Does a Decade Make a Difference? Changes in Pre- and In-service Preschool Teachers' Knowledge of Early Mathematical Development

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ABSTRACT

This study examines whether, in the wake of considerable research since 2007 on the importance of supporting early mathematical development, students in early childhood teacher education programs gained more knowledge in the subsequent decade in this essential area of development. The analysis compares data on pre- and in-service teachers' knowledge of mathematical development gathered during 2008 as measured by the Knowledge of Mathematical Development Survey (KMDS) and compares it to data gathered in 2017-2018. Results showed that while the KMDS mean scores of students in each of the education groups (beginning versus seniors versus math course) statistically differed for each collection year, there was no statistically significant difference between 2008 and 2017-2018 collection years for beginners. However, there was a statistically significant difference between 2008 and 2017-2018 collection years in average scores in the seniors and math course groups, resulting in lower mean scores in 2017-2018 than those in 2008.

KEYWORDS

Early mathematics, early childhood education, preservice, in-service, teachers

In 2007, Duncan and colleagues published an influential longitudinal study across three countries on kindergarten-entry predictors of academic success at third and fifth grade. Controlling for socioeconomic status and mother's education, the authors concluded that early math skills at entry to kindergarten had the greatest predictive power.

Long before Duncan et al. (2007) contemplated such analyses, appeals for increased classroom support for early mathematical development appeared in academic journals (American Educational Research Association, 2005; Clements, 2001; Ginsburg & Golbeck, 2004), at national conferences and meetings (Clements, 2004; Copley & Padron, 1998; National Research Council, 2005),

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in position papers and standards (Administration for Children and Families, 2005; National Association for the Education of Young Children & National Council of Teachers of Mathematics, 2002; National Council of Teachers of Mathematics, 2000) and in myriad of well-regarded texts on education (Baroody, 1987; Bowman et al., 2001; Ginsburg et al., 1998; Ginsburg et al., 2006). In 2001, the National Research Council and Mathematics Learning Study Committee stated, "The responsiveness of preschool teachers to the developmental level of a child in the domain of mathematics, helping to put in place the concepts that are prerequisites to success in first grade arithmetic, can provide the foundation for performance in the school years" (p. 83).

Since 2007, numerous studies have added to the evidence base on the importance of early mathematical development to later academic achievement (Duncan & Magnuson, 2011; Geary et al., 2013; Jordan et al., 2009; Kwok et al., 2021). In 2010, multiple research teams published additional analyses of the 2007 longitudinal study, resulting in similar outcomes with additional subgroup effects (Foster, 2010; Grimm et al., 2010; Hooper et al., 2010; Pagani et al., 2010).

Relatedly, research studies have shown that early childhood programs that increase math skills can have lasting effects on academic achievement (Gormley et al., 2018; Mattera et al., 2021). Researchers have found beneficial effects of quality mathematics curriculum on language and literacy outcomes (Sarama et al., 2012) and large effect sizes of math interventions on early math development (Joo et al., 2020; Wang et al., 2016). Researchers continue to investigate which mathematical skills are important (Fyfe et al., 2019; Nguyen et al., 2016).

Specific to Head Start, studies have found that Black and Latino students may be the recipients of the largest gains in math and language skills among children attending Head Start programs (National Academies of Sciences, Engineering, and Medicine, 2023). However, the authors of these reports express concern that enrollment in early childhood programs does not always result in better outcomes for children (p. 67). The quality of the program, including its curriculum and teachers' abilities to support learning, has significant effects on child

outcomes.

Prior to 2007, numerous studies examined mathematical activities in early childhood classrooms. Many of these concluded that math activities that involved teacher and child interactions were not common (Ginsburg et al., 1999; Graham et al., 1997; Sarama et al., 2004). As an example, in a study on 26 preschool classrooms (Klibanoff et al., 2006), authors reported that activities around cardinality (including object counting) existed in all classrooms, but verbal counting sequence activities in fewer than 70%. Fewer than 31% of classrooms engaged children in number ordering activities (e.g., "What number comes after seven?"). Significantly, researchers found the growth in math knowledge over the course of the school year was positively related to the amount of math talk in the classroom (pp. 62-64), making the lack of math talk problematic.

Studies published since 2007 also demonstrate concern about the level of support for early mathematical development in preschool classrooms. In a 2021 study of 27 prekindergarten classrooms, results showed that teachers displayed low- to medium-support of early mathematics learning (Cerezci, 2021). In a study of 30 private preschool centers, Bachman et al. (2018) found that 4- to 5-year-old children were exposed to an average of two minutes of math per day.

Concern has also been expressed before and after Duncan and colleagues' (2007) publication that teacher educators may not have the experience or information necessary to prepare early childhood teachers to provide mathematically rich environments and instruction (Ginsburg et al., 2006). Wright and colleagues (2021) reported that prekindergarten teacher accreditation policies across the United States are not aligned with state standard expectations, with only eight of 64 certification programs including a course on mathematics. An example of the misalignment between standards and teacher preparation programs is the inclusion of a new mathematics standard (Standard 8) and related teacher performance expectations in California (California Commission on Teacher Credentialing [CTC], 2023a). Teacher preparation programs for preschool through 3rd grade in the state must now prepare teachers to demonstrate that they can,

Plan and implement mathematics instruction

appropriate to children's age, grade, and developmental levels (including children's linguistic, cognitive, social and emotional strengths and learning needs) that is grounded in an understanding of California's most current Mathematics Standards and Framework and the most current Preschool Learning Foundations and Curriculum Framework. (CTC, 2023b)

Relatedly, in a recent study across eight states (including two of the states included in this study) that surveyed early childhood education programs in community colleges and universities across each state, Copeman Pettig and colleagues (2018) revealed that faculty reported teaching math content in courses for practitioners who worked with preschoolers at "higher rates than they reported feeling capable of teaching that content" (p. 19). This indicates that teacher educators may have been recruited to teach math courses or encouraged to include more math content in their curriculum and development courses despite their (self-reported) insufficient skill level in understanding and teaching about early math development. In a recent review of early childhood teaching credential programs, Schachner et al. (2023) noted that mathematics teaching was one of the domains with the least amount of coursework.

Given the considerable evidence for supporting early mathematical development, coupled with concerns about insufficient classroom support for that development, is there evidence that we have increased instruction on mathematical development for our pre- and in-service teachers? This study compares data on pre- and in-service teachers' knowledge of mathematical development gathered during 2008 and compares it to data gathered in 2017-2018, a ten-year span. Because teachers also gain knowledge through their everyday interactions in the classroom with colleagues and children, the study also examines whether two or more years of classroom teaching experience influences their knowledge of mathematical development.

The timing of this study is particularly significant given that many teacher education programs were derailed beginning in the spring of 2020, so measurement prior to COVID's interference in instruction is a valuable window on a time when fewer confounds (i.e., absence of in-person observa-

tion and practicum courses and the high frequency of online instruction in early childhood education teacher preparation programs) may have affected instruction. Consequently, teachers who completed some or all or their coursework and/or began their teaching careers during COVID may have experienced differences in preparation for teaching. The instrument utilized in this study is the Knowledge of Mathematical Development Survey (KMDS).

Knowledge of Mathematical Development Survey

In order to provide effective support for early mathematical development, research suggests teachers must develop a) comprehensive knowledge of mathematical content and concepts (Litowski et al., 2020; Ma, 1999); b) an awareness of young children's mathematical development, including developmental trajectories that build on past knowledge and build the foundation for future knowledge (Sarama & Clements, 2009; Turrou et al., 2021); and c) pedagogy that engages children and advances development through the use of meaningful representations and activities (Baroody et al., 2006; Clements et al., 2023; Seo & Ginsburg, 2004). It has also been argued that effective teaching of mathematics also requires respect for the mathematical thinking of the child (Ball, 1993; Ginsburg, 2016). This suggests that curriculum and development courses must include math-specific content and pedagogy, and that teacher educators must possess this knowledge themselves if they are to provide instruction in this domain.

"Effectively supporting early mathematical development in the preschool classroom also requires teachers to attend to children's interests and provide meaningful opportunities for their engagement."

Effectively supporting early mathematical development in the preschool classroom also requires teachers to attend to children's interests and provide meaningful opportunities for their engagement. Mathematical activities in these supportive classrooms are integrated, playful, useful, fun, and culturally inclusive (Stipek & Johnson, 2020). All in all, when combined with the need for deep understanding of mathematical concepts and appropriate pedagogy, this is a steep ask of early childhood teachers and their educators.

Considerable attention has been paid to the mathematics pedagogical and content knowledge of elementary school teachers. Hill and colleagues' (2008) seminal paper on pedagogical content knowledge has been cited over 3000 times. However, to date, the measurement of pre- and in-service preschool teachers' knowledge of mathematical development has been limited. Researchers have used interviews (McCray & Chen, 2012; Rosenfeld, 2012), achievement tests of mathematical pedagogical content knowledge (Dunekacke et al., 2015; Dunekacke et al., 2016), and questionnaires (Anders & Rossbach, 2015). This paper explores preand in-service early childhood teachers' knowledge of mathematical development in the year after Duncan and colleagues' (2007) publication and ten years later.

The KMDS was developed in 2007. The 20item survey includes questions on the verbal counting sequence, object counting, ordinal number words, addition and subtraction, division of sets (fair/equal sharing), and written number symbols and words. Each item requires the respondent to choose which activity typically comes first in development (e.g., Saying the counting words in order from 1-10 [i.e., "1, 2, 3, 4, 5, 6, 7, 8, 9, 10] or Saying the counting words in order from six to ten [i.e., "6, 7, 8, 9, 10]), or mark "Same" or "I don't know." Rationale for selecting the items reflected two assumptions: they should be (a) representative of empirical research on early mathematical development and (b) indicative of activities that can and do occur in preschool classrooms (Platas, 2014). The development of numeracy skills, including those described in the items on the KMDS, represent the most predictive early math skills on later academic achievement (Chu et al., 2015; Duncan et al., 2007; Duncan & Magnuson, 2011; Nguyen et al., 2016). The instrument validation and reliability were supported through two pilot studies (N = 20; N = 55) and a validation study with 346 pre- and in-service preschool teachers (citation omitted). The instrument has subsequently been used in several studies as described below.

Cox (2011) surveyed 207 teachers from 51 preschools examining dimensions of math anxiety and knowledge and beliefs about children's mathematical development and classroom mathematics curriculum. The range of preschool classroom experience was 1 to 30+ years, with 89% with 2 or more years. Cronbach's alpha for the KMDS for this sample was .88, with an average score of 11 correct out of 20. Participants' KMDS scores were positively correlated with their beliefs that support for math development is age-appropriate in preschool (r = .25, p \leq .001) and that math development is an important goal in the early years. There was no relation between the KMDS score and math anxiety by category (high positive affect, high negative affect, or mixed), although there was a trend with higher KMDS scores present in the high positive affect group.

Using the KMDS in a study that examined differences between 98 preservice and 77 in-service preschool teachers' knowledge of and beliefs about early mathematical development, Kim (2013) found significant differences between the KMDS scores (α = .81) of the two groups (M = 12.27 and 15.80, respectively; F(1,173) = 47.79, p < .001) resulting in a large effect size (η 2 = .22). Teachers in the in-service group all had either a bachelor's or master's degree, and 87% had participated in a professional development course on preschool mathematics in the previous three years.

Lange et al. (2022) utilized the KMDS to measure change in knowledge of mathematical development during 23 preschool teachers' engagement in a STEM professional learning model (control = 24). Results showed a statistically nonsignificant increase of .39 in scores from pretest to posttest. However, comparisons between posttest scores of the treatment group versus the control group resulted in a difference of 1.77 points, an effect size of d = .45.

The present study contributes to our knowledge base on teacher preparation programs and their ability to support current and future teachers'

understanding of early mathematical development. In particular, this study examines whether, in the wake of considerable research since 2007 on the importance of supporting early mathematical development in the early years, early childhood teacher education programs have improved instruction over the intervening ten years on this essential area of development. This study uses the KMDS to asks the following research questions:

- 1. Is there a difference between 2008 and 2017-2018 in knowledge of mathematical development as measured by the KMDS in the following groups:
- Beginning: Students at the beginning of a post-secondary early childhood education degree
- Seniors: Students at the end of a BA/BS in early childhood education with no math course
- Math course: Upper division and master's students at the end or at completion of a 3-unit semester early math development course
- 2. Does two or more years of experience in preschool classrooms make a difference in the KMDS score within each of the three groups in 2008 and 2017-2018?

Methodology

Data Collection

The study was approved by the author's university Institutional Review Board as well as a community college research review board. Instructors of courses were contacted via e-mail and provided with a description of the study and a request for permission for the author to recruit participants and survey students. In 2008, all instructors contacted granted access. In 2017-2018, two instructors stated insufficient time left in the semester to allow survey administration and two indicated that the majority of the students enrolled were taking the requested class for general education units. The remaining instructors granted access to their classrooms and students. Participants were given a \$10 gift card as an incentive and assured that instructors would not be notified of participation status. Gift cards were funded through a competitive university research mini-grant. Return rate for

completed surveys ranged from 75% to 100% per classroom with an average in 2008 of 97% and in 2017-2018 of 98%. Completion of the surveys took 10-25 minutes. The last page of the survey requested demographic information, including previous and current employment in the field of early childhood education, information on enrollment in a mathematical development course, year of birth, and ethnicity. Students were allowed to choose multiple ethnicities.

Participants

In 2008, 346 participants were recruited through a stratified purposeful sampling method in order to obtain participants with differing experience, education, and exposure to an early math development course. Participants included students from four community colleges in California (seven classrooms), three universities in California (six classrooms) and four MA/BA mathematical development courses in two states (western and eastern United States).

In 2017-2018, 338 participants were recruited through an identical sampling method as 2008. Participants included students from three community colleges in California (three classrooms), four universities in California (eight classrooms), and four MA/BA mathematical development courses in three states (western and eastern United States; three from the same states and systems as in 2008).

For the purposes of these study, students who had a complete score for the KMDS and were categorized as beginning (first- and second-year students enrolled in child development entry courses at community colleges and four-year universities), seniors (seniors with no math course), and math course (graduate master's and undergraduate upper division students who had completed a 3-semester unit math development course) were included in the analyses. To reduce ambiguity and confounds, students who indicated that they had at one time or were currently taking a math course (18 students in 2008 and 11 in 2017-2018) but were not enrolled in the math courses surveyed, were not included in the analysis. Cronbach's alpha for the KMDS for combined years 2008 and 2017-2018 was .76, indicating good reliability.

Demographics

The average student age in 2008 was 27.37 years (N = 252; SD = 9.502) and in 2017-2018 24.28 years (N= 268; SD = 6.704). This difference was significant t(518) = 4.254, p = <.001. Eleven students in 2017-2018 did not provide a birth year.

Students who identified as female in 2008 and 2017-2018 (N= 236 and 258, respectively) far exceeded the number who identified as male (N = 20 and 20, respectively). There was no significant change in gender identification from 2008 to 2017-2018 χ 2(1) = .786, p = .870, two sided.

Between 2008 and 2017-2018, the proportion of students across ethnicities changed significantly, specifically in the percentages of Latino, and White students $\chi 2(5) = 51.770$, p < .001. Figure one shows the distribution of students in each reported ethnicity for each of the two reported years. The black portion of each bar represents the percentage of students in the beginning group, the medium gray the number of students in the senior group, and

the light gray the number of students in the math course group.

Analyses by group showed that among seniors there was a change in the percentage of Asian and White students $\chi 2(5) = 21.111$, p < .001. In the math course groups there was a change in the percentage of Latino and White students $\chi 2(4) = 37.774$, p < .001. There was no change in ethnicity in the beginning group.

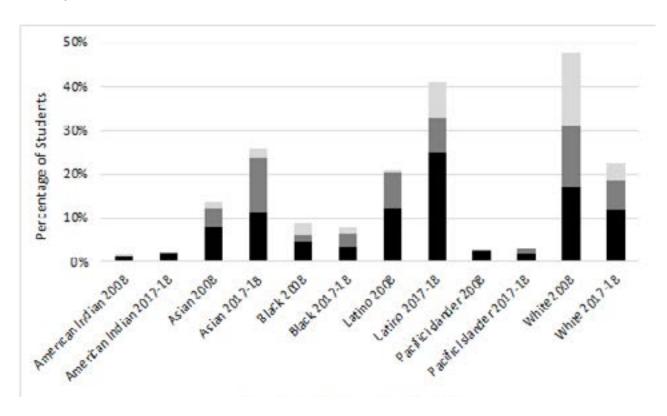
Results

Differences between groups' KMDS scores within years were examined through analysis of variance (ANOVA) with appropriate post hoc tests. Differences between (a) 2008 and 2017-2018 KMDS scores and (b) effects of two or more years of experience within groups in 2008 and 2017-2018 were measured by a univariate analysis.

Research Question #1:

Is there a difference in students' knowledge of math-





-ematical development as measured by the KMDS in 2008 or 2017-2018 between education groups?

Table 1 shows that there were statistically significant differences in 2008 between all groups in the mean KMDS scores, with scores increasing from beginning students to seniors to those who had taken a math course. In 2017-2018, mean KMDS scores increased in the same direction as 2008, with statistically significant differences between the mean scores of the beginning group and the math course group and between the seniors group and the math course group (see Table 1). However, the difference in mean KMDS scores between the beginning and seniors groups in 2017-2018 did not reach significance.

As noted in Table 2, there was no statistical difference between 2008 and 2017-2018 KMDS mean scores in beginning students; there were statistically significant differences between 2008 and 2017-2018 KMDS mean scores in seniors and math

course students.

The significant difference in KMDS scores between 2008 and 2017-2018 in the math course group warranted further analysis, in particular because of the differences in student levels (undergraduate versus graduate) within this group. The range of KMDS mean scores in 2008 in this group was 14.33 to 15.82. In 2017-2018, the range was 12.47 to 15.08. To further explore where these differences arose, ANOVA was used to examine the variance within each year among courses included in the math course group. These 3-unit courses included courses that served only master's students, only bachelor's students, and some that included both bachelor's and master's students. In separate analyses by year, there were no statistically significant differences in the mean KMDS scores between these courses.

TABLE 1 *KMDS Score Means in 2008 and 2017-2018 between groups*

Group	N	Mean (SD)	<i>p</i> -value			
•			Beginning	Seniors		
		2008^{1}				
Beginning	121	11.18 (3.89)				
Seniors	73	12.81 (2.70)	.002			
Math course	64	15.30 (2.27)	<.001	<.001		
		2017-20182				
Beginning	149	10.58 (3.56)				
Seniors	84	11.63 (3.42)	.070			
Math course	46	13.54 (2.68)	<.001	.007		

1Levene Statistic significant; Tamhane's T2 post hoc test used

TABLE 2Differences in KMDS Score Means between 2008 and 2017-2018

		N	Mear	<i>p</i> -value	
	2008	2017-2018	2008	2017-2018	
Beginning	121	149	11.18 (3.89)	10.58 (3.56)	n.s.
Math course	64	46	15:30 (2:27)	13:54 (2:68)	<.001

However, in a comparison of mean KMDS scores from all math courses in 2008 and 2017-2018 combined, there were significant differences between Course H and courses A, B and C. Note

that Table 3, with mean scores in descending order, shows that undergraduate versus graduate student status does not necessarily dictate the course KMDS score mean.

TABLE 3ANOVA by Math course by year and graduate status1

Course	BA/BS (B)/ Master's (M)	Year	N	Mean (SD)	A	В	С	D	E	F	G
A	M	2008	17	15.82			<i>p</i> -	value			
В	B/M	2008	16	(1.59) 15.50	n.s.						
C	M	2008	19	(1.97) 15.26	n.s.	n.s.					
D	В	2017-2018	12	(2.62) 15.08	n.s.	n.s.	n.s.				
E	B/M	2017-2018	9	(1.31) 14.33	n.s.	n.s.	n.s.	n.s.			
F	M	2008	12	(2.74) 14.33	n.s.	n.s.	n.s.	n.s.	n.s.		
G	В	2017-2018	5	(2.81) 12.67	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	
Н	В	2017-2018	18	(1.86) 12.47	.001	.008	.012	n.s.	n.s.	n.s.	n.s.
				(3.01)							

1Levene Statistic non-significant; Bonferroni post hoc test used

Research Question #2

Does two or more years of experience in classrooms make a difference in the KMDS mean scores within each of the three groups in 2008 and 2017-2018?

A combined univariate analysis of students in 2008 and 2017-2018 with and without two or more years of classroom experience for the three student groups showed a statistically significant difference only in those students in the math course group. On average, KMDS mean scores increased by 1.02, for those 2008 and 2017-2018 students who had taken a math course and had two or more years of classroom experience when compared to those who had less or no experience (t[40.06] = 2.09, p = .039, η 2 = 0.039). When separated into cohorts, neither 2008 nor 2017-2018 student groups with and without two or more years of classroom experience reached a statistically significant difference in KMDS mean scores.

Discussion

This study was initiated to examine whether there were differences in pre- and in-service teachers' knowledge of mathematical development in 2008 when compared to 2017-2018. Given the extensive research on the importance of supporting young children's mathematical development over the decades, and in particular since 2007, it could be expected that teacher education programs and teacher educators would have provided increased instruction and resources to their students around mathematical development and pedagogy.

The results showed that while the KMDS mean scores of students in each of the education groups (beginning versus seniors versus math course) statistically differed for each collection year, there was not a statistically significant difference between 2008 and 2017-2018 collection years for beginners. However, there was a statistically significant difference between 2008 and 2017-2018 collection years in average scores in the seniors and math course groups. Surprisingly, the senior and math course groups achieved a lower KMDS mean score in 2017-2018 than those in 2008.

Because of the differences between bachelor senior and masters students (i.e., admissions requirements and cost) included in the math course group,

further investigation into whether there was a difference in scores by undergraduate versus graduate programs versus mixed programs was warranted. In an analysis by year and graduate status, with one exception, there was no significant difference between the scores. The one exception was an undergraduate math course that resulted in the lowest of all of the KMDS averages, but was only statistically different from three of the highest performing courses. As indicated in Table 3, KMDS scores did not differ by graduate program status.

In an attempt to explain the differences between 2008 and 2017-2018 in the senior and math course groups, I refer back to Copeman Pettig and colleagues' 2018 study that found that teacher educators across eight states (including two of the states in this study) reported that they were teaching math content in courses beyond their comfort level. While we do not have a comparable study in 2008, it could be reasonable to expect that those who included math development in their curriculum and development courses and/or were teaching math development courses in 2008 were likely to be more comfortable as the pressure to include early math development courses in early childhood education programs had not yet begun in the field, better ensuring that those who taught those courses were more familiar with mathematical development. In support of this notion, of the four instructors teaching the surveyed math development courses in 2008, all had authored journal articles on early mathematical development. Despite a considerable search in 2008, it was very difficult to find early math development courses. Frequently courses were listed in college and university course catalogs but had not been taught for some time. By the time 2017-2018 rolled around, many more math courses were being taught, but only one of the instructors of the courses surveyed had authored journal articles on the topic (with one exception, the 2008 instructors were not teaching that year).

As to the findings that two or more years of preschool teaching increases knowledge of mathematical development only for those who had taken a math course, it appears that teaching alone does not provide sufficient support for teachers in gaining knowledge of early mathematical development. However, for students who had completed a math development course, it seems that their ability to

put that knowledge to practice serves to increase their understanding of that development even more.

Early childhood teachers want what is best for the children for whom they provide care and education. However, their efforts are stymied by a lack of engagement in early mathematical development and pedagogy during their teacher education paths. Early childhood teacher educators also want what is best for the teachers they prepare. However, they themselves frequently are likewise not well prepared to support or understand mathematical development (Ryan et al., 2014). Researchers examining ways to support children in their mathematical development throughout their education have suggestions. Aligning teacher requirements and preparation (and pay) between preschool and elementary school teachers could provide a path forward. This could engender shared expertise within teacher preparation programs, where faculty understanding of content knowledge, child development, and children's mathematical development converge (Stein & Coburn, 2023).

Efforts are underway in the United States to provide better support for the understanding of mathematical development in teacher education and training programs (Association of Mathematics Teacher Educators, 2017; McCormick et al., 2020). States are beginning to increase their teacher training standards in early mathematics, partly in response to more robust early math standards for young children (California Commission on Teacher Credentialing, 2023a; Math In Pre-kindergarten Through Twelfth Grade Act, 2023; Scherer et al., 2020). We have also learned that providing support for early academic skills like mathematics in preschool without a plan for sustaining that support in later years can result in a failure to support children as they build on these foundational skills (Bailey et al., 2020; Clements et al., 2013; Stipek et al., 2017). This coordination requires both teacher standards and academic standards to be aligned from preschool through high school. Unfortunately, longstanding traditions result in the housing of these policy systems separately (Whitaker et al., 2022). Programs like Head Start that engage in activities that support coordination between these programs and elementary schools have been shown to increase children's language and mathematics skills

(Cook & Coley, 2019). Although perhaps an optimistic perspective, there is growing recognition that acquiring the skills to support children's early mathematical development is an essential outcome of teacher education programs. Perhaps the next decade will make a difference.

"We have also learned that providing support for early academic skills like mathematics in preschool without a plan for sustaining that support in later years can result in a failure to support children as they build on these foundational skills"

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