RESEARCH ARTICLES

Novel Interpretations of Academic Growth

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MetaMetrics

Integrating a construct theory with Rasch measurement not only places persons and tasks on a common scale, but it also resolves the indeterminacy of scale location and unit size when the scale is anchored in an operationalized task continuum based on the construct theory. Such an approach has several advantages for understanding academic growth as evidenced in a series of empirical examples, which demonstrate how to: a) conjointly interpret student reading growth in the context of reading materials concomitantly used during instruction; b) interpret a reading growth trajectory in light of future (e.g., postsecondary) reading requirements; c) forecast individual reading comprehension rates relative to both contemporary and future text complexity requirements; and d) create growth velocity norms for average academic growth in reading or mathematics achievement.

Keywords: achievement; reading; mathematics; longitudinal; growth; norms

During the 1980's two measurement companies in the United States introduced a fundamental innovation in the scaling of student reading ability that moved the world closer to an absolute framework for the measurement of reading comprehension. The strategy entailed combining the Rasch measurement model with an operationalized reading construct theory. As with other item response theory (IRT) models, the Rasch model makes it possible to place persons and tasks (items) on a common scale, but certain scale properties (location, unit size) are arbitrary (i.e., they vary with changes in the person sample and/or items). The key innovation involved two steps that anchored the scale and defined its unit size in terms of an empirical text complexity continuum.

The first step was to define and validate a construct model that operationalized the reading difficulty of texts in terms of specific semantic and syntactic features of texts that are effective proxies for the cognitive demand experienced by readers while reading. Secondly, it was demonstrated that the empirical difficulties of a well-defined, text-based item type could be nearly perfectly predicted by the complexities of the texts associated with the items. Once students and items were measured via the Rasch model, the construct theory was used to calibrate the items to the text complexity continuum. This produced a direct correspondence between the person measures and the text measures.

The company now known as Questar Assessment, Inc. was the first to use this type of approach. They developed the Degrees of Reading Power® (DRP®) Program, which reports student reading measures from criterion-referenced tests on a proprietary DRP Scale of Text Complexity, which it uses to measure the reading difficulty of printed material (Bruning, 1985). Nelson, Perfetti, D. Liben and Liben (2012) described the scale as follows:

> DRP text difficulty is expressed in DRP units on a continuous scale with a theoretical range from 0 to 100. In practice, commonly encountered English text ranges from about 25 to 85 DRP units, with higher values representing more difficult text. (p. 11)

Questar defined a DRP *prose comprehension model* based on the application of the Bormuth (1969) readability formula to measure text complexity. Their reference item type was a text-embedded, cloze item (i.e., based on a text passage with certain words removed) administered according to a specific protocol. The unit size of the DRP scale was specified in terms of a transformation of the Bormuth text complexity measure, R. Research has shown that the DRP scale places both student reading ability and text complexity on a common well-defined, unidimensional scale that remains invariant over time. Thus, research supports the claim that the DRP tests "are like measures in the natural sciences." (B. L. Koslin, Zeno, & Koslin, 1987; p. 171)

At nearly the same time, a second company pursued the same fundamental idea. MetaMetrics® developed The Lexile® Framework for Reading to measure both readers and texts on a common scale. They independently developed a *construct-specification equation* to operationalize text complexity and predict item difficulties (Stenner & Smith, 1982; Stenner, Smith & Burdick, 1983). They also developed a well-defined reference item type (consisting of a text passage followed by a cloze-like, sentence-completion stem) and demonstrated that the empirical difficulties of such items could be nearly perfectly predicted by the difficulties of the associated texts (Stenner, D. R. Smith, Horabin & Smith, 1987). They coupled this construct model with a Rasch measurement model to place both a student's reading ability and a text's readability on a common invariant scale.

In order to define a logical unit for the Lexile scale, MetaMetrics chose to explicitly anchor its scale at two points on the text-complexity continuum. Based on its anchoring, a Lexile scale unit equals 1/1000 of the difference between the readability of certain specific basal primers and the readability of an online adult encyclopedia (Stenner, H. Burdick, Sanford, & Burdick, 2007). This approach provided a well-defined unit of measurement that retains its absolute size across different applications of measurement. It may be noted that this method is directly analogous to the way the meter was standardized based on the length of the meridian quadrant (i.e., the distance from the North Pole to the equator) through Paris (Legendre, 1805). It is also precisely analogous to the way that temperature scales are anchored.

Because the Lexile Framework and the DRP are based on a Rasch measurement model, they are examples of conjoint measurement. Conjoint measurement makes it possible to simultaneously scale two variables that jointly predict an outcome. For example, reader ability and text difficulty jointly predict reading comprehension; so, both the reader measure and the text difficulty measure can be placed on a common scale. Thus, both the Lexile Framework and the DRP can be utilized to generate student scores that are reported on a text difficulty continuum, giving the scores supplemental interpretability anchored in a real-world context. Since their creation, both systems have been widely implemented in the United States. The

primary use of both these systems to date appears to have been the matching of students with texts of appropriate difficulty.

In 2004, MetaMetrics launched The Quantile® Framework for Mathematics, a measurement system for mathematical understanding, which uses Rasch measurement to conjointly scale both persons and items and anchors the resulting scale in a real-world task continuum. The Quantile Framework uses a quantified mathematics lesson continuum as the real-world context for anchoring the developmental scale (Sanford-Moore et al., 2014). As a companion scale to the Lexile Framework, the Quantile Framework demonstrates that the strategy of combining Rasch measurement with construct theories and anchoring the resulting scales in real-world task continua is a viable method for behavioral science measurement which generalizes to multiple constructs. As was the case with the Lexile Framework, the Quantile Framework was primarily designed to link assessment with instruction (MetaMetrics, 2009).

The purpose of this paper is to demonstrate, through several examples, that interpretations of student academic growth benefit from the use of Rasch-based measurement scales that have been anchored in a real-world task continuum by means of construct theory. These examples benefit from the fact that one state had the foresight and commitment to utilize such scales over a long period of time. The state of North Carolina (NC) began linking its reading assessment scales to the Lexile Framework for Reading starting with the first edition of its end-of-grade assessments (introduced in 1993) and continuing with subsequent editions of reading tests up to the current day. Similarly, the state began linking its mathematics assessments to the Quantile Framework starting with the third edition (introduced in 2006) of their mathematics end-of-grade tests and continuing to the present day. In addition, the state began linking its high school content area tests in 2008, providing a basis to extend the longitudinal measurement of reading and mathematics achievement on common scales beyond the elementary and middle school years.

These measurement innovations adopted by North Carolina have several advantages for the interpretation of academic growth. As demonstrated in the examples, the benefits include: a) conjointly interpreting student reading growth in the context of reading materials concomitantly used during K-12 instruction; b) interpreting a reading growth trajectory in light of future (e.g., postsecondary) reading requirements; c) forecasting individual reading comprehension rates relative to both contemporary and future text complexity requirements; and d) creating growth velocity norms for average academic growth in reading and mathematics.

Theories about the developmental velocity of physical attributes can be traced to Aristotle, who observed that height increases fastest when individuals are young; over the intervening centuries, many studies of stature have confirmed and explicated this now well-accepted fact (Tanner, 2010). However, it was not until the emergence of educational and psychological measurement in the early part of the 20th century, that studies of individual academic growth became possible.

A central question in all studies of academic growth is what mathematical function to use for modeling individual growth and the decision necessarily reflects assumptions about learning rate (i.e., growth velocity). In general there have been two traditions to address the question of functional form: a) the empirical tradition of fitting growth curves, which has been traced to Wishart (1938); and, the tradition of selecting a growth function based on an explicit theory of growth rate. In the latter approach, theories of learning rate have been adapted from chemical processes (Robertson, 1909) and the study of mortality (Gompertz, 1825; Winsor, 1932), among

others. Whether one works in a purely empirical tradition or is guided by substantive theory, all potential growth models must be subjected to empirical confirmation with longitudinal data.

A key research hypothesis in longitudinal studies relates to whether growth proceeds according to a straight line (with constant velocity) or curvilinear pattern (with variations in velocity and/or acceleration). Some researchers (e.g., Catts, Bridges, Little, & Tomblin, 2008; Guglielmi, 2008; Kieffer, 2012; Sonneschein, Stapleton, & Benson, 2010) found a straight-line growth model adequate for their purposes. However, Lee (2010) reported that American students' growth in reading and mathematics achievement during the K-12 school years is curvilinear, characterized by declining velocity over time. Researchers using more extensive longitudinal research designs have confirmed this finding using the empirical approach (e.g., Schulte, Stevens, Elliott, Tindal, & Nese, 2016; Williamson, 2015) as well as the theory-driven approach to growth (Cameron, Grimm, Steele, Castro-Schilo, & Grissmer, 2015). Moreover, Andrich and Styles (1994) provided psychometric evidence to substantiate intellectual growth spurts in early adolescence.

In America, academic growth predominantly occurs in the context of schooling and growth is presumably influenced by exposure to instructional content. Accordingly, it is illuminating to note that the difficulty of reading materials (Williamson, Koons, Sandvik, & Sanford-Moore, 2012) and the difficulty of mathematical skills and concepts (Sanford-Moore, Williamson, Bickel, Koons, Baker, & Price, 2014) also proceed across Grades K-12 in a curvilinear pattern characterized by positive velocity and deceleration.

The adoption of specific, previously-determined, growth curve results for the subsequent examples carries with it a set of implicit research questions, which I here make explicit.

1. Is NC aggregate reading growth curvilinear during Grades 3-8?
   a. What is the initial status of NC average reading growth in Grades 3-8?
   b. What is the initial velocity of reading growth, Grades 3-8?
   c. What is the acceleration of reading growth, Grades 3-8?
2. Is NC aggregate reading growth curvilinear during Grades 3-11?
   a. What is the initial status of NC average reading growth in Grades 3-11?
   b. What is the initial velocity of reading growth, Grades 3-11?
   c. What is the acceleration of reading growth, Grades 3-11?
3. Is NC aggregate mathematics growth curvilinear during Grades 3-11?
   a. What is the initial status of NC average mathematics growth in Grades 3-11?
   b. What is the initial velocity of mathematics growth, Grades 3-11?
   c. What is the acceleration of mathematics growth, Grades 3-11?

Answers to these three research questions were available from previous research. The aggregate, student growth curves used in the subsequent examples all exhibit a quadratic (curvilinear) functional form with positive initial velocity accompanied by deceleration across time. Specific parameter estimates are provided in the **Examples** section.

The featured examples themselves also have associated research questions, explicitly stated below:

   d. How does average NC student reading growth compare with proposed text-complexity standards widely adopted in the US?

e. How does average NC reading growth align with the text complexity of postsecondary reading materials?

f. Given the average reading growth of NC students, what reading comprehension rates are forecasted relative to K-12 text-complexity standards?

g. Given the average reading growth of NC students, what reading comprehension rates are forecasted relative to postsecondary text complexity?

h. What incremental velocities characterize the historical, average reading growth of NC students from the end of Grade 3 to the end of Grade 11?

i. What incremental velocities characterize the historical, average mathematics growth of NC students from the end of Grade 3 to the end of Grade 11?

Answers to research questions 4 through 9 are presented and explained in the **Examples** section.

Results from the current research study can be used by educators and students alike. To illustrate, consider a student (or group of students) progressing through school. Typically students are assessed on their reading and mathematics achievement annually. As they are assessed, students can compare their individual performance and growth to the average historical growth of previous students. Similarly, teachers can compare their students' individual growth as well as the group's aggregate growth with historical growth. Additionally, students and teachers benefit from the conjoint properties of the measures in the following ways. Based on the first three examples, student reading achievement is readily compared to the text-complexity of both K-12 and postsecondary texts; and, by monitoring students' forecasted comprehension rates, teachers can individualize the match between students and texts that the teacher may assign as students improve their reading abilities. Finally, as students accumulate a history of measured performance, educators can reference the velocity of reading growth and mathematics growth to historical growth rates determined from longitudinal data. These interpretive contexts offer new insights and perspectives that can facilitate instruction as well as program monitoring and evaluation.

DATA

The "data" for the subsequent examples consist of the parameter estimates from multilevel growth models estimated for various panels of students who participated in the North Carolina assessment program. As already mentioned, the parameter estimates are adopted from previous work (e.g., MetaMetrics, 2011; Williamson, 2014). The original student-level data, which were the basis for the fitted growth models, consisted of Lexile or Quantile measures that were obtained through linking the North Carolina assessment scales to the Lexile Framework and the Quantile Framework.

North Carolina assessments have well-documented technical characteristics (Bazemore & Van Dyk, 2004; North Carolina Department of Public Instruction, 2009; Sanford, 1996) and have successfully satisfied the requirements of the Elementary and Secondary Education Act (No Child Left Behind, 2002). In general, panels were comprised of longitudinal data spanning Grades 3-8, where the assessments were administered once a year at the end of each grade. For the velocity norms examples, additional waves of data were employed through Grade 11.

EXAMPLES

The first three examples are based on a multilevel unconditional quadratic growth model, which was fit to the longitudinal data from a North Carolina panel spanning Grades 3-8 in 2000-2005. Based on data from every student who had at least one reading measure during the six-year time frame, this curve provides a historical summary of average student reading growth for 98,515 students, representing 92.8% of the Spring 2005 eighth-grade cohort that defined the panel. The estimates of the intercept, velocity and curvature parameters for the average reading growth curve were 670.2L, 119.6L/year and -6.1L/year^2, respectively. In Figure 1, I provide a visual summary of the statewide average reading growth curve, the corresponding velocity curve and the acceleration curve for reading growth based on the multilevel analysis.

Note in Figure 1 the horizontal scale is graduated by grade, where the coding refers to the end of the respective year. So for example, the numeral 3 on the grade scale refers to the end of Grade 3. Furthermore, the time scale for the growth model was centered at the end of Grade 3; thus the velocity parameter estimate refers to the velocity at the end of Grade 3. The vertical axis is denominated in Lexile scale units. The meaning of the Lexile scale unit was described earlier.
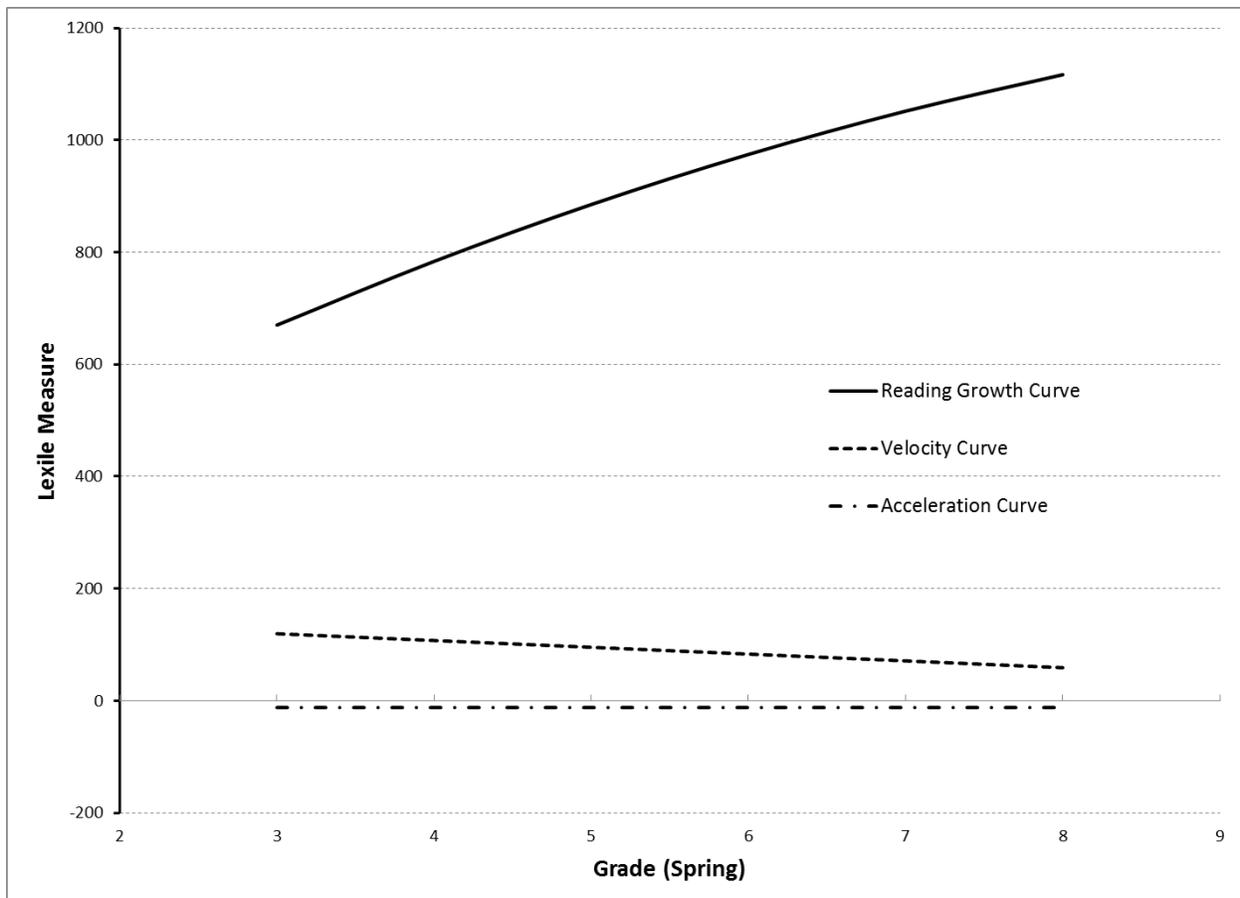

Figure 1. Average reading growth, velocity and acceleration curves for the 2000-2005 North Carolina panel (n = 98,515). The vertical axis graduates the growth curve in Lexile scale units, the velocity curve in Lexile units/year and the acceleration curve in Lexile units/year^2.

In Figure 1, notice that the growth curve begins around 670L at the end of Grade 3 and then rises quickly during the early grades; however, the curve decelerates across the Grade 3-8 time frame. The velocity curve in Figure 1 displays the fact that velocity is linearly related to time when the growth curve has a quadratic functional form. In this particular example, the velocity curve shows that velocity declines from approximately 120L/year at the end of Grade 3 to approximately 60L/year at the end of Grade 8. The slope of the velocity curve (-12.2L) is equal to the acceleration rate of the growth curve. Because the slope of the velocity curve is negative, growth is decelerating during the time frame. For a quadratic growth curve, the acceleration rate is manifested through the curvature parameter. Acceleration is constant and equal to twice the curvature parameter (i.e., -6.1L in this case). This is consistent with the constant negative elevation displayed for the acceleration curve in Figure 1. The growth, velocity and acceleration curves are relatively simple for a quadratic growth model; nevertheless, it is useful to display them in the fashion of Figure 1 because it provides a convenient and readily understandable summary of the key features of growth.

Student Growth in Reading versus the Common Core State Standards

The Common Core State Standards (CCSS) Initiative [National Governors Association Center for Best Practices (NGA Center) & the Council of Chief State School Officers (CCSSO), 2010] established quantitative text complexity standards for specific grade bands in the public schools. The standards are expressed as text complexity ranges denominated in terms of six text complexity metrics in common use in the United States. One of those metrics is the Lexile measure, which makes it possible to compare the text complexity standards of the CCSS to actual student reading achievement measured with the Lexile Framework. The CCSS College and Career Readiness Anchor Standards for Reading require that by the end of specific grades that demark the end of the CCSS grade bands (i.e., grades 3, 5, 8, 10, and 12), students must "read and comprehend literature, including stories, dramas, and poetry/poems, at the high end of the ... text complexity band independently and proficiently." (pp. 12, 37, 38) The upper end of the text complexity range for the Grade 11-12 grade band was labeled "CCR" by the CCSS to connote college and career readiness.

In Figure 2, I depict the 2000-2005 NC growth curve and the CCSS text complexity ranges for Grades 3, 5 and 8. The lower and upper boundaries of the CCSS text complexity ranges at the critical grades are represented by dots, which are connected by dashed lines to provide a visual reference as context for the growth trajectory. If student growth were commensurate with the CCSS text complexity standards, then one would expect to see the growth curve traversing a path that lies within the text complexity boundaries, rising near the upper end of the range by the specified grades, which denote the end of each grade band. In fact, the NC average growth curve approximates this behavior. Its intercept appears to be slightly above the mid-point of the text complexity range for the Grade 2-3 grade band and the curve rises nearer the upper boundary by the end of the Grade 6-8 band. If one imagines that the average growth curve is in fact the growth curve for an individual student, then it would seem that the student's growth is reasonably well aligned with the standards. Is it good enough? What does the growth curve imply about the actual reading experience that the student would have relative to the CCSS upper boundaries as he or she grows? I will come back to these questions in a subsequent example. First, I wish to introduce the idea that there are additional text

requirements that characterize reading experiences which students may encounter after they graduate high school. Consequently, student growth during the K-12 years has implications for reading experiences that students will encounter later.
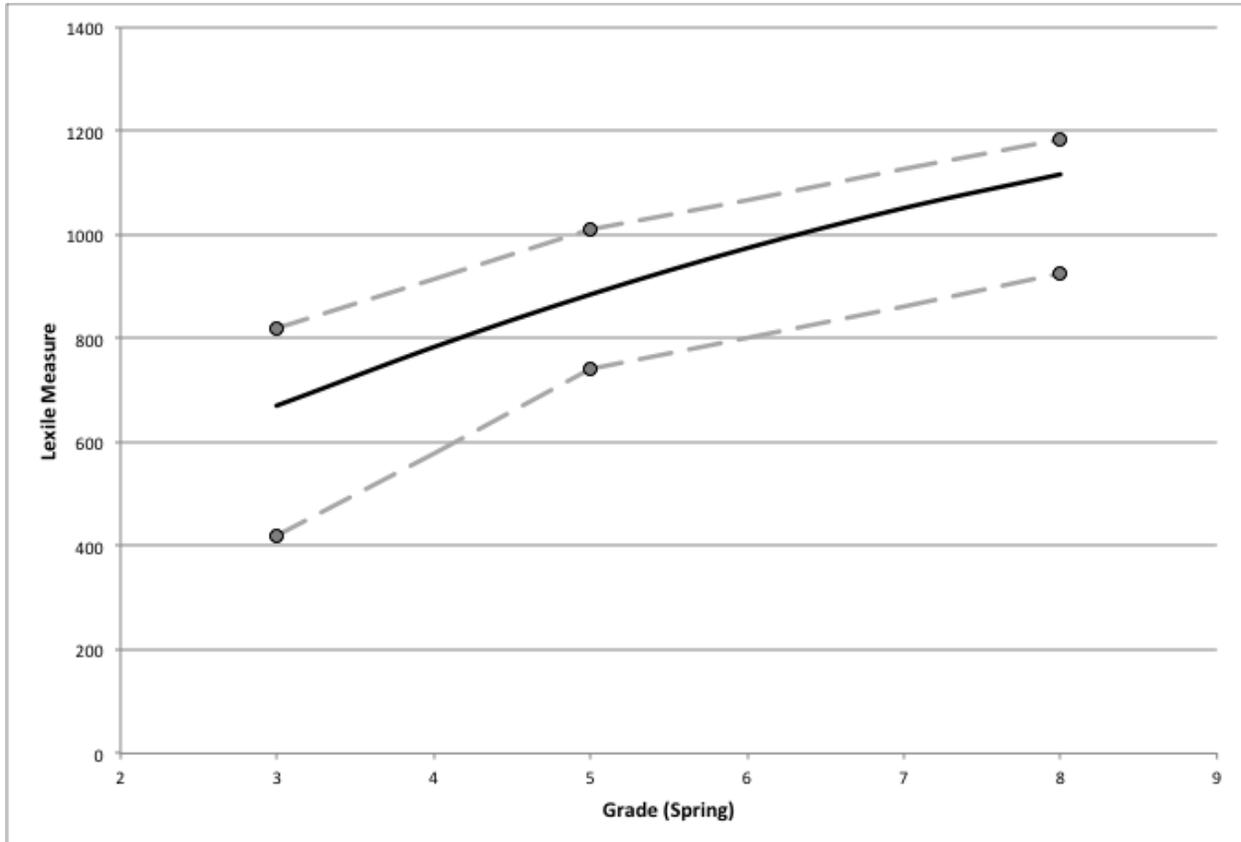
Figure 2. Reading growth relative to the Common Core State Standards (CCSS) text complexity ranges. The growth curve is the 2000-2005 North Carolina average growth curve (n = 98,515). The dots represent the CCSS Lexile range boundaries at grades 3, 5 and 8. The dashed lines provide a visual reference for the growth trajectory as it traverses the CCSS grade bands.

Reading Growth in Relation to Postsecondary Text Complexity

The objective of this example is to illustrate average student reading growth in relation to the text complexity of reading materials that students may encounter beyond high school. To accomplish this objective, I combine knowledge about the functional form of reading growth during K–12 with text complexity measures of postsecondary reading materials to construct an empirically-based model of student growth toward postsecondary performance aspirations. Such a model can be a useful first step toward understanding the possible long-term implications of growth.

Williamson (2008) elaborated a continuum of text complexity for reading materials associated with typical postsecondary endeavors (e.g., postsecondary education, the military, the workplace, citizenship). This work demonstrated substantial differences between the materials that high school students are expected to read and the materials they may encounter after high

school. The latter reflect a substantially higher text demand, or correspondingly, require a higher reading ability from students in their postsecondary lives. The median Lexile measures for five postsecondary text collections summarized by Williamson are: 1395L (university), 1295L (community college), 1260L (workplace), 1230L (citizenship) and 1180L (military).

Once again, I use the statewide average reading growth curve of the 2000-2005 NC panel. Using the fixed effects estimates from the multilevel analysis, the average reading growth curve is expressed as a mathematical equation: $E(L/T) = 670.2 + 119.6\ T - 6.1\ T^2$. This equation quantifies the estimated average achievement in any grade.

The 2000-2005 panel is comprised of 98,515 North Carolina public school students who were third graders in the spring of 1999-2000 and who progressed to the end of eighth grade in the spring of 2004-05. These students progressed from Grade 3 to 8 without repeating a grade and were included in the analysis if they had at least one reading measure during the six-year time frame. Consequently, the average growth curve of these students should provide a good illustration of typical student growth toward postsecondary expectations. All of the relevant information about the growth curve is summarized in the three parameter estimates: 670.2L (initial status—end of third grade), 119.6L (initial velocity), and -6.1L (curvature).

Data were not available prior to the end of Grade 3 or after the end of Grade 8. However, with some caution, the quadratic equation that characterizes the curve through the range of observed data can be used to estimate average performance before Grade 3 and after Grade 8. Simply evaluating the growth curve at the other time points suffices.

When extrapolating, it is important to use caution for at least two reasons. First, there are no actual data to check the assumption that growth from Grades K–2 and Grades 9–12 can be described by the same quadratic equation that describes growth from Grades 3–8. Second, the nature of a quadratic polynomial is that it has a maximum point or a minimum point, after which the curve reverses direction. When the curve is concave to the time axis (as is the case for the NC average growth curve), there will be a maximum point after which the curve turns downward. It is implausible that future performance will decline back to the third-grade level and below; this would be inconsistent with normal developmental growth.

There are (at least) three ways to address these concerns. The easiest way is to analytically check the quadratic equation to determine when the maximum point occurs. If it occurs outside the range of time to which one wishes to generalize, then there is less reason to worry that the depiction of growth may be inappropriate. As it turned out, the maximum for the 2000-2005 North Carolina growth curve occurred at Grade 12.9, almost a year beyond the end of twelfth grade, which is the last occasion for which average student achievement was projected.

A more direct way to avoid non-developmental behavior in a growth model is to adopt a different mathematical model for growth—e.g., one that cannot display a reversal in direction. A linear model with a transformed time scale is one possibility, such as: $r(t) = a + b \ln t$, which increases monotonically without bound. Another alternative is to select a model that is nonlinear in the parameters, such as the negative exponential: $r(t) = a - (a - b)e^{-ct}$, which increases monotonically to an asymptote. There are many possibilities (e.g., see Singer & Willett, 2003; or, Goldstein, 1979 for a variety of specific choices). Alternative models carry with them alternate interpretations of growth, may be more complex mathematically, and may require additional data to obtain satisfactory fit. Ultimately, the choice of most appropriate model is based on multiple considerations—e.g., substantive theory, available data, empirical fit, parsimony, and perhaps other requirements.

The third way to address the risks of extrapolation is to strategically collect more data to fill in the missing time points with student achievement information. Unfortunately, this is harder than it sounds for a variety of reasons, including the costs of collecting the information and the challenge of measuring the same construct over longer and longer periods of time. For the present example, the results may be regarded as provisional, bearing in mind that extrapolations to lower and higher grades may need to be revised based on future information.

With those cautions in mind, Figure 3 shows the results of combining the information from the text analyses and the information from the NC reading growth curve. There are several important things to notice about Figure 3.

Once again, the horizontal scale represents end-of-grade in school. On this scale, zero stands for the end of the kindergarten year. Subsequent Grades (1–12) are denoted as usual. Then the numerals 13 through 14 are used to denote the next two years of postsecondary experience. The vertical scale displays the Lexile measure, which is used to quantify *both* the students' average reading achievement and the median text difficulty of each text collection.
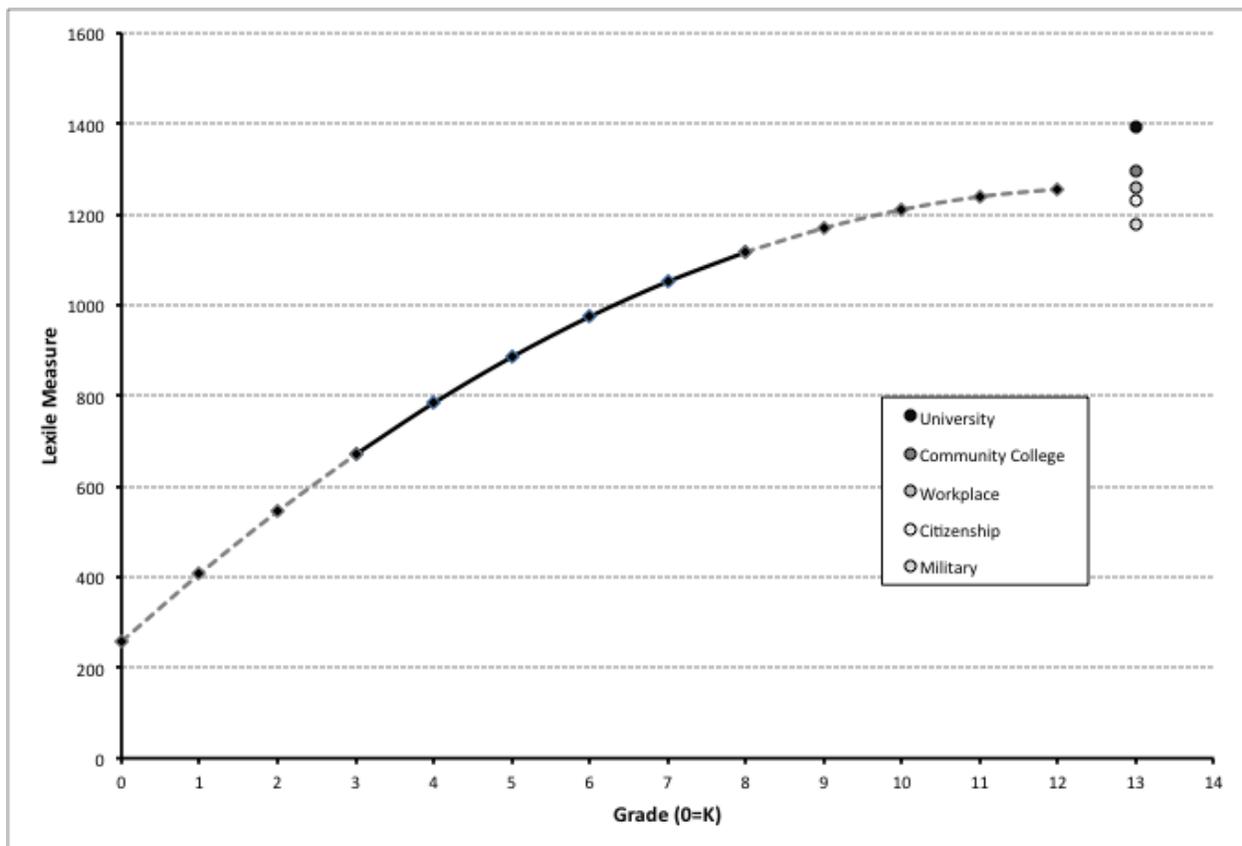

Figure 3. Average student growth in relation to postsecondary text complexity. The solid curve represents the 2000-2005 North Carolina average growth curve (n = 98,515). The dashed portions of the curve are mathematical extrapolations based on the quadratic equation for the average growth curve. The shaded dots in the upper right represent the median text complexities for the respective text collections listed in the legend (Williamson, 2008).

In the graph, diamonds are used to indicate the estimated average reading ability of students at the end of each grade. The estimates for Grades 3–8 are connected by the solid

empirical growth curve to represent the fact that they are based on the available data. The estimates for Grades K–2 and 9–12 are connected with dashed curves to represent the fact that they are theoretical extrapolations determined analytically from the quadratic equation for the empirical growth curve. As such, the dashed portions of the curve are only reasonable guesses based on the observed data, subject to future revision based on more complete longitudinal records[1]. The farther one goes from the observed data (Grades 3–8), the more one has to bear in mind the provisional nature of the projections. Finally, in the figure, the median text difficulties of the postsecondary text collections are arrayed vertically at Grade 13 to indicate that students face these expectations in the year following their exit from Grade 12.

The primary feature of the chart is the alignment of the projected twelfth-grade reader measure in conjunction with the postsecondary text measures.[2] It appears that the average growth trajectory of these students, if unaltered, will carry them to a reading level (1256L) that lies near the median text requirements of the workplace (1260L). Students with higher postsecondary aspirations (e.g., the community college, the university) need to be on a higher trajectory that tracks above the average growth curve depicted in the figure.

One must remember, however, that individual growth is variable and that students vary in their individual parameters of growth. That is, students have different beginning points, different initial velocities and different degrees of deceleration. Each of these features of growth results in a different individual trajectory, which may differ from the average growth trajectory. Thus, there are many possible ways to reach a given end point. For example, one student might begin at a higher level and exhibit modest but steady growth with little deceleration over time. Another might start out lower in reading ability but progress very rapidly with some deceleration over time. Both students might reach the same twelfth-grade reading ability through different individual growth curves. Williamson, Fitzgerald and Stenner (2014) discussed alternate growth trajectories in terms of the pedagogical and educational policy implications of directly targeting key features of growth (status, velocity and acceleration). For example, early-intervention reading programs can successfully influence initial reading status; increased deliberate practice might impact velocity; and, systematic exposure to summer school could be a viable strategy to moderate deceleration.

Forecasted Comprehension Rates Based on a Growth Curve

For this example, I return to the question of what kind of reading experience students are likely to have with particular levels of text complexity—e.g., the CCSS text standards or postsecondary text requirements. Again using the 2000-2005 NC average growth curve and supposing that the curve might describe the trajectory of a particular individual, it is possible to estimate the

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[1] Although there are assessments of U.S. students prior to the end of Grade 3 [e.g., the Early Childhood Longitudinal Study (ECLS)] and after the end of Grade 8 [e.g., National Education Longitudinal Studies (NELS)], they are generally available only for samples of individuals and reading measurements from these studies have not yet been brought onto a common scale.

[2] The median difficulty (1130L) of texts used near the end of high school (i.e., grades 11 and 12) is not shown in the figure, because it does not represent a postsecondary aspiration. High school texts are significantly easier to read on average than are citizenship materials, workplace materials, community college texts or university texts (Williamson, 2008).

individual's comprehension rate relative to texts the individual may encounter. To do this, it is necessary to have a general idea of how the Lexile Framework for Reading can be used to forecast reading comprehension given a reader of a particular reading ability and a text of a particular difficulty. Stenner, H. Burdick, Sanford and Burdick (2007) described the approach. In essence, one forecasts the comprehension rate by using the Rasch model equation, which expresses the reading outcome (comprehension) as a function of the exponentiated difference between the reader's ability and the text's difficulty. The Lexile Framework is designed so that an exact match between reader and text (i.e., reader ability equals text complexity, and so the difference between the two is zero) results in a comprehension rate of 75%. A comprehension rate of approximately 75% seems to be associated with successful reading experiences; whereas, a comprehension rate of 50% or lower results in frustration for the reader (Scholastic, Inc., 2007). MetaMetrics typically advises educators to choose texts that lie in a proximal zone ranging from 100L below the reader's ability to 50L above it when using the Lexile Framework to match readers with texts of appropriate difficulty. This proximal zone corresponds to comprehension rates that range from approximately 70% to 80%.

Consider a reader whose growth curve is equal to the 2000-2005 NC average growth curve. What happens when such a student reads a book that has text complexity equal to the upper end of the CCSS text complexity ranges? What happens when such a student reads a book that has a text complexity equal to the typical text complexity of postsecondary reading materials (1300L)? In the first scenario, the CCSS text demand changes from grade to grade as the student's reading ability (reflected by the growth curve) changes. In the second scenario, there is a fixed future, postsecondary target toward which the student is progressing. I address both situations in Table 1.

For each of the Grades 3, 5, 8, 10 and 12 (i.e., the transition grades between the CCSS grade bands), I tabulate in the first four rows of Table 1: a) the average student performance (estimated from the growth curve); b) the CCSS text complexity upper bound; c) the difference between the two; and, d) the resulting forecasted comprehension rate at the end of the grade.

TABLE 1

Forecasted Comprehension Rates Implied by the 2000-2005 North Carolina Average Reading Growth Curve Relative to a) the Common Core State Standards (CCSS) Grade Bands and Text Complexity Ranges and b) Median Postsecondary Text Complexity

	Grade				
	3	5	8	10	12
	Average Student Achievement Summarized by the Longitudinal Growth Curve				
NC (2000-2005)	670L	885L	1117L	1211L	1256L[a]
	CCSS Text Complexity Requirements				
CCSS	820L	1010L	1185L	1335L	1385L
Difference	-150L	-125L	-68L	-124L	-129L
Forecasted Comprehension	61%	63%	69%	63%	63%
	Median Postsecondary Text Complexity				
Postsecondary Texts (Median)	1300L	1300L	1300L	1300L	1300L
Difference	-630L	-415L	-183L	-89L	-44L
Forecasted Comprehension	15%	32%	57%	67%	71%

Note. A multilevel growth analysis (n = 98,515) was used to estimate the average reading achievement at the end of each respective CCSS grade band. The upper boundaries of the CCSS Lexile ranges associated with the respective grades are given in the row labeled *CCSS*. A reader who is well matched with a text at his or her Lexile measure is forecasted to have a 75% comprehension rate.

[a] The empirical data spanned Grades 3-11. The estimated average achievement at the end of Grade 12 is extrapolated from the growth curve.

In general, we expect a reader to have 75% comprehension of a well-targeted text (i.e., a text at the student's reading level). Because the CCSS text complexity standards represent a series of increasing aspirational goals, we can ask how well the average reader in our example might do relative to the changing text complexity standards as he or she grows. That is, what would be the student's comprehension rate when confronted with a text with the higher text complexity prescribed by the CCSS? Table 1 provides the answer. The forecasted comprehension rates rise from 61% (in Grade 3) to 69% (in Grade 8) during the empirical time frame for the panel. However the comprehension rate is forecasted to drop back to 63% during the high school years, if the individual continues on the same trajectory traversed during Grades 3-8. Although, the CCSS grade bands and text complexity ranges are designed to provide flexibility to accommodate readers with a wide range of abilities, this example suggests that the average student in the 2000-2005 panel may experience some challenge relative to texts at the

upper ends of the CCSS text complexity ranges (because all of the forecasted comprehension rates are less than 75%).

In the bottom half of Table 1, we can see that the hypothetical average student experiences increasing rates of comprehension while growing toward the fixed postsecondary text complexity target.   Although forecasted comprehension of the median (1300L) postsecondary text is understandably low (15%) when the student reads as a typical third grader, the forecasted comprehension rate steadily climbs to 71% by the end of Grade 12, based on the estimated average reading growth curve.

A nice feature of this analysis is that it can be replicated with any estimated growth curve, whether for an individual or for a group (e.g., an average growth curve). One only needs estimates of reading ability at each desired point in time, which can easily be determined from the mathematical equation for growth.

Incremental Velocity Norms for Average Reading and Mathematics Growth

Replicating or exceeding some specified previous student achievement level was the basis for educational expectations throughout most of the 20th century. Similarly, replicating or exceeding previous growth *rates* eventually emerged as a basis for student growth standards (North Carolina Department of Public Instruction, 1996).   Even so, the best implementation of educational growth standards to date has been based on year-to-year gains, without the benefit of an underlying longitudinal growth curve. Growth velocity norms did not emerge even for height or weight until the work of Tanner, Whitehouse and Takaishi (1966) in the United Kingdom and later in the United States (Roche & Himes, 1980; Baumgartner, Roche & Himes, 1986). In this next example, I use two parametric models for growth (one for reading, one for mathematics) derived from NC longitudinal data (MetaMetrics, 2011). I shall use the historical results to create incremental growth velocity norms for average reading and mathematics growth. The approach yields not only estimates of year-to-year gain, but estimates of growth between any two points within the design time frame running from the end of Grade 3 to the end of Grade 11.

The starting point is the realization that an historical aggregate growth curve provides a long-term summary of observed growth for a group of students. As such, it may be regarded as a norm for growth. If this norm were treated as a growth expectation for future panels of students, the implicit policy goal would be that future students *should* grow in a manner that is similar to previous historical growth. When regarded as a set of expectations for future growth, the growth curve represents a *growth standard.* Perhaps the easiest way to operationalize such a growth standard is by generating incremental growth velocity estimates from the average growth curve. It is relatively easy to do this. One needs only the parameter estimates for the average growth curve. In this case, there are two parametric models—one based on a ten-wave analysis of reading growth and the other based on a nine-wave analysis of mathematics growth. These two growth curves are salient because they each span Grades 3-11, the grades during which accountability assessments are most often implemented in the United States and the grades most often the focus of state accountability systems.

The estimated average reading growth curve is a function of time, $r(T) = 663.8 + 148.0\ T - 8.7\ T^2$. I can use it to estimate the expected amount of growth from one time point to another. For purposes of the example, let us interpret the time scale in terms of *grade in school* with the

understanding that the gains so calculated will represent the growth from one spring to another because testing took place at the end of the school year.

When I calculate the gain between adjacent grades, I have calculated the amount of change per unit of time—i.e., the incremental velocity. When I calculate the gain between any two grades more than one year apart, it produces an incremental estimate of the amount of growth that took place between those two grades.

In Table 2, I have tabulated the values of $r(k) - r(j)$ for all pairs of grades (j,k) such that $k > j$ where $j = 3, 4, \ldots , 10$ and $k = 4, 5, \ldots , 11$.   The resulting values are displayed in matrix form. Quantities along the diagonal represent the expected gain for each year-to-year transition: Grade 3 to Grade 4, Grade 4 to Grade 5, and so on. These are the incremental yearly, spring-to-spring growth velocity norms based on a population of 101,610 students. The off-diagonal elements of the table display the amount of growth between every other possible pair of grades. This information is useful because it captures longer-term growth expectations, spanning multiple grades.

To illustrate the interpretation of growth using Table 2, first consider the annual yearly growth expectations displayed along the diagonal. A fourth-grade teacher might reference the entry at the intersection of the row for Grade 3 and the column for Grade 4. The entry conveys the expectation for average reading growth between the end of Grade 3 and the end of Grade 4— namely during the fourth grade year. It is 139L. Similarly, the fifth-grade teacher would reference the entry at the intersection of the row for Grade 4 and the column for Grade 5 and learn that the average growth expected of fifth graders is 122L. The principal of a middle school serving students in Grades 6-8 would be interested in the total gain expected between the end of the fifth grade and the end of the eighth grade. Referring to the intersection of the row for Grade 5 and the column for Grade 8, the principal learns that the expectation for average reading growth for students who spend all three years at the middle school is 260L.

Similarly, the average mathematics growth curve can be expressed as: $m(T) = 586.0 + 100.6\,T - 3.0\,T^2$. Having evaluated the average mathematics growth curve at all grade-pairs, I displayed the results in Table 3. The interpretation of average mathematics growth in Table 3 follows in the same manner as for reading growth (Table 2).

In both Table 2 and Table 3 it is obvious that historical growth is typically greater in earlier grades and tapers off as grade increases. This is apparent as one scans along the diagonal from upper left to lower right. This pattern reflects the deceleration of growth and quantifies it in practical terms for educators.   However, the off-diagonal entries in the table reinforce the realization that long-term growth is the result of a cumulative growth process that endures across the developmental life-span.

TABLE 2
Incremental Velocity Norms for Average Reading Growth Denominated in Lexile Scale Units

	Student Achievement Estimated from the Average Reading Growth Curve								
	664L	803L	925L	1029L	1116L	1185L	1237L	1271L	1288L
End of Grade	3	4	5	6	7	8	9	10	11
3		139L	261L	365L	452L	521L	573L	607L	624L
4			122L	226L	313L	382L	434L	468L	485L
5				104L	191L	260L	312L	346L	363L
6					87L	156L	208L	242L	259L
7						69L	121L	155L	172L
8							52L	86L	103L
9								34L	51L
10									17L

Note. The table is based on an average reading growth curve (ten waves of measurement) for North Carolina students (n = 101,610), spanning grades 3-11 during the years 2002-2010. The fitted model is summarized by the equation: $E(L|T) = 663.8 + 148.0\,T - 8.7\,T^2$ where the time scale is centered at Grade 3 (i.e., T = Grade -3). Velocity increments for adjacent grades (i.e., spring-to-spring gains) are shown in the shaded diagonal.

TABLE 3
Incremental Velocity Norms for Average Mathematics Growth Denominated in Quantile Scale Units

Student Achievement Estimated from the Average Mathematics Growth Curve

	586Q	684Q	775Q	861Q	941Q	1014Q	1082Q	1144Q	1200Q
End of Grade	3	4	5	6	7	8	9	10	11
3		98Q	189Q	275Q	355Q	428Q	496Q	558Q	614Q
4			92Q	177Q	257Q	331Q	399Q	460Q	516Q
5				86Q	165Q	239Q	307Q	369Q	425Q
6					80Q	153Q	221Q	283Q	339Q
7						74Q	142Q	203Q	259Q
8							68Q	130Q	185Q
9								62Q	118Q
10									56Q

Note. The table is based on an average mathematics growth curve (nine waves of measurement) for North Carolina students (n = 101,650), spanning grades 3-11 during the years 2002-2010. The fitted model is summarized by the equation: $E(Q|T) = 586.0 + 100.6\,T - 3.0\,T^2$ where the time scale is centered at Grade 3 (i.e., T = Grade -3). Velocity increments for adjacent grades (i.e., spring-to-spring gains) are shown in the shaded diagonal.

CONCLUSION

In this paper, I have proposed novel interpretations of student academic growth based on conjoint measurement and longitudinal data analyses. In three examples, I illustrated how to interpret student reading achievement and growth in light of the text complexity associated with reading materials that students may encounter during schooling or in the postsecondary world. In the final example, I implemented a strategy to create incremental velocity norms for average academic growth and provided examples of velocity norms for reading growth and for mathematics growth, each based on over 100,000 students.

The first three examples highlighted the power of conjoint measurement when combined with the longitudinal perspective of student growth curves. We first saw how to compare student growth to changing text complexity requirements such as those expressed in the CCSS. Then, we saw a student growth curve juxtaposed with postsecondary text requirements and I suggested that alignment between the two is desirable. Next, we saw how the first two examples lead us to forecasted comprehension rates for readers who are themselves growing in their reading ability. Although these three examples featured reading ability relative to text complexity requirements, it is possible to provide similar examples for growth in mathematics ability relative to the complexity of mathematical skills and concepts.

Finally, we saw how parametric growth curves can strengthen the basis for setting growth standards based on longitudinal panel data, rather than the usual practice of setting year-to-year growth standards based on non-developmental (e.g., status projection) or short-term growth (e.g., gain score) formulations. Incremental velocity norms such as those presented here are an indispensable complement to traditional cross-sectional norms for interpreting student achievement because velocity norms a) base year-to-year gains on a longitudinal growth curve and b) make it possible to construct expectations of growth between any pair of grades.

Although, the growth velocity norms provided in this paper are for statewide average growth, they are easily extended to sub-populations. To briefly elaborate, one possibility for expanding growth standards is to disaggregate an historical average growth curve into multiple growth curves conditioned on initial status. For example, by grouping students into deciles based on initial performance, average growth curves can be estimated for each of the ten deciles. Once decile growth curves have been determined, incremental velocity norms can be established for each decile group simply by replicating Table 2 (or 3) for each group's aggregate growth curve. Conditioning growth standards on initial performance is a feature that has been desired in some accountability systems.

Furthermore, if common scales were universally used for educational constructs and longitudinal data were routinely collected and analyzed, then growth velocity standards could have even greater generalizability. Individual state norms, national norms, perhaps even international norms for academic growth velocity would become possibilities.

In the present study, the measurement of growth was constrained to Grades 3-11. An important policy challenge for educators is extending the measurement of reading and mathematics abilities beyond traditionally assessed grades. This entails devising ways to measure the same constructs over longer portions of the lifespan using a common scale so that we can accurately chart the academic growth of students from emerging readers and mathematicians, throughout formal instruction and schooling, and into adulthood. Our current educational measurement capabilities are focused on a fraction of the developmental lifespan and miss much of the growth that we might otherwise observe. Notably, we miss critical transitions

such as the entry into K-12 education and transitions into various postsecondary endeavors (e.g., higher education, the workplace). Similarly, we know little about the effects of aging on academic growth trajectories because we have not fully developed our capacity to measure reading and mathematics abilities across the life course using a common scale.

Improving the measurement and study of academic growth is more than a research agenda or a matter for the research and measurement community. Educational leaders and policy makers should commit resources to support the intellectual endeavor because it enriches and sustains the educational enterprise, possibly with residual benefits for long-term human intellectual capacity and quality of life. The advantages become palpable when conjoint measurement is brought to bear as a means to link assessment with instruction. As we have seen in these examples, when conjoint measurement is combined with longitudinal analyses of academic growth, unique insights and perspectives emerge to inform educational practice.

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