Fencing in the Goats: Adaptation of Modeling Framework for Emergent Bilinguals

I, Ji-Yeong
Iowa State University

Kait Ogden
Kate Mitchell Elementary

Ricardo Martinez
University of Nebraska-Lincoln

Betsy Araujo Grando
Iowa State University

Abstract

Through collaboration between a teacher and a researcher, we developed 5-Act Task Framework by utilizing modeling as vehicle, aiming to support Emergent Bilinguals (EBs) to engage in high-quality mathematics. A classroom episode is described to illustrate how 5-Act Task-based lessons were implemented effectively for EBs by connecting the students' language and real-life context. With the belief that EBs are capable of mathematical modeling with appropriate support, we implemented the 5-Act Task Framework that provides EBs with access to rigorous mathematics while developing language throughout the modeling process.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. How do you feel when your student speaks a language in class that you don't understand? What do you do (or would you do) when you hear a language that you don't understand?

2. How do you think Emergent Bilingual students engage with mathematical modeling? How can mathematical modeling be fully utilized by Emergent Bilingual students?

3. How do you think mathematics teachers usually support Emergent Bilingual students to solve mathematical modeling tasks? How do you want to support your Emergent Bilingual students in engaging in mathematical modeling tasks?

Acknowledgement: This work is supported by a 7-12 Classroom Research Grant from the National Council of Teachers of Mathematics-Mathematics Education Trust.

Ji-Yeong I (jiyeongi@iastate.edu) is an assistant professor of mathematics education at Iowa State University. She taught mathematics and science for diverse students in urban areas before she started her research career. Her research interests include Emergent Bilinguals, mathematical modeling and problem solving, and culturally responsive teaching.

Kait Ogden (kaitlin.ogden26@gmail.com) is a K-5 ESL teacher in Ames Community School District. She spent her first 5 years as a middle school ESL math and science teacher working in the Intensive English Language Center with newcomer students from diverse backgrounds. She continues to be involved with critical consciousness and equity work within the Ames district.

Ricardo Martinez (rmartinez21@unl.edu) is an Assistant Professor in Mathematics Education in the department of Teaching, Learning and Teacher Education at the University of Nebraska-Lincoln. His research centers paradigms of Critical Youth Studies in mathematics education.

Betsy Araujo Grando (betsya@iastate.edu) is a Ph.D. student in Human Computer Interaction at Iowa State University. Her interest is the use of educational technology for teaching math to Emergent Bilingual students. She has experience teaching mathematics abroad and working as an instructional designer.

Fencing in the Goats: Adaptation of Modeling Framework for Emergent Bilinguals

Ji-Yeong I, Kait Ogden, Richardo Martinez, and Betsy Araujo Grando

In order to have an excellent mathematics program, students need access to a high-quality curriculum along with high expectations (National Council of Teachers of Mathematics, 2014). However, it is no secret that teachers experience difficulty when guiding Emergent Bilinguals (EBs, a.k.a. English learners) to understand word problems. Interviews conducted by I et al. (2019) revealed that some teachers only want to use computational tasks for EBs because of the belief that simple computations do not pose a language barrier. However, with only easy computation tasks, EBs cannot engage in problem-solving or reasoning that cultivates a deeper understanding of high-level mathematics. For this reason, researchers (e.g., Moschkovich, 2010; Celedón-Pattichis & Ramirez, 2012) assert that EBs should have an equal opportunity to learn challenging mathematics. Making connections between mathematics and EBs' lives has been recommended to help EBs make sense of mathematics (e.g., Chval & Chavez, 2011).

Responding to these demands and recommendations, we designed modeling-based lessons for teaching high-quality mathematics to EBs in order to investigate how modeling can be used as a powerful tool to teach mathematics to EBs. The discussions around modeling imply that modeling must include challenging tasks while incorporating situations connected to students' lives, which complements recommendations for teaching EBs. However, teachers may hesitate to use modeling when teaching EBs due to the heavy language demand associated with modeling tasks. However, well-designed modeling lessons can engage EBs by utilizing social interactions and real-life contexts that are particularly crucial for EBs (Anhalt, 2014). With the belief that EBs can do modeling with appropriate support, we aim to develop a framework that empowers EBs to have access to rigorous mathematics.

5-Act Task: Modeling for Ebs

The Common Core State Standards for Mathematics (CCSSM) includes modeling as a standard of mathematical practice as well as a high school domain. Modeling is defined as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (NGA & CCSSI, 2010, p. 72). These inclusions of modeling represent two different functions of modeling: modeling as vehicle and modeling as content (Galbraith, 2012). Modeling as vehicle is using modeling as a tool to teach mathematical concepts and procedures while modeling as content treats modeling as a curriculum itself and enables students to use mathematics to solve an ill-defined, authentic, and real-life problem. These two functions of modeling may be complementary in a lesson when a teacher has a goal of teaching a specific concept and using a modeling activity to teach that concept. Our focus in this study is modeling as vehicle because our goal is to implement modeling as an instructional approach that can be used on a daily basis by teachers. And throughout this paper, the term *modeling* refers to modeling as a vehicle to learn and teach a mathematical concept, which is more related to the modeling standard of mathematical practice, "apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (NGA & CCSSI, 2010, p. 7)

To develop the modeling framework, we adapted the following 3-Act Task framework by Dan Meyer (2011):

> Act 1: A story is given and students identify the conflict of a real-life story.
> Act 2: Students choose tools, resources, and make assumptions they need to solve the conflict.
> Act 3: Students build models and find a solution to the problem and continue with an extension.

Instead of heavy text, 3-Act Tasks often utilize multimedia and visuals to describe a problem within a real-life story. We added more actions to this framework to help EBs make sense of the given story (which may include cultural biases or language barriers) and to support EBs in expressing their mathematical thinking. Hence, we developed 5-Act Task by adding Act 0 to support EBs' ability to make sense of a problem before starting Act 1 and by adding Act 4 at the end to ensure EBs share their process (see Appendix).

Act 0 is crucial in that it reflects a set-up stage needed to engage in teaching EBs (I & de Araujo, 2019), where teachers support EBs' understanding of both language and mathematics embedded in Act 1. For example, teachers assess EBs' prior knowledge related to the story and provide appropriate scaffolding. Act 0 benefits all students because it helps them better comprehend the problem context (Jackson et al., 2013), but it is essential for EBs because unknown language or cultural biases may prevent them from finding mathematical entry points. Act 1 begins with a story, either shown in a video or demonstrated in class and ends when the questions/problems are posed. The real-life stories have conflicts that provoke students' curiosity, leading to questions being posed with or without the teacher's guidance. The creation of a question builds on quantities in the conflict of the problem, which leads to the development of the models used in the next phases. In Acts 0 and 1, sharing and valuing language and culture is crucial for EBs. Revisiting Bane (1992), Nieto (2018) reminds us that multilinguals like to hear their multiple languages in school: "when culture and language are acknowledged by the school, students are able to reclaim the voice they need to continue their education successfully" (p. 120). Hence, we should not just focus on delivering meaning(s) of a word, but also ask EBs to share their stories, using their language as a means to connect to their culture. When a teacher acknowledges students' multiple languages, students can begin to see their language, culture, and themselves as part of the classroom (Macedo & Bartolomé, 1999).

In Act 2, students are asked to find information and make assumptions needed to answer the question created in Act 1. Act 2 provides students with another chance to have ownership over how they approach the task because they can select and explore whatever tools/information they think they need to solve the problem, rather than being given all necessary information and quantities. Act 3 represents key parts of the solving process in that students formulate, compute, interpret, and validate their models. Based on the result of Act 3, students may return to Act 0 (e.g., to be reminded the meaning of a word), to Act 1 (e.g., to justify the result in the context), or to Act 2 (e.g., to gather more information). The role of Act 0 is reemphasized in Act 3 in that Act 0 gives EBs the background to figure if both the model and solution make sense in the context, and if not, they modify either their models or the previous steps, which reflects the iterative process of modeling. Finally, Act 4 starts when EBs communicate their solutions and reasoning behind their models. Because Act 4 involves productive language modes (writing and speaking), this step can be challenging for EBs. However, adding this step is critical because it provides an opportunity for students to develop their bilingual proficiency. To reduce EBs' anxiety speaking English, teachers should design Act 4 carefully by applying various strategies such as having group/pair presentations, allowing EBs to explain with visuals and gestures, giving enough wait time for students to prepare what to say in English, and writing sentence frames on the board.

Setting and Task Selection

In this project, collaboration between a teacher and a researcher plays a crucial role in that the collective expertise balances each other's needs: The teacher's knowledge of students complements the researcher's knowledge of modeling. The middle school teacher (Ogden) acted as a researcher-teacher, and the researcher (I) acted as a teacher-learner-investigator, as they co-developed, co-taught, and co-analyzed the lessons (Jung & Brady, 2016). The study was conducted in a classroom consisting of 11 EBs in grades 7 and 8, who came from Sudan, Myanmar, Thailand, Congo, Guatemala, and Honduras. Eight students arrived in the U.S. as refugees, and seven of 11 students spoke more than two languages. All the students had been in the U.S. for less than two years and were assessed by their school district to be at the beginning level of English proficiency. Also, most of them had not experienced formal education for at least two years, and three students were illiterate in their own language.

The teacher chose a mathematical concept that she was required to teach in the semester and the researcher suggested tasks, and then they discussed, selected a task to be used, and co-developed a detailed lesson plan. While co-developing the lesson introduced in this paper, a pattern task was chosen because it encompassed algebraic reasoning and generalization that were connected to the target concept, creating equations. The task has been posed with various contexts such as combinations of tables and chairs, cows and pens, and goats (or other animals) and fence sections. Selecting a context was imperative because a real-world integration is a core of modeling, and familiarity with a context provides EBs with access to mathematics. Goats were chosen because the teacher noticed goats were the animal closely related to the EBs' prior experiences. Pigs were not considered to respect some students' religious beliefs, and we excluded chickens because chickens do not usually reside alone when fenced. Although authenticity was considered, we cannot deny the base case of the scenario of fencing goats in a row may sound artificial, but this setting was necessary to achieve the objective of generalizing an expression, following our view of modeling as vehicle.

5-Act Task Lesson

In this section, we introduce one of the 5 Act Task lessons developed and implemented in this study. The mathematics and language objectives were "write expressions/equations from a real-life situation using increasing patterns and functional patterns" and "use precise terms to explain their algebraic thinking using the expressions they developed," respectively. We started the lesson by grouping students and placing a shared large piece of chart paper and markers in each workspace. Then, we began Act 0 by showing the class a picture of a goat and asked questions to initiate a conversation. The following dialogue illustrates this moment:

> **Teacher**: Does anybody have goats in their country? [one student raised a hand and all said "Yes!"] and you had to take care of them?
> **Students**: [all answered] Yes! [one student said "My country has it, but not own."]
> **Researcher**: What do you call goats in your language?
> **Teacher**: Raise your hand [she gestured by raising her hand] if you know how to say *goat* in your language.
> **Students**: Me, Me, Me [With hands raised, students' body language shows excitement]
> **Student 1**: *mbuzi*
> **Researcher**: *m-buzi* [repeating what Student 1 said]
> **Student 2**: *cabra*
> **Teacher**: *cab… cabra*
> **Researcher**: In Korean, it is called *yumso* [multiple students repeat *yumso*]

All students were excited to share their languages and experiences related to goats. The word *goat* was said in Arabic (maeiz ماعز), Burmese (sate ဆိတ်), Korean (yumso 염소), Spanish (cabra), and Swahili (mbuzi). To introduce the story in Act 1, Researcher (this paper's first author) used a document camera to show a goat paper cutout and four popsicle sticks around the goat (Figure 1, left) in saying, “If you have one goat, you need four sections of fence.” Afterward, Teacher (the second author) wrote *G* and *S* on the whiteboard and also wrote 1 and 4 associated with each letter. Then, we asked what the letters *G* and *S* were for, and students replied that *G* represented the number of goats, and *S* the number of fence sections.

We continued to demonstrate more cases with goat cutouts and popsicle sticks, but this time, we asked a question, “If you have two (or three) goats, how many sections of fence do you need?” without placing all popsicle sticks (Figure 1, middle and right). Then, we gave a more challenging question, “What if you have six goats in a row?” “What if you have 20 goats?,” without showing the case. After students answered with many numbers, we asked the following questions for Act 2: “What do you need to find an answer?” and “What information/kind of stuff do you need?” We provided various tools, such as counters, popsicle sticks, multiplication charts, and goat cutouts, and said, “You can use any tools you want.” Each group chose their tools and wrote out their ideas on the shared chart paper. However, we did not provide more than four goat paper cutouts and six sticks to prevent students from counting. Then, Act 3 began. We walked around to monitor and help students. We observed one group arranged goats and then sticks around them, and another group placed only sticks without goats.

Figure 1
Three cases with one to three goats and the sections of fence needed

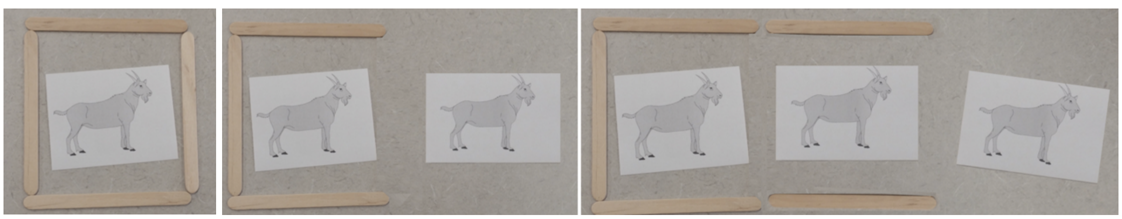
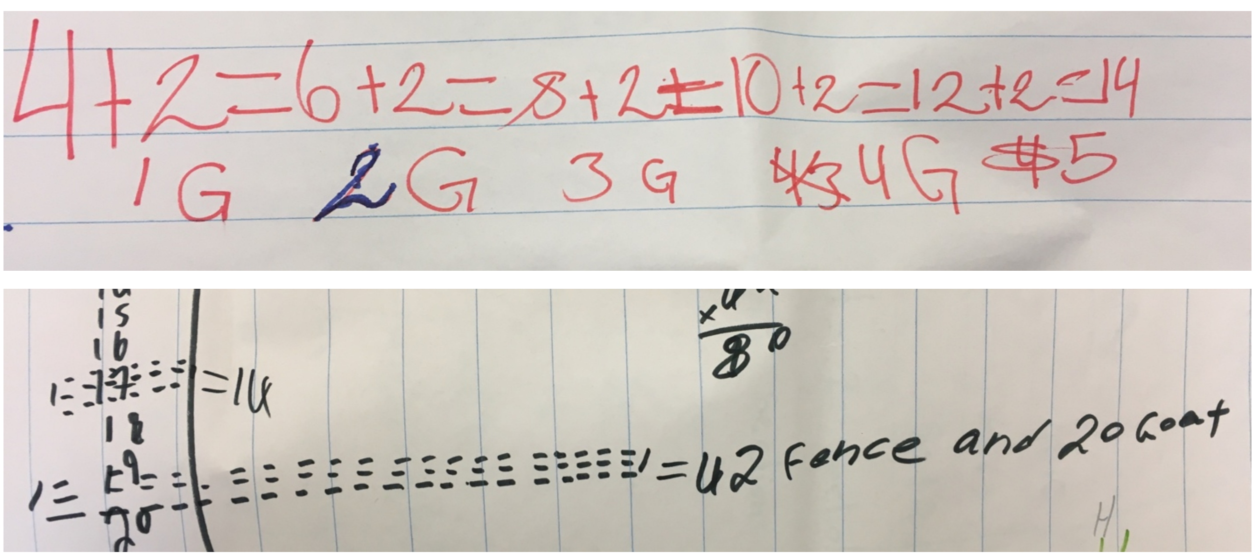


Figure 2
EBs’ initial modeling before teachers’ guidance



In other groups, one student started writing equations on the chart paper without using manipulatives (see Figure 2 top). Although he misused equal signs (4+2 is not equal to 6+2), the student found adding two was the repeating pattern for the number of fence sections. Then, one of his group members stopped him and showed his drawing model (Figure 2 bottom), using line segments to represent both goats and fence sections. This student found a correct answer by counting the line segments. To promote algebraic reasoning beyond rote counting, we gave the final task, "What if you have *G* goats?" The students first struggled with this generalization process, so we provided some guidance, such as "why don't you use a table?" and "why don't you list equations showing the relationship from one goat?" With guidance and a reminder of Acts 0 and 1, the students could build models for *G* and finally figure out the expression of the relationship between *G* and the number of fence sections needed (Figure 3).

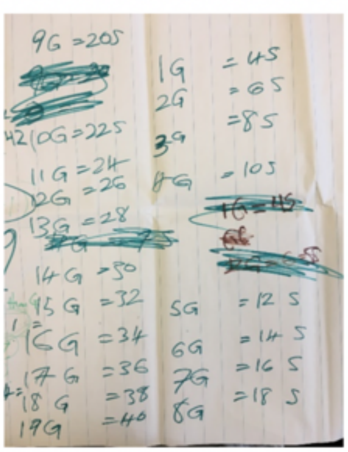
The model in the left side of Figure 3 used a list of equations with *G* for the number of goats and *S* for the number of fence sections. Note that in this notation, the equals sign (=) does not represent the quantity equivalence, but instead shows the same circumstances or conditional situation (e.g., for 2 goats, we need 6 sections). The right side of Figure 3 shows another model, a two-column table. The first row has two variables and the numbers below were used to find a pattern between *G* and *S*. From the table, the EBs saw the repeated pattern of variant and invariant parts and figured out how the variant part can be replaced with *G* – 1.

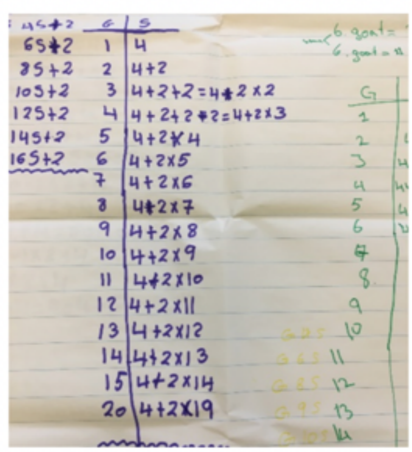
At the end of the lesson (Act 4), students reflected on their learning by writing in their journal where they were encouraged to use the language of their choice, as well as drawings and mathematical expressions. Since some EBs were not yet comfortable writing in English or even in their own language, we provided students with keywords and sentence frames on the whiteboard. One student wrote in his journal, "I like this math, challenging math today. I learned how to do expression." This particular student also verbally expressed his excitement while finding expressions with variables because he had hardly been given challenging tasks in prior math classes. Others wrote sentences with words (goats, fence, expression, number) and the expression they found, using either English or their native language.

Discussion and Implications

Although the lesson described in this paper implemented a 5-Act Task in view of modeling as vehicle, the 5 Act Task Framework can be used for teaching modeling as content. Checking and supporting students' understanding in mathematics and language (Act 0) and communicating about solutions (Act 4) are essential for EBs in any type of modeling.

Figure 3
EB's modeling after teachers' guidance





Through this study, we found several keys to implementing modeling as vehicle to teach EBs challenging mathematics with space for language development. First, it is crucial to spend enough time on assessing and building students' understanding of the problem statement and situations by developing shared vocabulary, prior knowledge, and contextual understanding. Act 0 enhances every other Act and the modeling process because whenever teachers see students struggle, they can refer to Act 0 to redirect and remind students of the contextual clues they understood in Act 0. Since EBs have different cultural experiences from non-EBs, integrating contexts that EBs are familiar with into mathematics lessons is crucial when applying modeling for EBs. In this lesson, we chose the context of goats because goats are the most common domestic animal in the countries the EBs came from. Moreover, by contributing their own story and language, each EB had an opportunity to act as a valuable member of the class. Finally, the reflection process (Act 4) is often ignored when working with EBs due to time restrictions and EBs' difficulty in verbal presentations, but sharing what students found and learned is an important component of learning and language development.

The 5-Act Task Framework enables teachers to empower EBs in experiencing meaningful communication and deepening mathematical understanding through the modeling process. With this Framework, teachers can use modeling in multiple ways to effectively teach EBs while using rich cognitively-demanding tasks.

References

Anhalt, C. O. (2014). Scaffolding in mathematical modeling for ELLs. In M. Civil & E. Turner (Eds.), *Common Core State Standards in Mathematics for English language learners* (pp. 111–126). TESOL International Association.

Bane, M. (1992). *Voices from the inside: A report on schooling from inside the classroom*. Institute for Education in Transformation at the Claremont Graduate School.

Celedon-Pattichis, S., & Ramirez, N. G. (Eds.). (2012). *Beyond good teaching: Advancing mathematics education for ELLs*. National Council of Teachers of Mathematics.

Chval, K. B., & Chavez, O. (2011). Designing math lessons for English language learners. *Mathematics Teaching in the Middle School*, *17*(5), 261–265.

Galbraith, P. (2012). Models of modelling: Genres, purposes or perspectives. *Journal of Mathematical Modelling and Application*, *1*(5), 3–16.

I, J. Y., Chang, H., & Son, J.-W. (2019). *Rethinking the teaching mathematics for emergent bilinguals: Korean teacher perspectives and practices in culture, language, and mathematics*. Springer. http://doi.org/10.1007/978-981-15-0966-7

I, J. Y., & de Araujo, Z. (2019). An examination of monolingual preservice teachers' set-up of cognitively demanding mathematics tasks with emergent multilingual students. *Research in Mathematics Education*, *21*(2), 208-228. https://doi.org/10.1080/14794802.2019.1615980

Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. *Journal for Research in Mathematics Education*, *44*(4), 646–682. https://doi.org/10.5951/jresematheduc.44.4.0646

Jung, H., & Brady, C. (2016). Roles of a teacher and researcher during in situ professional development around the implementation of mathematical modeling tasks. *Journal of Mathematics Teacher Education*, *19*(2–3), 277–295. https://doi.org/10.1007/s10857-015-9335-6

Macedo, D., & Bartolomé, L. I. (1999). *Dancing with bigotry*: *Beyond the politics of tolerance.* Palgrave McMillan.

Moschkovich, J. N. (Ed.). (2010). *Language and mathematics education: Multiple perspectives and directions for research*. Information Age Publishing.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. National Governors Association Center for Best Practices, Council of Chief State School Officers.

Nieto, S. (2018). *Language, culture, and teaching: Critical perspectives*. Routledge.

Appendix

5-Act Task modified for engaging EBs in mathematical modeling

	Description	Purpose
Act 0	Assessing/scaffolding before Act 1	To support EBs by providing background knowledge of the problem's language, context, and mathematics.
Act 1	A story with conflict via multimedia/visuals/physical movements and problem posing based on the story	To engage EBs in a relevant real-life story towards an authentic understanding of the mathematical situation and to empower EBs by having them pose their own problem(s).
Act 2	Student-driven collection of information to solve the problem they posed	To give EBs agency to reflect on the story and develop their own assumptions to solve them.
Act 3	Model construction with solution(s)	To provide EBs opportunity to visualize/model their mathematical process to analyze and improve their decisions in the real world.
Act 4	Collective communication of solutions/thinking process	To support EBs as they develop English proficiency while also building mathematical competency.