Learning to Leverage Obstacles, Resources, and Strategies in Math Classes With Multilingual Learners

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Abstract

Mathematics teachers must be ready for diverse classrooms, where students who are multilingual learners (MLs) bring new dimensions to the teaching and learning. While MLs face obstacles to learning particular to their linguistic and cultural background, they also bring resources and strengths to bear that can be applied to teaching and learning. We have developed a Challenge-Based Instructional activity to help teachers leverage their experiences of teaching math to ML students, to better understand the obstacles and resources, and to select more effective pedagogical strategies particular to this context. This paper reports the benefits teachers gain in implementing this activity.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What obstacles do multilingual learners (MLs) face in the English-taught mathematics classroom?

2. How can MLs' linguistic and cultural knowledge serve as resources for their mathematics learning during mathematics instruction?

3. How does teachers' knowledge of MLs' obstacles and resources affect their selection of teaching strategies?

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Mathematics teachers in the U.S. have an increasing wealth of cultures and languages at hand in their classrooms. Although most teachers in the U.S. use only English to teach mathematics, children who speak a language other than English will often be more comfortable using a language other than English to communicate in the mathematics classrooms (Chval & Chavez, 2011; Gutstein et al., 1997). Consequently, teachers must have a deep understanding of multilingual learners (ML[1]) and learn how to teach mathematics within linguistically complex situations. Our research and observations with middle school teachers indicate that teachers can achieve this learning by envisioning and discussing specific teaching instances involving MLs. To help teachers develop a deeper understanding of the complexities of culture and language encountered when teaching specific mathematics content in the multilingual classroom, we created a unique professional development activity, called the "Teaching Multilingual Learners (TML) Project." In this paper, we present how we implemented the TML project and share findings from it.

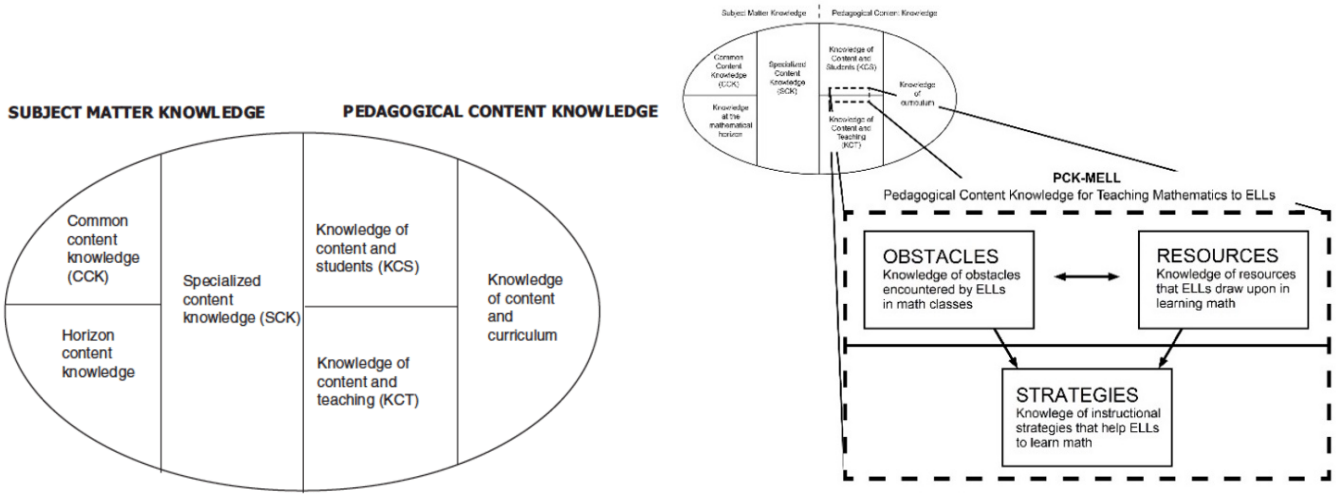
Theoretical Background and Overview of the TML Project

We designed the TML Project to develop pedagogical content knowledge (PCK) for teaching mathematics in diverse classrooms, specifically when MLs are present.

[1] Drawing from John (2019), we use the term *multilingual learner* in this paper, instead of the previously used terms *English language learner* or *emergent bilingual*, broadly to refer to a student who "can be an immigrant, a child of an immigrant who is bilingual, a permanent resident, a naturalized citizen, or an international student, who attends the academic English classroom today" and may come with "differing language abilities, learning styles, learning attitudes, and opportunities to communicate in English" along with "varying levels of inhibition, risk, and self-confidence" and "countries, cultures, sociocultural, sociopolitical, and socioeconomic backgrounds, all of which impact their learning significantly" (p. 41).

Figure 1

MKT (Ball, Thames, & Phelps, 2008; left) and Pedagogical Content Knowledge for teaching mathematics to ELLs (Wilson, 2016) framed within MKT (right) used by permission.



To do this we adopted Wilson's (2016) model of *pedagogical content knowledge for teaching mathematics to English Language Learners* (PCK-MELL) as a framework. The PCK-MELL model (Figure 1) draws on Shulman's seminal theorization of PCK (1986) and locates this knowledge within Ball, Thames, and Phelps's (2008) familiar model of *mathematical knowledge for teaching* (MKT). According to the PCK-MELL model, effective mathematics teachers of multilingual students draw upon three special aspects of knowledge related to their knowledge of content, students, and teaching, including knowledge of: A) Obstacles encountered by MLs in mathematics classes that are taught in English; B) Resources that MLs draw upon both to do and to communicate mathematics in these classes; and C) Instructional strategies that teachers may use to support MLs in mathematics, which is informed by teachers' knowledge of obstacles and resources. Wilson (2016) elaborates these domains of PCK-MELL in detail. For instance, *Obstacles* that ELLs face may include high-level speech formats of teaching (i.e., lecturing) and word problems that are linguistically complex by the multiplicity or multiple meanings (polysemy) of words. In contrast, *Resources* that ELLs draw upon in mathematics classrooms may include fluency in their non-English language to grasp and express concepts taught in English (by way of cognates, for instance), gesturing, and prior mathematical knowledge. Examples of *Strategies* include: teachers' usage of students' prior (mathematical, but also social and cultural) knowledge for teaching, using visual supports and displays (gestures, pictures, objects, word walls, etc.) and using students' own in-class writings and speech for teaching (Chval & Chávez, 2011).

A goal of the TML project was for teachers also to perceive MLs through an affordance lens: to see *difference not deficit* (Lewis, 2014) in MLs. The project was designed to help teachers to grasp the importance of the MLs' bilingualism and their diverse cultural backgrounds as resources in learning and communicating mathematics.

Phases of the TML Project

To accomplish these goals, we adapted a discovery learning method that has been used effectively in engineering education: *challenge-based instruction* (CBI). CBI uses a series of six Phases called the *Legacy Cycle* (Crown, Fuentes, & Freeman, 2012; Schwartz, Lin, Brophy, & Bransford, 1999), in which students are first given a *challenging problem*; they then *generate ideas* about the problem; they consider the challenge from *multiple perspectives*; they *research and revise* a plan for solving the challenge; they *test their mettle* by trying out

their ideas and solutions; and finally, they *go public* by displaying their findings. We explain below the phases of the project as they were designed to be carried out in the TML project.

Phase 1: The Challenge. At the outset of the TML project, the teachers were formed into teams of two or three "assessment item-writers" and given a version of the following challenge:

> *You and your colleagues in the Psychometrics Department of the State Education Agency (SEA) have been tasked with drafting the next generation of teacher licensure exams. Policymakers are calling for exams that are more reflective of the actual content and actual tasks of teaching mathematics. Your group will work closely with the Equity Taskforce of the SEA to ensure that the exam items respond to a particular need: teaching mathematics to multilingual learners. This project will require your team to create three multiple-choice assessment items designed to measure whether prospective teachers who are about to receive their teaching licenses know how to teach specific mathematical concepts from school mathematics to students who are multilingual learners.*

The item-writing task given above played the central cognitive challenge in the TML project. It elicited interest in the topic and motivated effort toward the following tasks.

Phases 2 and 3: Generate Ideas and Multiple Perspectives. Per the CBI Legacy Cycle, completing the challenge required that teachers first generate ideas about teaching mathematics to MLs. Without precisely defining them or offering examples, we required that teachers initially thought about (brainstorm) obstacles faced by MLs and resources possessed and used by MLs in math classes. This was so that teachers could begin to envision the work of teaching MLs and discovering what knowledge would be needed. After developing brainstormed lists of obstacles, resources and strategies, teachers were instructed to consult with additional, text-based and other, sources of ideas about teaching mathematics to MLs. Furthermore, we required that teachers contextualize the teaching of MLs in a specific school mathematics topic.

Phase 4: Research and Revise. In this stage, the teachers wrote their items, submitted them to their group members and often (but not necessarily) to the professor for comment and criticism, and then revised their assessment items. Furthermore, to facilitate their careful consideration of more and less formidable obstacles for MLs, and of more and less powerful resources within these MLs' grasp or available teaching strategies, teams of item-writers were asked to justify the correctness of the "correct" answer options for their multiple-choice items and to explain the incorrectness of all "distractor" options in detail.

Phases 5 and 6: Test Your Mettle and Go Public. In this stage, teachers submitted their licensure items for discussion in the larger group. Discussing items with peers, by justifying their "correct" or "incorrect" answers, was an important procedural part of the TML project. When teachers were asked to "Test their Mettle" and "Go Public" with their assessment items, they were encouraged to articulate their thinking about teaching mathematics to MLs, and they responded to the myriad "What if?" questions their peers raised that challenged their thinking or altered the "correct answer" by modifications to the variables in their items.

Implementing the TML Project

This section illustrates our actual implementation of the TML project with participants.

Participants and context. While we originally planned to use the TML project with pre-service teachers, this paper's implementation context is from a week-long summer mathematics professional development institute for 35 in-service middle school math teachers from different U.S. school districts along the U.S.-Mexico border.

Teachers were predominantly from cities with large percentages of people who are of Latino/as or of Hispanic heritage, and who worked at public schools that have large numbers of MLs from Spanish-speaking backgrounds. Hence, it was imperative to us that teachers gained a deeper-than-surface familiarity with complexities that would arise when teaching in linguistically and culturally diverse classrooms. Many of the teacher participants had novice to expert fluency in the Spanish language, and some also reported to have been MLs during their primary or secondary schooling. Consequently, the outcomes of this study mainly concerned MLs from Spanish-speaking traditions. However, we believe that the core ideas and findings from

TML project could be adapted for usage in other linguistic or ethnic contexts.

Modifications of TML Project. Our actual implementations of the phases of the TML project have varied slightly depending on the professional development context and whether in- or pre-service teachers participated. During this summer implementation, the TML project was introduced and completed entirely on one day of a week-long mathematics professional-development institute. Completing the project in one day required modifications to the phases of the project (explained further below) and may have limited the breadth and depth to which teachers could develop their thinking. For example, the generating ideas, researching and revising phases were essentially seamless. Spreading this work over two or three days would have allowed for more research and collaboration. Yet, completing the TML project over one day seemed to provide a self-contained and focused look at teaching mathematics with MLs.

Outcomes of the TML Project

In this section, we illustrate in-service teachers' thinking about the *Obstacles*, *Resources*, and *Strategies* involved with MLs in mathematics. We first explain the item-writing tasks that we used to prepare teachers for the main work of the TML project, both to contextualize the outcomes and to help readers consider implementing the project.

At the start of the day, we began Phase 1 by setting up the challenge, informing teachers that they would be working in groups of four people as "educational licensure test-developers." In our prior work on the TML project with PSTs, we had observed that participants can struggle with the abstract thinking required for writing items intended to measure teachers' knowledge about teaching mathematics. Hence, to introduce the kind of thinking that they would be doing as well as to bring MLs into the focus, we discussed in whole-group an example test item from a teacher licensure exam, an item concerned with MLs. By analyzing such a test item, teachers could deduce the kind of teacher knowledge that the item was intended to measure, and how the distractor options failed to capture that knowledge. But an important result of looking at the selected test item was also in demonstrating the lack of relevancy of some such items to the actual work of teaching; teachers commented that the item seemed to come from a textbook and not from the classroom. This observation motivated them to try to write more authentic items!

Furthermore, we introduced teachers to the two, seemingly opposing, PCK-MELL domains of *Obstacles* and *Resources* by having different groups read and think about teaching vignettes found in the literature concerning MLs: a vignette concerning *Obstacles* that MLs face in math classes (many examples exist) and a vignette concerning MLs' usage of language as a *Resource* (a nice example is given in Moschkovich, 2005, p. 133).

Teachers' Knowledge of Obstacles, Resources, and Strategies

Phases 2 and 3 involved having individual groups brainstorm lists of either *Obstacles* or *Resources* in place with MLs, or *Strategies* for teaching MLs mathematics that were then shared with the whole group for discussion and revision. The outcomes of this initial brainstorming task were insightful in several ways. To begin with, compared to similar lists evinced by PSTs with whom we have completed the TML project, the number of items in each PCK-MELL category that the in-service teachers were able to identify were much more extensive, an indication of the in-service teachers' more advanced teaching and learning experiences.

With respect to obstacles faced by MLs, teachers identified different ways in which language was a problem ("academic vocabulary vs. social language"). They also elaborated the extent to which language was seen within the instruction phase of teaching, as well as throughout the assessment phase by their own statement that, "Everything is verbal." Teachers identified problematic linguistic structures that cause students difficulty (see Martiniello, 2009): homonyms, homographs. But they also named several non-linguistic obstacles to MLs' learning of mathematics (e.g., self-confidence), as well as obstacles found in students' school and home environments (e.g., the threats of bullying, or economic hardship). For example, teachers recognized that some MLs faced bullying at school or faced

instability and poverty at home, which could be formidable obstacles to learning.

With respect to resources that their MLs drew upon, the number of resources teachers articulatedexceeded the number of obstacles, which had not previously happened in our work with PSTs. Examples of resources named included: prior knowledge, cognates, peer tutoring, visuals, word walls, gesturing, and native language use. This may be a positive indication that participating teachers in this group were better able to see the unique assets that MLs possess, beyond their challenges alone. But it was also evident that the resources and strategies lists were not disjoint; several resources were also listed as strategies, and this overlap was addressed during whole-group discussion. As teachers explained it, *when a teaching strategy has been implemented, that strategy often results in a resource that MLs then utilize for their own learning*. Word walls and peer tutoring were examples: Teachers used them as strategies and MLs used them as resources. Indeed, peers who also speak the same languages can be among the greatest resources for MLs in mathematics classes, where translating concepts into a different language benefits both the ML and the bilingual peer.

Operationalizing Obstacles, Resources, and Strategies: Writing Test Items

At this point (and after a good lunch!) teachers were ready for Phases 4 – 6 of the TML project: writing test items in groups and sharing with others. Three selected assessment items are presented from teacher groups that focused, respectively, on Obstacles, Resources, and Strategies.

TML Example 1: Obstacles That MLs Face in Mathematics Classes

Teacher group #7 focused their work on obstacles MLs face and they created the following situation involving an ML in the classroom of a teacher who used "long lectures:"

> *Mrs. Yanez, known for her long lectures, has a new ELL [English language learner] student in her class. Chum Lee comes with a strong concept of math, but very limited in the English Language. Which obstacle would affect this student the most?*
>
> *A. Everything is verbal*
> *B. The confidence level*
> *C. Academic Langauge*
> D. *Comprehension*

This simply stated item presents a complex situation and decision for the respondent. The teachers who wrote the item intended to present a situation in which the ML was of a different linguistic and cultural background than the instructor: Mrs. Yanez is evidently Latinx, while Chum Lee is Asian. Furthermore, Chum Lee has limited knowledge of the English language, but is perceived as being strong in math.

The potential obstacles that would affect Chum Lee illustrate the different perspectives the teachers who wrote the item took, and they are also reflective of obstacles discussed in the earlier brainstorming session of Phase 2. Some obstacles here seem to be related to Chum Lee's external classroom environment and some to his own prior experience. For instance, "Everything is Verbal" may refer to the nature of Mrs. Yanez's instruction or to the emphasis in standardized state assessments, and hence in Mrs. Yanez's class, on solving word problems. Contrarily, "Confidence Level," "Academic Vocabulary" and "Comprehension" seem to imply limitations found within Chum Lee's prior experiences that the teachers envisioned as obstacles for this student. As teachers discussed this item, the relative difficulty posed to Chum Lee by any of the four given options was deliberated by all: each item could be a real obstacle. Some teachers found it difficult to choose the greatest obstacle. To others the choice was clear.

TML Example 2: Resources That Multilingual Learners Call Upon in Learning Mathematics

Teacher group #3 focused their work on resources that MLs employ in their classroom involving mathematical and grammatical complexity perhaps familiar to teachers of algebra:

> *Mr. Zamora is teaching 7^{th} grade students sequences. He knows the students will have difficulty comprehending why $3n$ means 3 times n. Which of the following resources will help address the students' difficulty?*
>
> *A. Prior knowledge*
> *B. Peer tutoring*
> *C. Vocabulary*
> *D. Visual Model*

The item describes a challenging mathematical situation for both MLs and students who speak English as their first language. This is due to a symbolic representation of a concept that students may not be familiar with regardless of their language background. However, the teachers who developed this item considered that MLs may find an even greater challenge than those students who speak English as their first language, because the explanation of "3*n*" could heavily depend on a verbal description. The answer options presented as potential resources for MLs showed different perspectives taken by participating teachers, as in Example 1. The choices also reflected earlier brainstorming (as described in Phase 2), where teachers engaged in potential resources possessed uniquely by MLs.

Furthermore, from the perspective of PCK-MELL, we noted how closely teachers' knowledge of obstacles students faced was linked with the ways that teachers expressed knowledge of resources that MLs could draw on to learn mathematics. The situation begins with an anticipated "difficulty" to which a list of optional "resources" could be applied as solutions. We also noticed that, among the resources offered, two of them could be advantages already in MLs' possession (prior knowledge and vocabulary), and one was a resource that teacher could offered (peer tutoring), the fourth option (visual model) could originate with the ML or the teacher could provide. These options also reflected the apparent intersection of MLs' resources and teachers' strategies discussed above. But, the authors of this item offered us a real challenge: selecting which of these resources would best help the ML to arrive at an understanding of the targeted symbol is not easy. In group discussion of this item, it was noted that, "one size does not fit all," and different resources would vary in value to different MLs.

TML Example 3: Strategies for teaching MLs mathematics

Teacher group #5 focused their work on the strategies for teaching MLs. The following item presented the integration of four instructional strategies suggested for the benefit of an ML student learning about unit rate and slope:

Which of the following strategies would benefit an ELL [English language learner] student learning about unit rate/slope?

A. The teacher will model an example for the student to implement on their own
B. Provide students with manipulatives to reinforce the concept
C. Provide a word wall as a reference
D. Small group instruction to provide guided practice on the concept

This item demonstrates the value of the Phase 2 brainstorming to the TML project: each answer option had been specifically suggested during the brainstorming phase. The four answer options portrayed diverse instructional strategies to facilitate the learning process for MLs, which included both teacher- and peer-led components, as well as visual supports and displays. Finally, it became evident in whole-group discussion that all students, and not only MLs, would benefit from teachers' usage of these strategies. Teachers acknowledge that almost all students are supported in mathematics through the strategies set forward. Therefore, discussion of items like this one focused on carefully selecting the strategy that, beyond being helpful for all students, was singularly essential for MLs.

Findings of the TML Project

Based upon our implementations of the TML project, we made two observations relevant to teachers' gaining and sharing of knowledge about MLs in the mathematics classroom. First, the TML project has the potential to highlight multilingualism as a resource for students. In-service teacher participants developed rich lists of resources associated with MLs. In our work with PSTs, the process of finding and generating ideas about MLs' resources in specific mathematics contexts has been more challenging for them than producing ideas about MLs' obstacles or even strategies for teaching them. This may be related to a tendency to perceive nontraditional students, such as MLs, through a deficit lens rather than an asset lens (Gutiérrez, 2008; Lewis, 2014). The TML project enables us to address this imbalanced perspective and to bring into teachers' sight the particular advantages and resources that MLs have in learning mathematics.

Secondly, teachers thought about MLs strategically. Multiple-choice items written by in-service teachers

tended to require a decision between better and worse instructional strategies or teaching decisions. Even items that were intended to indicate knowledge of obstacles or resources that MLs encounter were frequently framed as decisions about what the teacher should or should not do in the circumstances. Compared to PSTs we have worked with, it seemed that in-service teachers showed strength in translating knowledge about obstacles and resources into actionable strategies, which is also implied by the two downward arrows from obstacles and resources to strategies (Figure 1). Furthermore, most teacher items presented an obstacle (not resource) followed by multiple choices between strategic decisions. Hence, we observed that the action from obstacles to strategies seemed to be stronger than from resources to strategies. This observation may be related to the aforementioned challenge that some participants have had in identifying MLs' resources. But this difference may also be explained by observing that teachers' possession of actual classroom experience may equip them with more strategic perspectives oriented toward handling challenging situations in the classroom.

Suggestions for Using the TML Project

The major goal of the study was to understand how the implementation of the TML project might assist teachers to develop a deeper understanding of the complexities of culture and language commonly encountered when teaching mathematics in classrooms with MLs. The discussions that occurred between in-service teacher participants in our implementations of the TML project were very lively. Framing issues as multiple-choice questions that have only one correct answer elevates, perhaps artificially, but we believe beneficially, the stakes. Participants really want to know, "What is the right answer?" Nevertheless, we suggest some cautions and areas for improvement of the TML project. First, the facilitator should give careful attention during the activity at the outset of the project in framing and preparing teachers for the item-writing task, which can be challenging for participants at a meta-cognitive level. As mentioned earlier, participants can have initial difficulty in understanding that they are instructed to write multiple-choice questions about teaching specific mathematics topics and not multiple-choice questions that are merely mathematics problems. PSTs and in-service teachers completing this project may need more time and more examples.

Second, the items that teachers in our sample produced, which were often classroom problem situations, possibly including hypothetical student work or MLs' spoken responses, were primarily derived from teachers' subjective personal experiences. This was expected and intentional, given the one day of the professional development that was allotted for the activity. Our results indicate that the teachers understood the needs of the MLs, the obstacles MLs encountered, and how the resources and strategies teachers identified and used can impact the whole learning experience. A more prolonged activity would allow for a deeper Research and Revise phase of the project in which teachers could compare their own experiences against relevant results found in the research and practitioner literature. After completing the TML project, teachers may even benefit from considering other aspects of their work, such as lesson-planning, through the lens offered by the PCK-MELL framework.

Conclusion

We find that when teachers write and discuss multiple choice items in the TML project, implicit beliefs about what they should know and do with MLs are brought forward for detailed consideration and discussion. The focus may be expanded to what teachers of other particular groups, traditionally marginalized groups for instance, need to know and do to be effective teachers. There are many ways to frame the challenge, and the results should produce lively and fruitful discussion.

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Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What obstacles do multilingual learners (MLs) face?

2. What linguistic and cultural resources do MLs possess and access, and how can teachers leverage these factors to make strategic instructional decisions that positively impact mathematics learning?

3. How would you modify the TML project to address a different aspect of mathematics teacher knowledge? For instance, how could this project assess knowledge for equitable mathematics teaching more generally or mathematical knowledge for social justice teaching specifically?

4. How would you modify the TML project to uncover obstracles, resources and strategies related to different special student populations, such as MLs from non-Spanish-speaking backgrounds, hearing-impaired students, or gifted students?