the terminology, associate it with an object (a dot, a line segment, and an arc), and translate procedures.

Secondly, it *illustrates* the effectiveness of the teaching techniques presented in the two lessons. Lesson 1 uses sticks to model angles and polygons, and questioning techniques. Lesson 2 uses drawings, color codes, and student participation at the board. Neither of the teachers used printed materials, like worksheets or any type of technologies. In fact, from the video, it is not clear if these classrooms had any electricity at all. The pedagogical technique itself does not make one lesson better than the other. It suggests that it is a very special kind of teacher's knowledge that integrates the mathematics with the pedagogy in an effective way (Ball & Bass, 2000).

A third reason is that it *unveils* the role of language (in this case, Spanish) when teaching mathematics. In Lesson 1, children are looking for characteristics in squares and rhombi, one says "tiene cuatro lados rectos" [it has four right sides *or* it has four straight sides]. In Spanish the word "recto" has multiple meanings outside mathematics which I will not discuss here, but even within mathematics the word "recto(a)" is used to describe both "lineas rectas" or "segmentos rectos" [straight lines, straight segments] and "angulos rectos" [right angles]. It is not clear what the child means by "lados rectos" [straight or right sides]; it could be that she is now defining straight sides as those that form a right angle. We can conjecture only that the learners are using the word "right" in the sense of "up*right*" sides to mean sides that form 90-degree angles. (The term "right angle" comes from the Latin *angulus rectus*, where *rectus* means upright and relates to being perpendicular to a horizontal baseline, thus suggesting a person standing upright.) This would explain why the rhombus does not have "up-right" sides. If this is the case, the students do have the correct understanding but they are using a conventional word used for angles in an unconventional way. This less conventional way to talk about the characteristics of an angle in terms of the direction of its sides confirms the assertion that the concept of angle is a difficult concept to articulate for young children.

Reflections for English Learners Teacher Preparation

Future plans for this kind of analysis of teaching practices in Latin America are to help to the preparation of English learners in this country. One concern of using this approach to tackle mathematics education for English learners is that instructional practices occur in very different classroom, cultural and social environments. In particular, Latin American classrooms that are monolingual lack many of the complexities of U.S. mathematics classrooms with a large proportion of English learners. So, the question is, how can a study of monolingual classrooms from other countries help with the challenges teachers face in U.S. classrooms with a large proportion of English learners? The straightforward answer is grounded in research: students' previous knowledge of mathematics is fundamental to constructing new understanding, and the only way to know *what* and *how* students know mathematics before attending school in the U.S. is by observing and understanding the instruction in their native language (Moschkovich, 2007). Teachers can generally learn about their students' prior knowledge by experience, studying the curriculum at the lower levels, or attending school in their country of origin.

One way bilingual teachers acquire this knowledge is by informal talks with parents or with someone from the student's homeland. Even bilingual teachers that attended school in Latin America did not learn the mathematics with the purpose of teaching others - that is, they may know algorithms and general vocabulary, but they do not know necessarily how these early concepts were acquired and taught. Now, a second question arises: if knowing what and how children learned previously is important, why do we have to analyze entire mathematical lessons? In other words, what justifies the analysis of classroom practices? The answer is also grounded in research in that students' prior learning experiences are better understood when the learning is analyzed in the context of instructional dynamic (Ball & Forzani, 2007). Instructional dynamic takes into account the interactions among teachers, students, and content, in various environments. The common factor in the

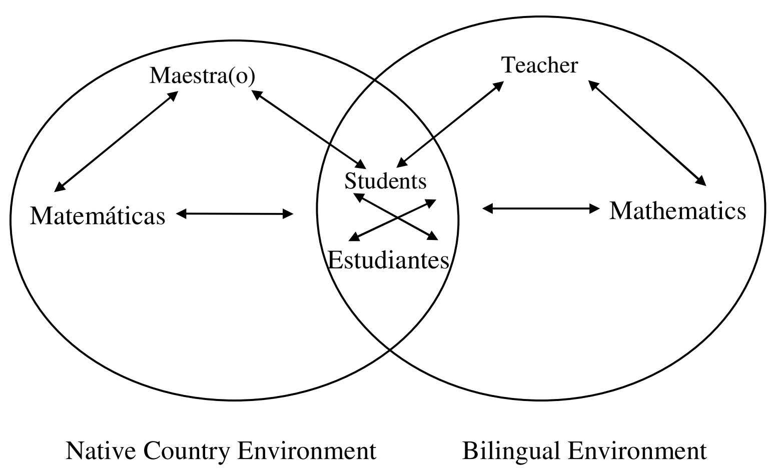

Figure 1. Instructional dynamic model (adapted from Ball & Forzani, 2007)

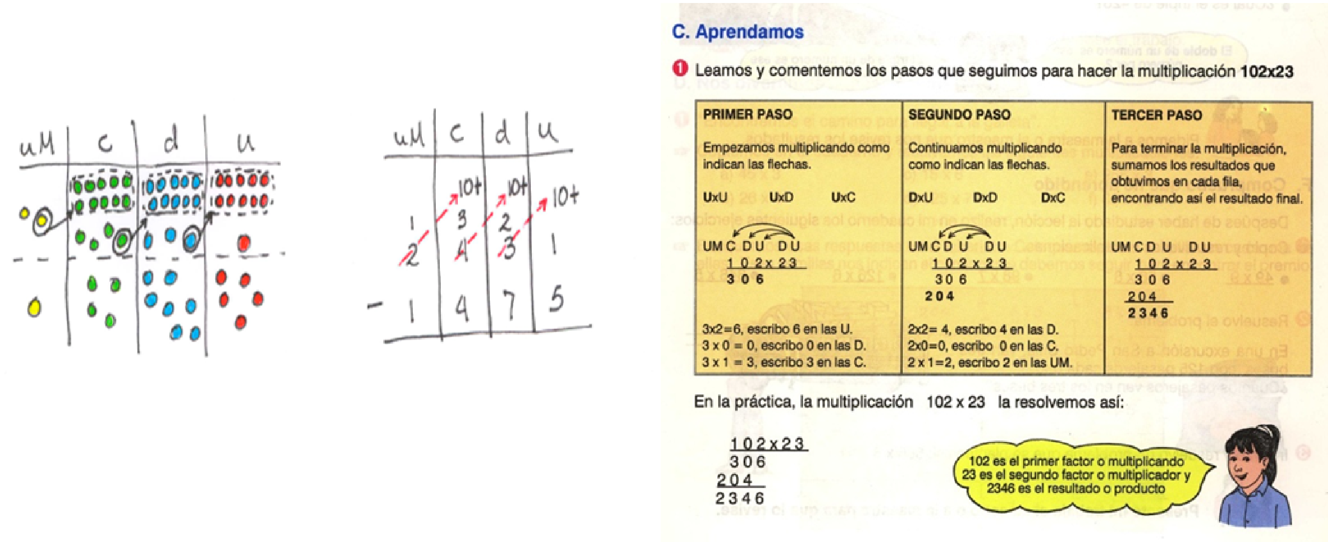

Figure 2. Subtraction algorithm work sample similar to *Matematica 3, Educacion Basica General*, Ministerio de Educacion (MEDUC), Panama, 2003. Multiplication algorithm from *Matematica 3, Escuela Morazanica*, Secretaria de Educacion, Honduras, 1997.

two instructional dynamics is the set of students (see Figure 1). The hypothesis for further research is that bilingual teachers' knowledge of students' instructional dynamic in their native country will help in the transition to the instructional dynamic in their current country of residence.

One more source of data that can be useful for understanding English learners' previous experiences in mathematics is the examination of textbooks. Figure 2 provides examples of the way third-grade Central American textbooks present the algorithms of subtraction and multiplication. Examining alternative algorithms can be a powerful tool to understand U.S. textbook procedures better. More instructional materials, lesson plans, and students' notebooks can be collected to enrich the other sources of data and to use them in the preparation of prospective and practicing mathematics teachers of English learners. This can be accomplished using the same type of activity from Sowder, Sowder, & Nickerson (2010) for preparing prospective teachers about the use of different algorithms by analyzing *nine* different ways children subtract 79 from 364, which includes the equal additions method used in many countries. The researchers claim, "when teachers realize that they must understand methods other than the ones they use, they are motivated to learn to attend to students' reasoning" (ibid, p. 54). The subtraction algorithm presented in Figure 2 can be added to the collection as yet another way to subtract using the abacus as a tool to demonstrate the regrouping process.

Closing Thoughts

Teaching quality in Latin American countries varies as in other parts of the world, and studying teaching practices suggests that variation is not necessarily associated with the students' social economic status or teachers' available instructional resources. This paper's lessons illustrate how teachers in rural areas lacking basic technology and manufactured teaching tools may use their knowledge of student concept development to support an environment for exploration in mathematics. More generally, analysis of teaching practices and materials can inform preparation of future teachers of English learners by increasing their awareness of the type of learning experiences some of their students may have had in their native countries.

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"Different solutions, interpretations, and approaches that are mathematically sound must be celebrated and integrated into class deliberations about problems. All members of the classroom group must accept the responsibility to engage with and support one another throughout the learning experience."

-- Position Statement on Equity in Mathematics Education (NCTM, 2008)

Discussion And Reflection Enhancement (DARE)
Post-Reading Questions

1. Describe the effect of the two teachers' pedagogical approaches on student learning. Which approach do you believe is more effective and why?

2. What are specific examples in the classroom dialogue excerpts of language issues that can inform the teaching of Latino/a students in the U.S.?

3. How can the use of teaching practices comparisons help teachers of Latino/a students?

4. What are some examples of alternative algorithms and how could they help Latino/a students learn the algorithms used in the U.S.?

5. Try this: Next time you teach geometry, give students two sticks and ask them to make a *right* angle. If any of your Latino students make a 180º angle, can you tell if they are interpreting the word "right" as in "straight" – since in Spanish (recto), the word "right" can mean straight? (Note: A straight angle in Spanish is called "ángulo llano o colineal.")

6. As a U.S. teacher, do you find it surprising and/or inspiring that Latin American classrooms with very limited resources have had high achievement? Why?

"DARE to Reach ALL Students!"

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"Students who are not native speakers of English, students with disabilities, females, and many nonwhite students have traditionally been far more likely than their counterparts in other demographic groups to be the victims of low expectations."

– Principles and Standards for School Mathematics (NCTM, 2000)